# Energy-efficient scheduling

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Joint Lab on Petascale Computing

# Energy: a crucial issue

- Data centers
  - 330, 000, 000, 000 Watts hour in 2007: more than France
  - 533, 000, 000 tons of $CO_2$: in the top ten countries
- Exascale computers ($10^{18}$ floating operations per second)
  - Need effort for feasibility
  - 1% of power saved $\rightsquigarrow$ 1 million dollar per year
- Lambda user
  - 1 billion personal computers
  - 500, 000, 000, 000, 000 Watts hour per year
- $\rightsquigarrow$ crucial for both environmental and economical reasons

# Energy: a crucial issue

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# Power dissipation of a processor

- $P = P_{\text{leak}} + P_{\text{dyn}}$
  - $P_{\text{leak}}$: constant
  - $P_{\text{dyn}} = B \times V^2 \times f$

    constant ($B$), supply voltage ($V$), frequency ($f$)

- Standard approximation: $P = P_{\text{leak}} + f^{\alpha}$ $(2 \leq \alpha \leq 3)$
- Energy $E = P \times time$
- **D**ynamic **V**oltage and **F**requency **S**caling
  - Real life: discrete speeds
  - Continuous speeds can be emulated

# Outline

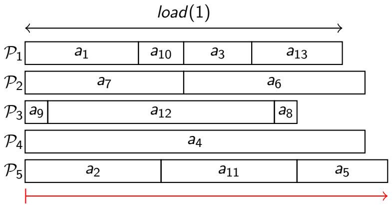
1. Revisiting the greedy algorithm for independent jobs
2. Reclaiming the slack of a schedule
3. Tri-criteria problem: execution time, reliability, energy
4. Checkpointing and energy consumption
5. Conclusion

# Framework

- Scheduling independent jobs
- GREEDY algorithm: assign next job to least-loaded processor
- Two variants:
  ONLINE-GREEDY: assign jobs on the fly
  OFFLINE-GREEDY: sort jobs before execution

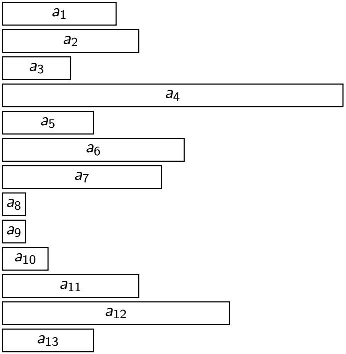
# Classical problem

- $n$ independent jobs $\{J_i\}_{1 \leq i \leq n}$, $a_i$ = size of $J_i$
- $p$ processors $\{\mathcal{P}_q\}_{1 \leq q \leq p}$
- allocation function $alloc : \{J_i\} \rightarrow \{\mathcal{P}_q\}$
- load of $\mathcal{P}_q$ = $load(q) = \sum_{\{i \mid alloc(J_i)=\mathcal{P}_q\}} a_i$




Execution time:

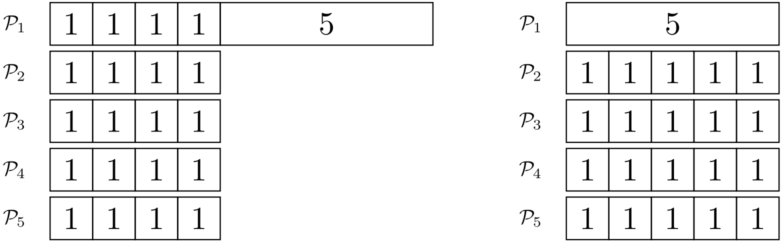
$\max_{1 \leq q \leq p} \quad load(q)$



# OnLine-Greedy

**Theorem**

OnLine-Greedy *is a* $2 - \frac{1}{p}$ *approximation (tight bound)*

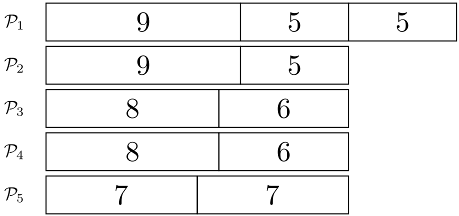



OnLine-Greedy Optimal solution

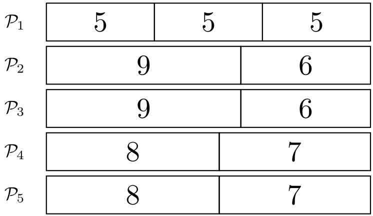
# OffLine-Greedy

**Theorem**

OffLine-Greedy *is a* $\frac{4}{3} - \frac{1}{3p}$ *approximation (tight bound)*




OffLine-Greedy




Optimal solution

# Bi-criteria problem

- Minimizing (dynamic) power consumption:
  $\Rightarrow$ use slowest possible speed $P_{dyn} = f^{\alpha} = f^3$

- Bi-criteria problem:
  Given bound $M = 1$ on execution time,
  minimize power consumption while meeting the bound

# Bi-criteria problem statement

- $n$ independent jobs $\{J_i\}_{1 \leq i \leq n}$, $a_i$ = size of $J_i$
- $p$ processors $\{\mathcal{P}_q\}_{1 \leq q \leq p}$
- allocation function $alloc : \{J_i\} \rightarrow \{\mathcal{P}_q\}$
- load of $\mathcal{P}_q = load(q) = \sum_{\{i \mid alloc(J_i)=\mathcal{P}_q\}} a_i$

$(load(q))^3$ power dissipated by $\mathcal{P}_q$

$\sum_{q=1}^{p} (load(q))^3$ **Power**

$\max_{1 \leq q \leq p} load(q)$ **Execution time**

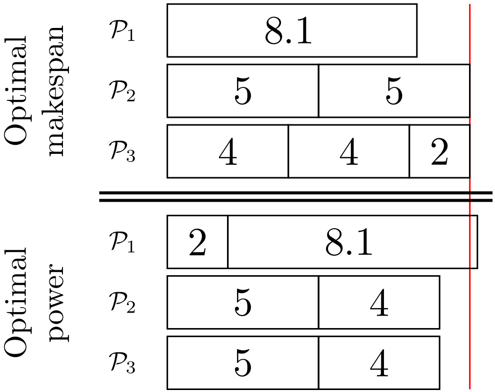
# Same GREEDY algorithm . . .

- Strategy: assign next job to least-loaded processor

- Natural for execution-time
  - smallest increment of maximum load
  - minimize objective value for currently processed jobs

- Natural for power too
  - smallest increment of total power (convexity)
  - minimize objective value for currently processed jobs

# ... but different optimal solution!




- Makespan 10, power 2531.441
- Makespan 10.1, power 2488.301

# GREEDY and $L_r$ norms

$$N_r = \left( \sum_{q=1}^{p} (load(q))^r \right)^{\frac{1}{r}}$$

- Execution time $N_\infty = \lim_{r \to \infty} N_r = \max_{1 \leq q \leq p} load(q)$
- Power $(N_3)^3$

# Known results

$N_2$, OffLine-Greedy

- Chandra and Wong 1975: upper and lower bounds
- Leung and Wei 1995: tight approximation factor

$N_3$, OffLine-Greedy

- Chandra and Wong 1975: upper and lower bounds

$N_r$

- Alon et al. 1997: PTAS for offline problem
- Avidor et al. 1998: upper bound $2 - \Theta(\frac{\ln r}{r})$ for OnLine-Greedy

# Contribution

$N_3$

- Tight approximation factor for ONLINE-GREEDY
- Tight approximation factor for OFFLINE-GREEDY

- Greedy for power fully solved!

# Approximation for OnLine-Greedy

$$\frac{P_{\text{online}}}{P_{\text{opt}}} \leq \underbrace{\frac{\frac{1}{p^3}\left((1+(p-1)\beta)^3+(p-1)(1-\beta)^3\right)}{\beta^3+\frac{(1-\beta)^3}{(p-1)^2}}}_{f_p^{(\text{on})}(\beta)}$$

## Theorem

- *$f_p^{(\text{on})}$ has a single maximum in $\beta_p^{(\text{on})} \in [\frac{1}{p}, 1]$*
- OnLine-Greedy *is a $f_p^{(\text{on})}(\beta_p^{(\text{on})})$ approximation*
- *This approximation factor is tight*

# Approximation for OffLine-Greedy

$$\frac{P_{\text{offline}}}{P_{\text{opt}}} \leq \underbrace{\frac{\frac{1}{p^3}\left(\left(1+\frac{(p-1)\beta}{3}\right)^3+(p-1)\left(1-\frac{\beta}{3}\right)^3\right)}{\beta^3+\frac{(1-\beta)^3}{(p-1)^2}}}_{f_p^{(\text{off})}(\beta)}$$

**Theorem**

- $f_p^{(\text{off})}$ *has a single maximum in* $\beta_p^{(\text{off})} \in [\frac{1}{p}, 1]$
- OffLine-Greedy *is a* $f_p^{(\text{off})}(\beta_p^{(\text{off})})$ *approximation*
- *This approximation factor is tight*

# Numerical values of approximation ratios

| $p$ | OnLine-Greedy | OffLine-Greedy |
|---|---|---|
| 2 | 1.866 | 1.086 |
| 3 | 2.008 | 1.081 |
| 4 | 2.021 | 1.070 |
| 5 | 2.001 | 1.061 |
| 6 | 1.973 | 1.054 |
| 7 | 1.943 | 1.048 |
| 8 | 1.915 | 1.043 |
| 64 | 1.461 | 1.006 |
| 512 | 1.217 | 1.00083 |
| 2048 | 1.104 | 1.00010 |
| $2^{24}$ | 1.006 | 1.000000025 |

# Large values of $p$

Asymptotic approximation factor

| | | |
|---|---|---|
| OnLine-Greedy | $\frac{4}{3}$ | 1 |
| OffLine-Greedy | 2 | 1 |

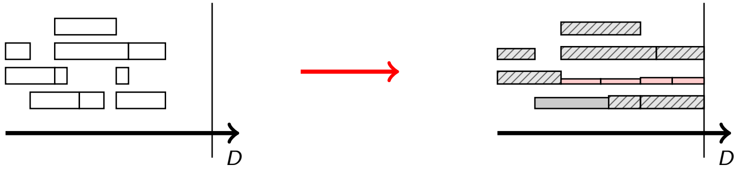
↑ *optimal*

# Outline

# Motivation

- Mapping of tasks is given (ordered list for each processor and dependencies between tasks)
- If deadline not tight, why not take our time?
- Slack: unused time slots

Goal: efficiently use speed scaling (DVFS)



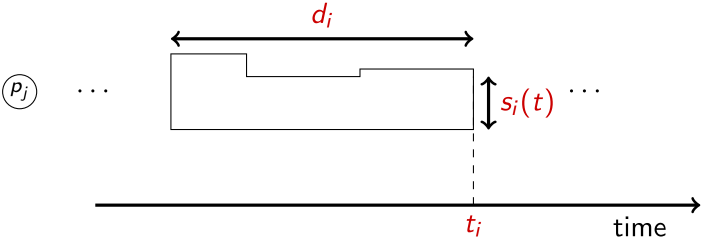
# Speed models

| | | Change speed | |
|---|---|---|---|
| | | **Anytime** | **Beginning of tasks** |
| Type of speeds | $[s_{\min}, s_{\max}]$ | CONTINUOUS | - |
| | $\{s_1, \ldots, s_m\}$ | VDD-HOPPING | DISCRETE, INCREMENTAL |

- CONTINUOUS: great for theory
- Other "discrete" models more realistic
- VDD-HOPPING simulates CONTINUOUS
- INCREMENTAL is a special case of DISCRETE with equally-spaced speeds: for all $1 \leq q < m$, $s_{q+1} - s_q = \delta$

# Tasks

- DAG: $\mathcal{G} = (V, E)$
- $n = |V|$ tasks $T_i$ of weight $w_i = \int_{t_i - d_i}^{t_i} s_i(t)dt$
- $d_i$: task duration; $t_i$: time of end of execution of $T_i$




Parameters for $T_i$ scheduled on processor $p_j$

# Makespan

*Assume $T_i$ is executed at constant speed $s_i$*

$$d_i = \mathcal{E}xe(w_i, s_i) = \frac{w_i}{s_i}$$

$$t_j + d_i \leq t_i \text{ for each } (T_j, T_i) \in E$$

Constraint on makespan:
$t_i \leq D$ for each $T_i \in V$

# Energy

Energy to execute task $T_i$ once at speed $s_i$:

$$E_i(s_i) = d_i s_i^3 = w_i s_i^2$$

$\rightarrow$ Dynamic part of classical energy models

**Bi-criteria problem**

- Constraint on deadline: $t_i \leq D$ for each $T_i \in V$
- Minimize energy consumption: $\sum_{i=1}^{n} w_i \times s_i^2$

# Complexity results

Minimizing energy with fixed mapping on $p$ processors:

- CONTINUOUS: Polynomial for some special graphs, geometric optimization in the general case
- DISCRETE: NP-complete (reduction from 2-partition); approximation algorithm
- INCREMENTAL: NP-complete (reduction from 2-partition); approximation algorithm
- VDD-HOPPING: Polynomial (linear programming)

# Summary

- Results for CONTINUOUS, but not very practical
- In real life, DISCRETE model (DVFS)
- VDD-HOPPING: good alternative, mixing two consecutive modes, smoothes out the discrete nature of modes
- INCREMENTAL: alternate (and simpler in practice) solution, with one unique speed during task execution; can be made arbitrarily efficient

# Outline

# Framework

- DAG: $\mathcal{G} = (V, E)$
- $n = |V|$ tasks $T_i$ of weight $w_i$

- $p$ identical processors fully connected
- DVFS: interval of available continuous speeds $[s_{\min}, s_{\max}]$
- One speed per task

- (I will not discuss results for the VDD-HOPPING model)

# Makespan

Execution time of $T_i$ at speed $s_i$:

$$d_i = \frac{w_i}{s_i}$$

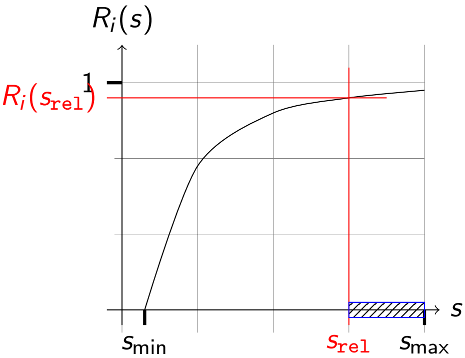
If $T_i$ is executed twice on the same processor at speeds $s_i$ and $s_i'$:

$$d_i = \frac{w_i}{s_i} + \frac{w_i}{s_i'}$$

Constraint on makespan:
end of execution before deadline $D$

# Reliability

- *Transient fault*: local, no impact on the rest of the system
- Reliability $R_i$ of task $T_i$ as a function of speed $s$
- Threshold reliability (and hence speed $s_{\mathtt{rel}}$)



Re-execution: a task is re-executed *on the same processor, just after its first execution*

With two executions, reliability $R_i$ of task $T_i$ is:

$$R_i = 1 - (1 - R_i(s_i))(1 - R_i(s_i'))$$

Constraint on reliability:

RELIABILITY: $R_i \geq R_i(s_{\mathtt{rel}})$, and at most one re-execution

# Energy

- Energy to execute task $T_i$ once at speed $s_i$:

$$E_i(s_i) = w_i s_i^2$$

$\rightarrow$ Dynamic part of classical energy models

- With re-executions, it is natural to take the worst-case scenario:

$$\textsc{Energy} : E_i = w_i \left(s_i^2 + s_i'^2\right)$$

# Tri-Crit-Cont

Given $\mathcal{G} = (V, E)$

Find

- A schedule of the tasks
- A set of tasks $I = \{i \mid T_i \text{ is executed twice}\}$
- Execution speed $s_i$ for each task $T_i$
- Re-execution speed $s'_i$ for each task in $I$

such that

$$\sum_{i \in I} w_i (s_i^2 + s_i'^2) + \sum_{i \notin I} w_i s_i^2$$

is minimized, while meeting reliability and deadline constraints

# Complexity results

- One speed per task
- Re-execution at same speed as first execution, i.e., $s_i = s'_i$

- TRI-CRIT-CONT is NP-hard even for a linear chain, but not known to be in NP (because of CONTINUOUS model)
- Polynomial-time solution for a fork

# Energy-reducing heuristics

Two steps:

- Mapping (NP-hard) $\rightarrow$ List scheduling
- Speed scaling + re-execution (NP-hard) $\rightarrow$ Energy reducing

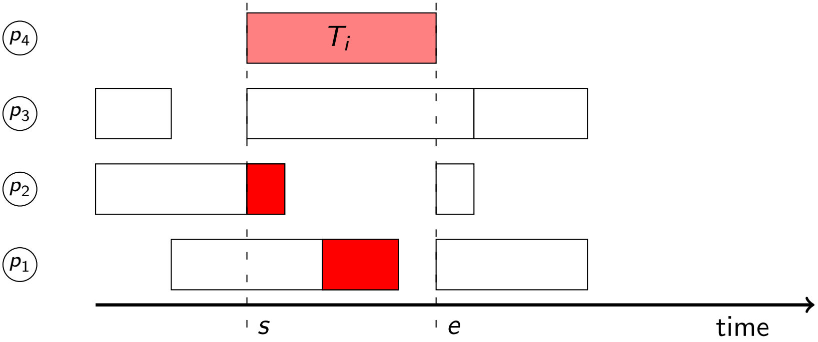
- The list-scheduling heuristic maps tasks onto processors at speed $s_{\max}$, and we keep this mapping in step two
- Step two = slack reclamation! Use of deceleration and re-execution

# Deceleration and re-execution

- Deceleration: select a set of tasks that we execute at speed $\max(s_{\texttt{rel}}, s_{\max}\frac{\max_{i=1..n} t_i}{D})$: slowest possible speed meeting both reliability and deadline constraints
- Re-execution: greedily select tasks for re-execution

# Super-weight (SW) of a task

- SW: sum of the weights of the tasks (including $T_i$) whose execution interval is included into $T_i$'s execution interval
- SW of task slowed down = estimation of the total amount of work that can be slowed down together with that task



# Selected heuristics

- **A.SUS-Crit**: efficient on DAGs with low degree of parallelism
  - Set the speed of every task to $\max(s_{\mathtt{rel}}, s_{\max} \frac{\max_{i=1..n} t_i}{D})$
  - Sort the tasks of every *critical path* according to their **SW** and try to re-execute them
  - Sort all the tasks according to their **weight** and try to re-execute them

- **B.SUS-Crit-Slow**: good for highly parallel tasks: re-execute, then decelerate
  - Sort the tasks of every *critical path* according to their **SW** and try to re-execute them. If not possible, then try to slow them down
  - Sort all tasks according to their **weight** and try to re-execute them. If not possible, then try to slow them down

# Results

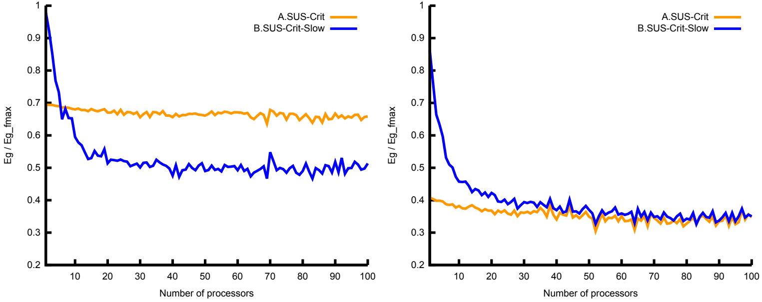
We compare the impact of:

- the number of processors $p$
- the ratio $D$ of the deadline over the minimum deadline $D_{min}$ (given by the list-scheduling heuristic at speed $s_{max}$)

on the output of each heuristic

Results normalized by heuristic running each task at speed $s_{max}$; the lower the better

# Results




With increasing $p$, $D = 1.2$ (left), $D = 2.4$ (right)

- A better when number of processors is small
- B better when number of processors is large
- Superiority of B for tight deadlines: decelerates critical tasks that cannot be re-executed

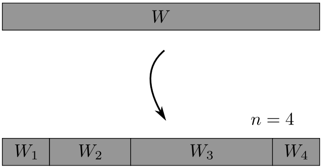
# Summary

- Tri-criteria energy/makespan/reliability optimization problem
- Various theoretical results
- Two-step approach for polynomial-time heuristics:
  - List-scheduling heuristic
  - Energy-reducing heuristics
- Two complementary energy-reducing heuristics for TRI-CRIT-CONT

# Outline

# Framework

- Execution of a divisible task ($W$ operations)
- Failures may occur
  - Transient faults
  - Resilience through checkpointing
- Objective: minimize expected energy given a deadline bound
- Decisions before execution:
  - Chunks: how many ($n$)? which sizes ($W_i$ for chunk $i$)?
  - Speeds of each chunk: first run ($s_i$)? re-execution ($\sigma_i$)?



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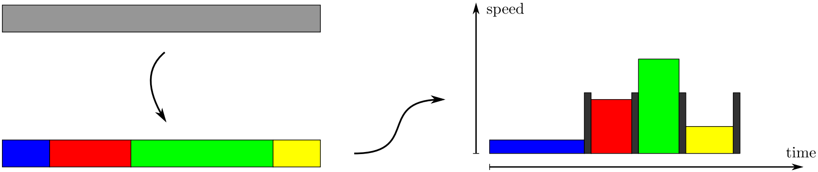

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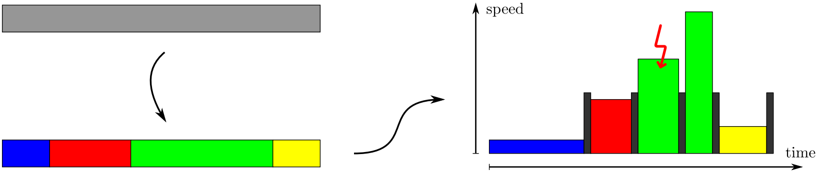


# Models

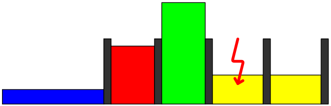
- Chunks

Single chunk

VS

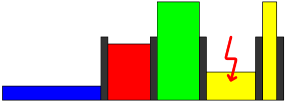
Multiple chunks

- Speed per chunk



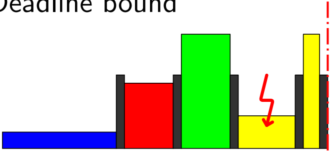
Single speed

VS



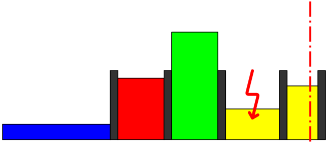
Multiple speeds

- Deadline bound



Hard (∼ Worst-case)

VS



Soft (Expected)

# Summary of results: single chunk

- Single speed
  - $s \mapsto \mathbb{E}(E)$ convex (expected energy consumption)
  - $s \mapsto \mathbb{E}(T)$ (expected execution time) and $s \mapsto T_{wc}$ (worst-case execution time) decreasing

$\rightarrow$ Expression of $s$ and $\mathbb{E}(E)$ (function of $\lambda, W, s, E_c, T_c$)

- Multiple speeds
  - Energy minimized when deadline tight
  - $\leadsto \sigma$ expressed as a function of $s$

$\rightarrow$ Minimization of single-variable function

# Summary of results: multiple chunks

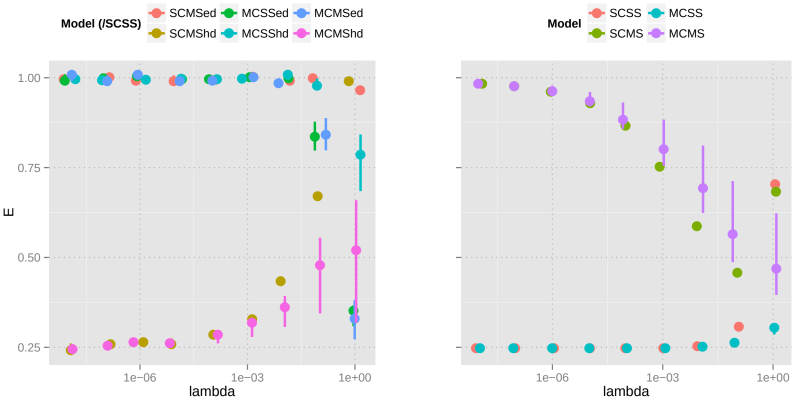
- Single speed
  - Equal-sized chunks, executed at same speed
  - Bound on $s$ given $n$

$\rightarrow$ Minimization of double-variable function

- Multiple speeds
  - Conjecture: equal-sized chunks, same first-execution / re-execution speeds
  - $\sigma$ as a function of $s$, bound on $s$ given $n$

$\rightarrow$ Minimization of double-variable function

# Comparison




Conclusions for realistic values of $\lambda$:

- Expected deadline constraint $\rightarrow$ use 1 chunk, 1 speed
- Hard deadline constraint $\rightarrow$ use multiple speeds anytime, multiple chunks if frequent failures

# Outline

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2. Reclaiming the slack of a schedule
3. Tri-criteria problem: execution time, reliability, energy
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# Conclusion

- OnLine-Greedy and OffLine-Greedy for power: tight approximation factor for any $p$, extends long series of papers and completely solves $N_3$ minimization problem ☺
- Different energy models, from continuous to discrete (through VDD-hopping and incremental)
- Tri-criteria heuristics with replication to deal with reliability
- Checkpointing techniques for reliability while minimizing energy consumption

What we had: 

What we aim at: 

Energy-efficient scheduling
+
frequency scaling