Multi-users, multi-organizations, multi-objectives : a single approach

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Collection of results of 3 papers with:
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Krzysztof Rzadca (Polish-Japanese computing school, Warsaw)
Fanny Pascual (LIP6, Paris)
Erik Saule (Grenoble university)

Aussois, may 19, 2008

Goal

The evolution of high-performance execution platforms leads to physical or logical distributed entities (organizations) which have their own « local » rules, each organization is composed of multiple users that compete for the resources, and they aim to optimize their own objectives…

Construct a framework for studying such problems.

Work partially supported by the Coregrid Network of Excellence of the EC.

content

Computational model

A set of users have some (parallel) applications to execute on a (parallel) machine.

The « machine » belongs or not to multiple organizations.

The objectives of the users are not always the same.

Multi-users optimization

Let us start by a simple case : several users compete for resources belonging to the same organization.

System centered problems (Cmax, load-balancing)
Users centered (minsum, maxstretch, flowtime)

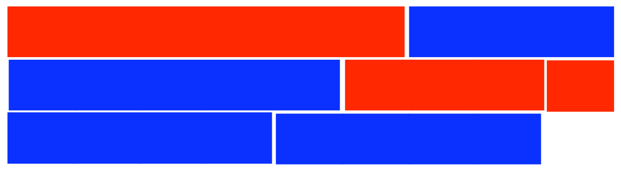
Motivation:
Take the diversity of users' wishes/needs into account

A simple example

Blue (4 tasks duration 3,4,4 and 5) has a program to compile (Cmax)

Red (3 tasks duration 1,3 and 6) is running experiments (ΣCi)

m=3 (machines)



Global LPT schedule

Cmax = 9

ΣCi = 6+8+9 = 23

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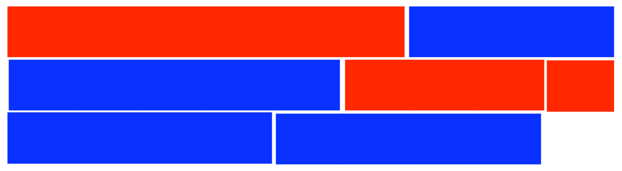

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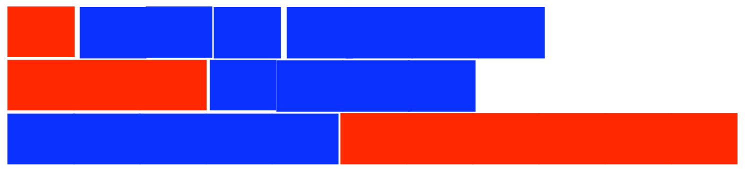


SPT schedule for red

Cmax = 8

ΣCi = 1+3+11 = 15

Description of the problem

Instance: k users, user u submit n(u) tasks,
processing time of task i belonging to u: $p_i(u)$
Completion time: $C_i(u)$
Each user can choose his-her objective among:
$C_{max}(u) = \max(C_i(u))$ or $\Sigma C_i(u)$ weighted or not

Multi-user scheduling problem:
$MUSP(k':\Sigma C_i;k'':C_{max})$ where $k'+k''=k$

Complexity

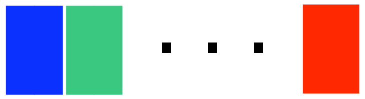
[Agnetis et al. 2004], case m=1
MUSP(2:ΣC_i) is NP-hard in the ordinary sense
MUSP(2:C_{max}) and MUSP(1:ΣC_i;1:C_{max}) are polynomial

Thus, on m machines, all variants of this problem are NP-hard
We are looking for approximation (multi-objective)

MUSP(k:C_{max})

Inapproximability:
no algorithm better than (1,2,…,k)

Proof: consider the instance where each user has one unit task ($p_i(u)=1$) on one machine ($m=1$).
$C_{max}^*(u) = 1$ and there is no other choice than:



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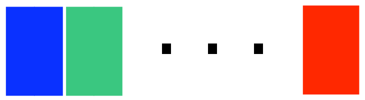
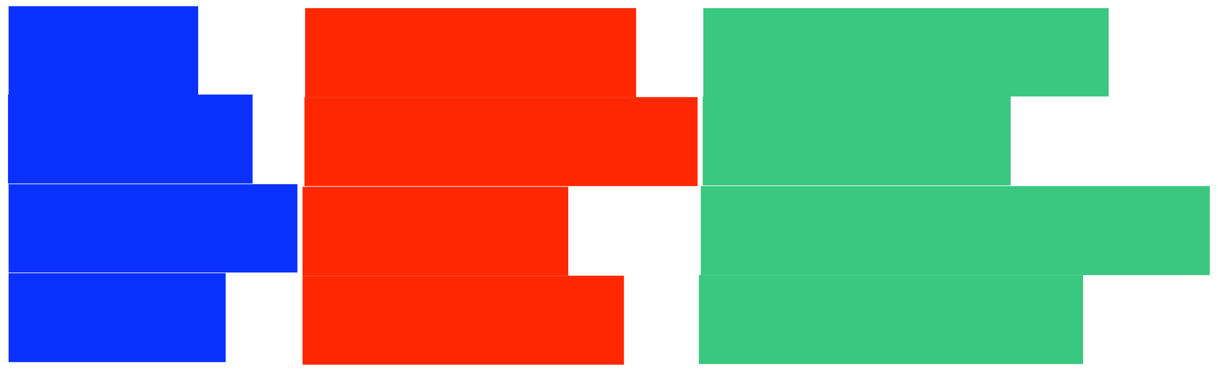


Thus, there exists a user u whose $C_{max}(u) = k$

MUSP(k:C_{max})

Algorithm (multiCmax):
Given a ρ-approximation schedule σ for each user
$C_{max}(u) \leq \rho C_{max}{}^{*}(u)$
Sort the users by increasing values of $C_{max}(u)$



Analysis:
multiCmax is a $(\rho, 2\rho, \ldots, k\rho)$-approximation.

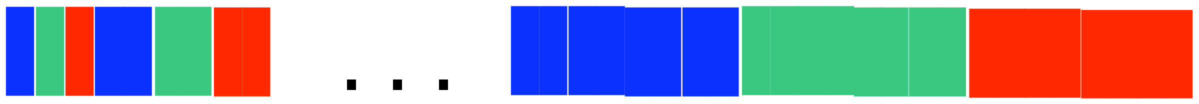
MUSP(k:ΣC_i)

Inapproximability: no algorithm better than $((k+1)/2,(k+2)/2,\dots,k)$

Proof: consider the instance where each user has x Tasks $p_i(u) = 2^{i-1}$.

Optimal schedule: $\Sigma C_i^* = 2^{x+1} - (x+2)$

SPT is Pareto Optimal (3 users blue, green and red):

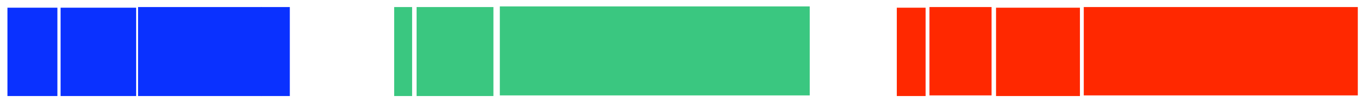


For all u, $\Sigma C_i^{SPT}(u) = k(2^x - (x+1)) + (2^x - 1)\, u$

Ratio to the optimal = $(k+u)/2$ for large x

MUSP(k:ΣC_i)

Algorithm (single machine) Aggreg:
Let $\sigma(u)$ be the schedule for user u.
Construct a schedule by increasing order of $\Sigma C_i(\sigma(u))$
(global SPT)



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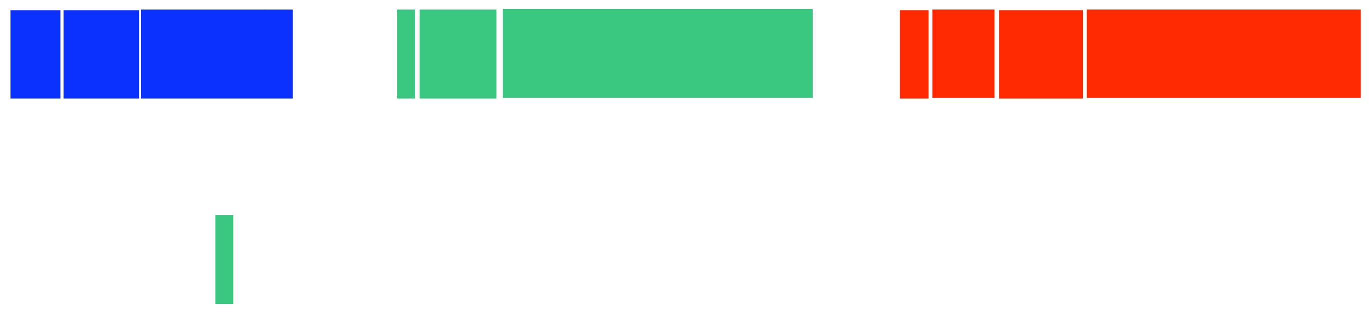

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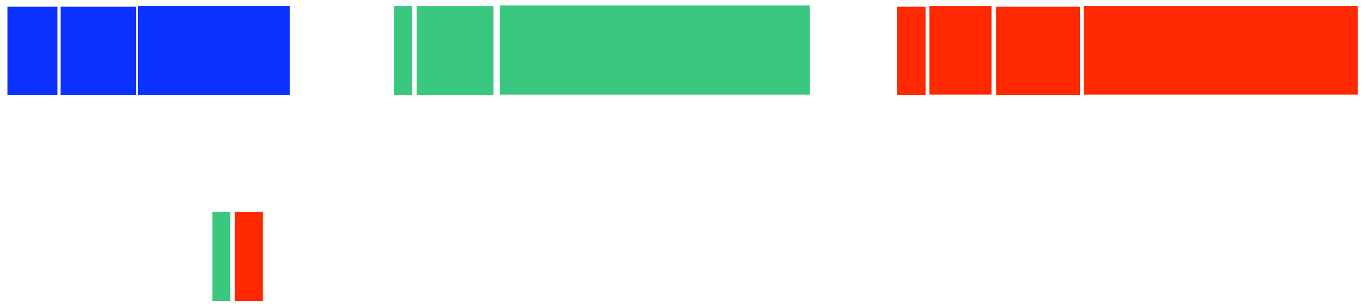

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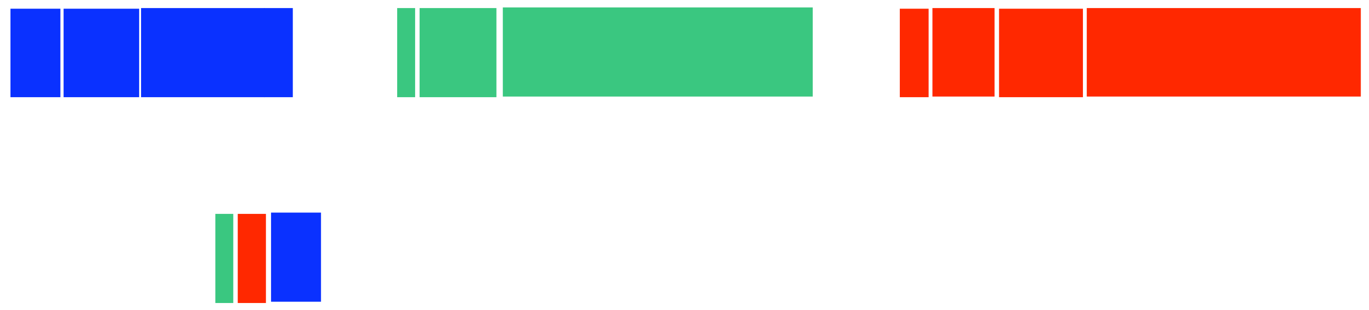

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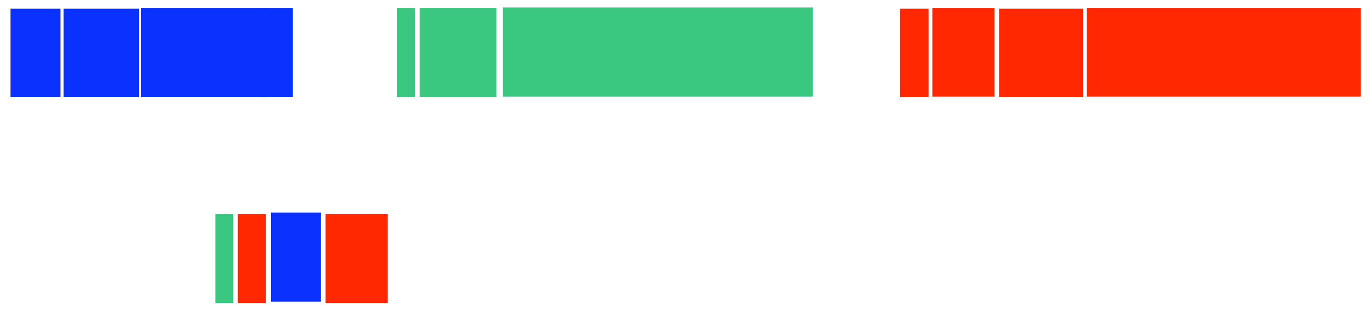

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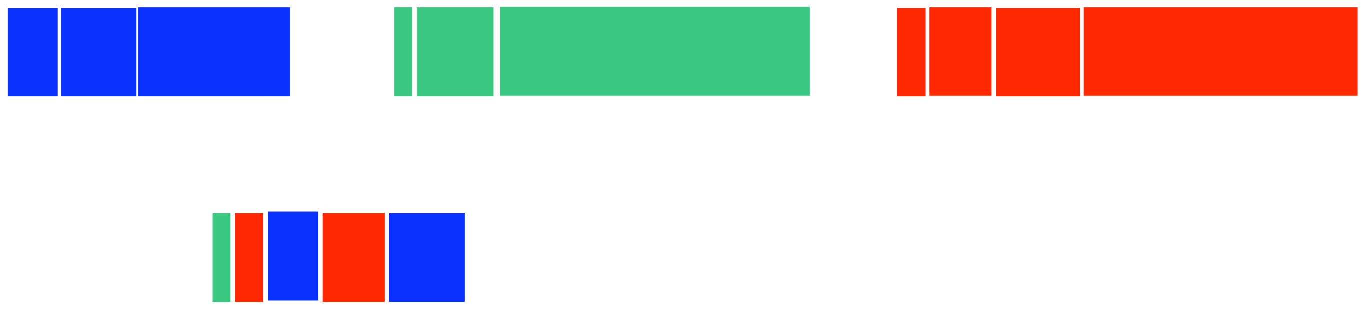

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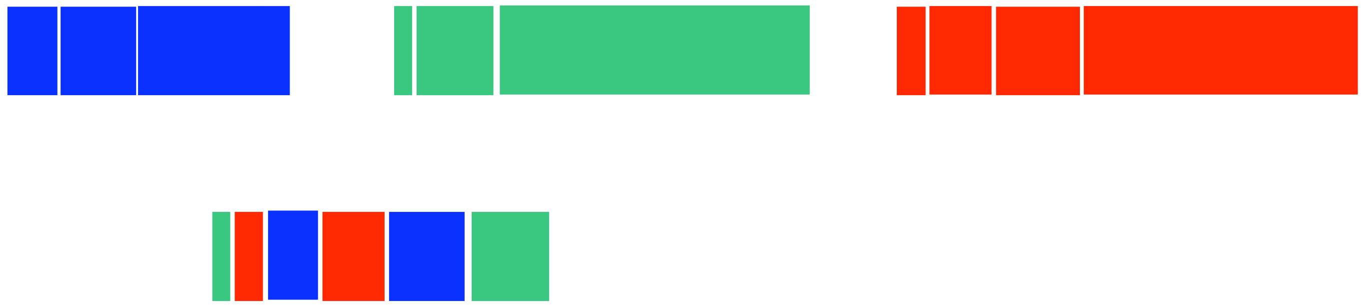

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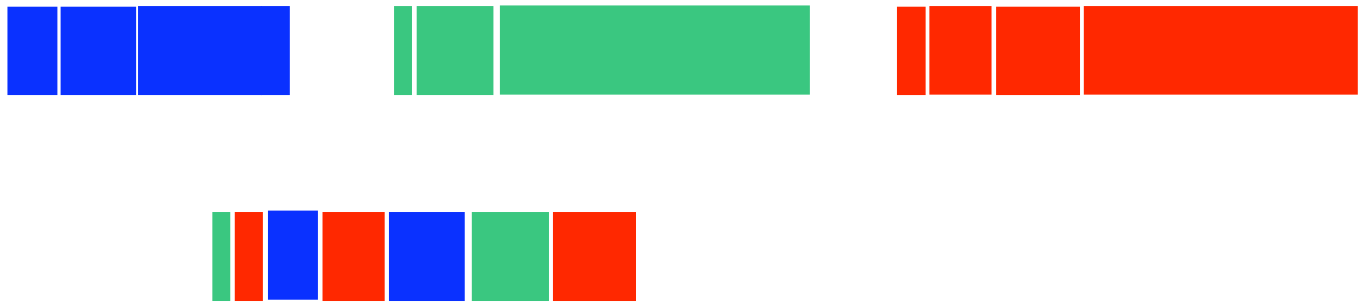

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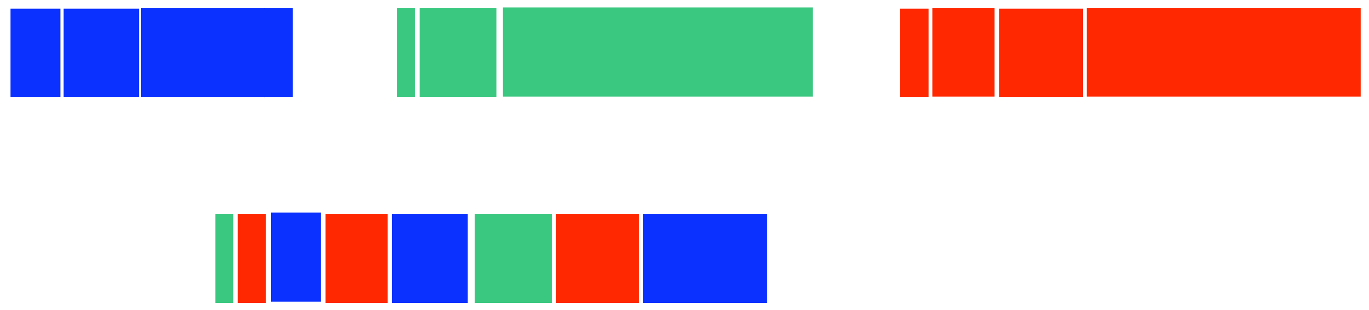

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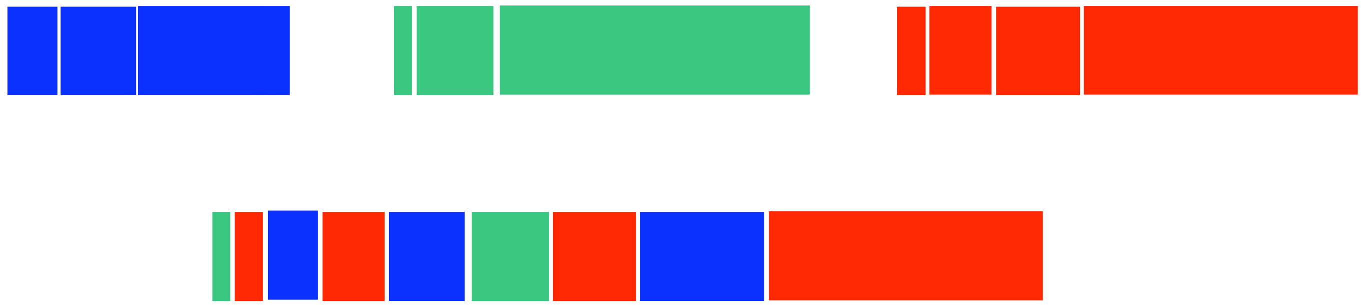

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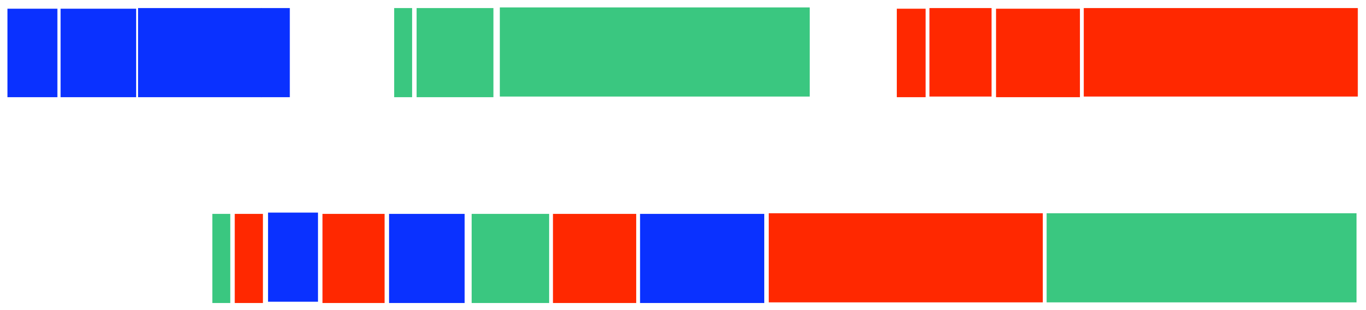
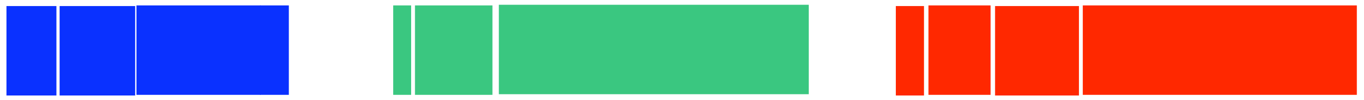


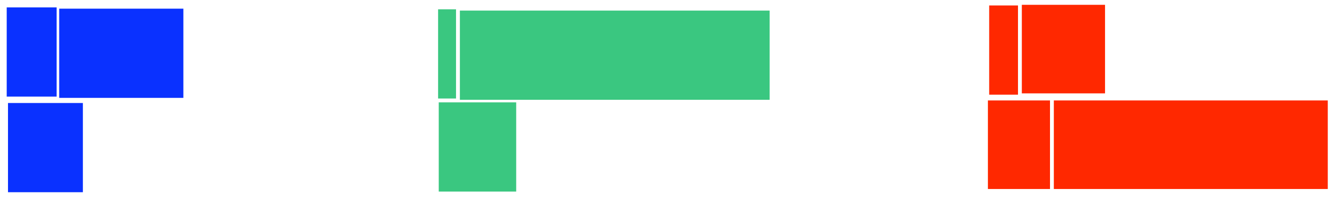
Analysis: Aggreg is (k,k,…,k)-approximation

MUSP(k:ΣC_i)

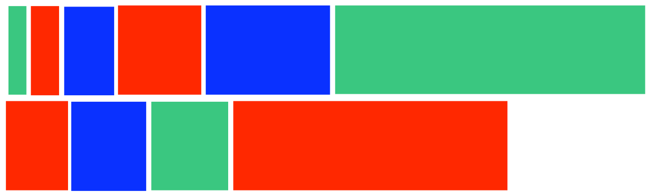
Algorithm (extension to m machines):
The previous property still holds on each machine
(using SPT individually on each machine)



Local SPT



Merge on each machine



MUSP(k:ΣC_i)

Algorithm (extension to m machines):
The previous property still holds on each machine (using SPT individually on each machine)

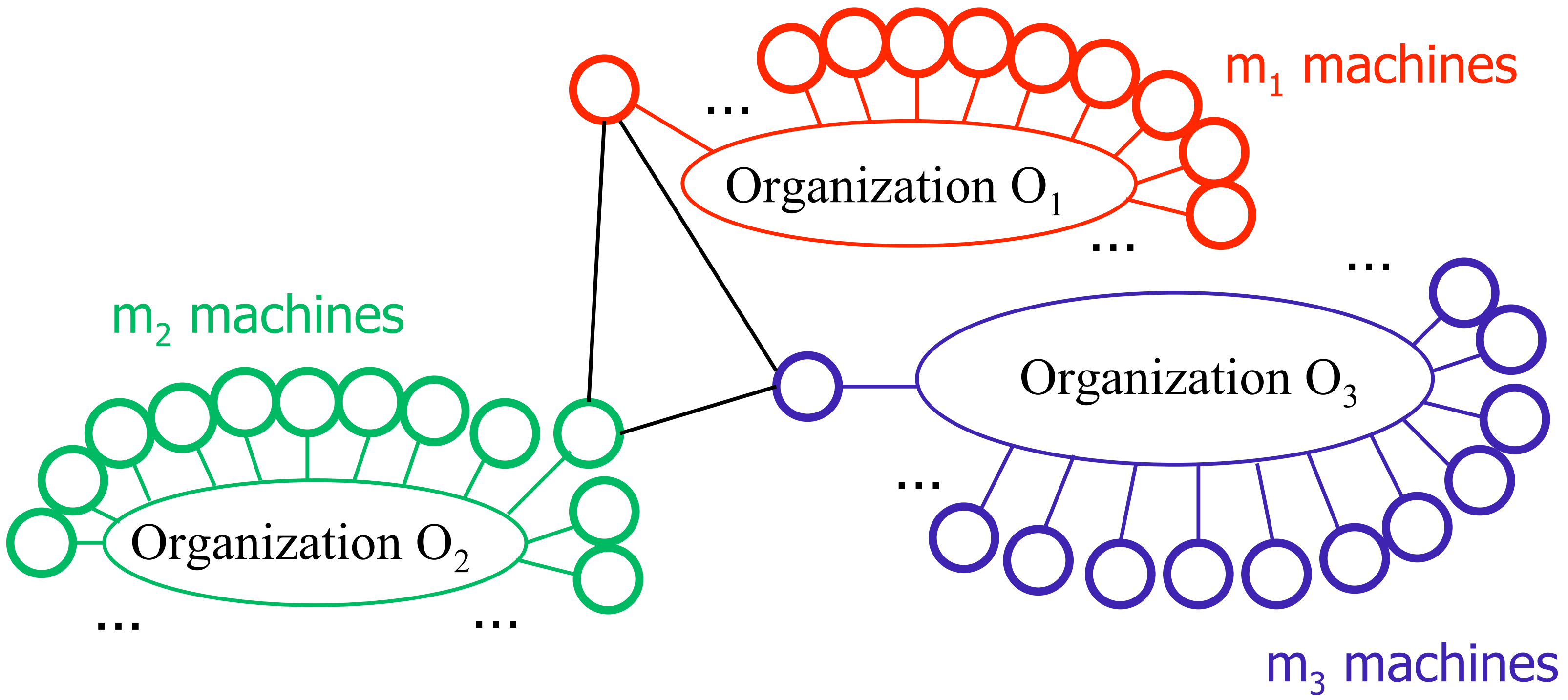
Analysis:
we obtain the same bound as before.

Mixed case MUSP(k':ΣC_i;$(k-k')$:C_{max})

A similar analysis can be done, see the paper with Erik Saule for more details

Complicating the model: Multi-organizations

Context: computational grids



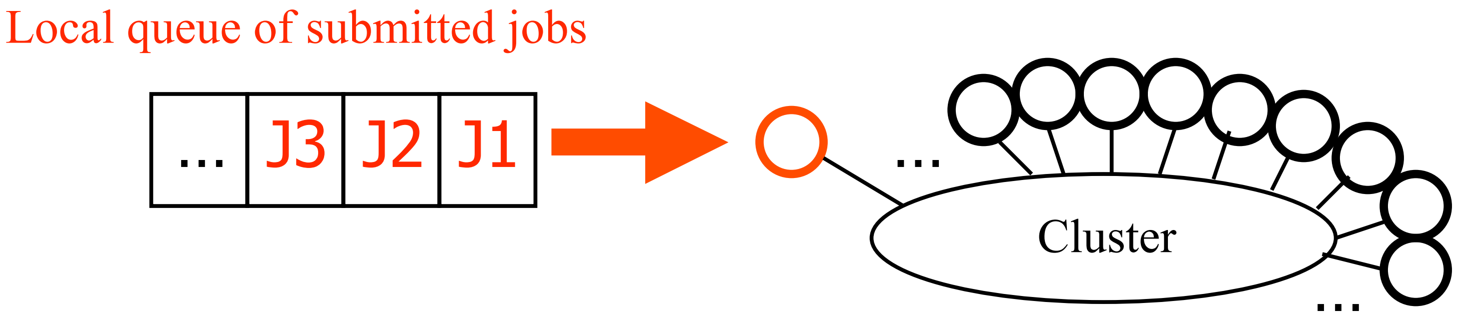
Collection of independent clusters managed locally by an « organization ».

Preliminary:

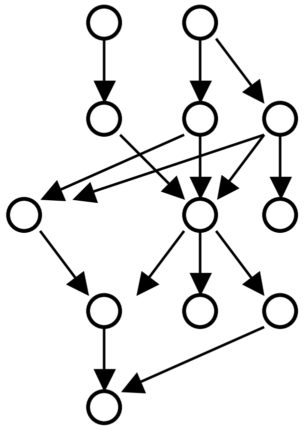
Single resource: cluster

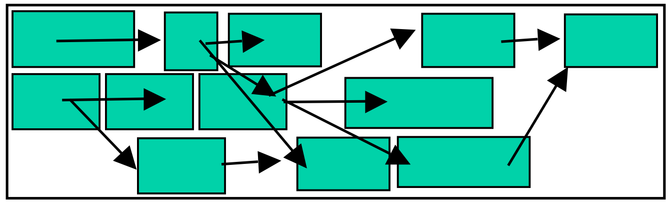
Independent applications are submitted locally on a cluster. The are represented by a precedence task graph.

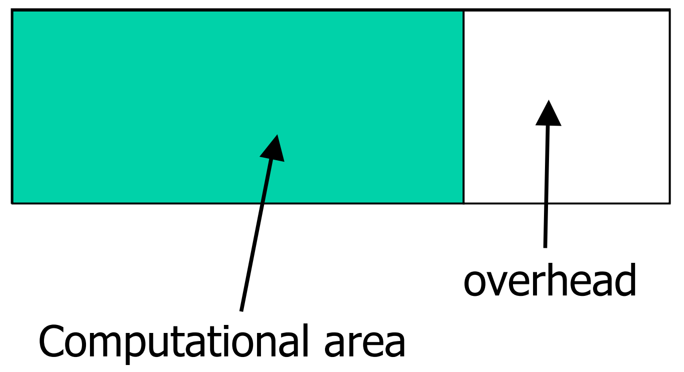
An application is viewed as a usual sequential task or as a parallel rigid job (see [Feitelson and Rudolph] for more details and classification).


Local queue of submitted jobs
...
J3
J2
J1
...
Cluster
...

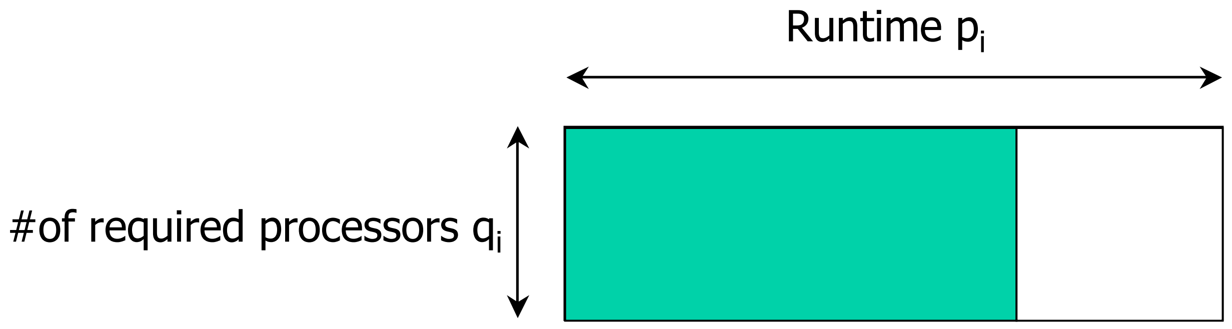
Job

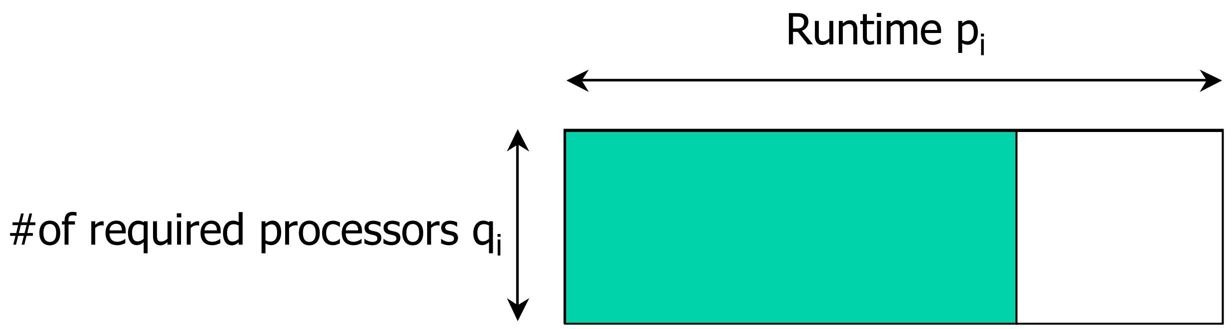






Rigid jobs: the number of processors is fixed.

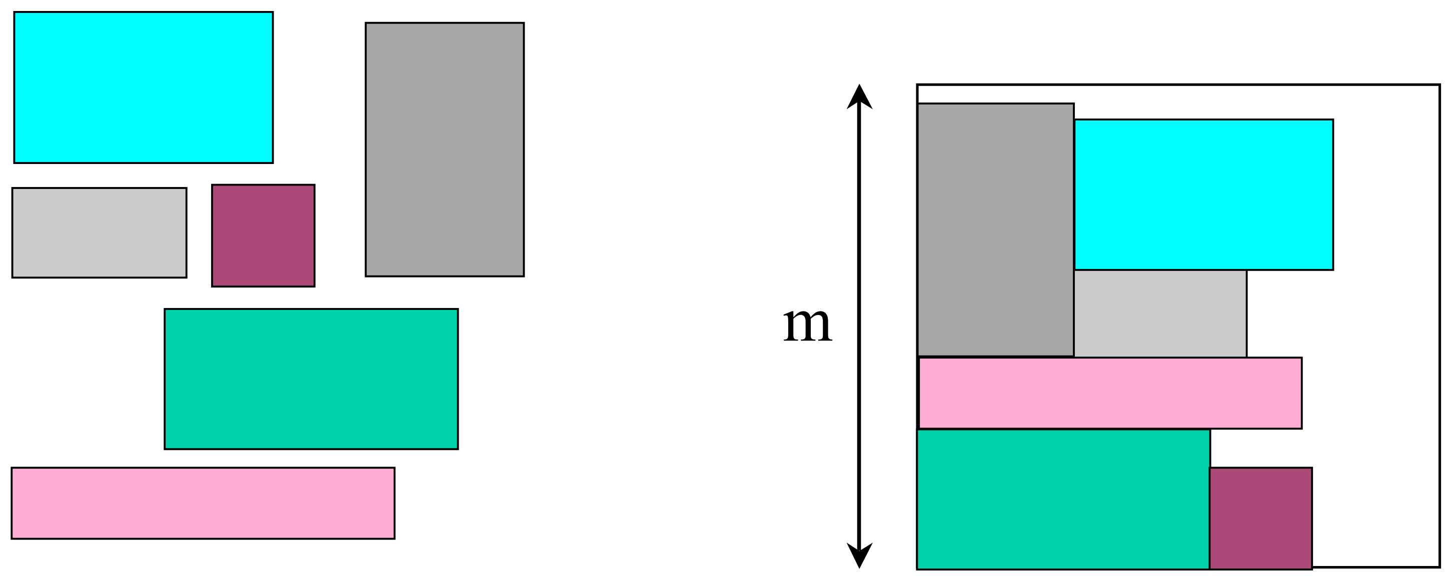

Runtime p_i
#of required processors q_i



Useful definitions:
high jobs (those which require more than m/2 processors)
low jobs (the others).

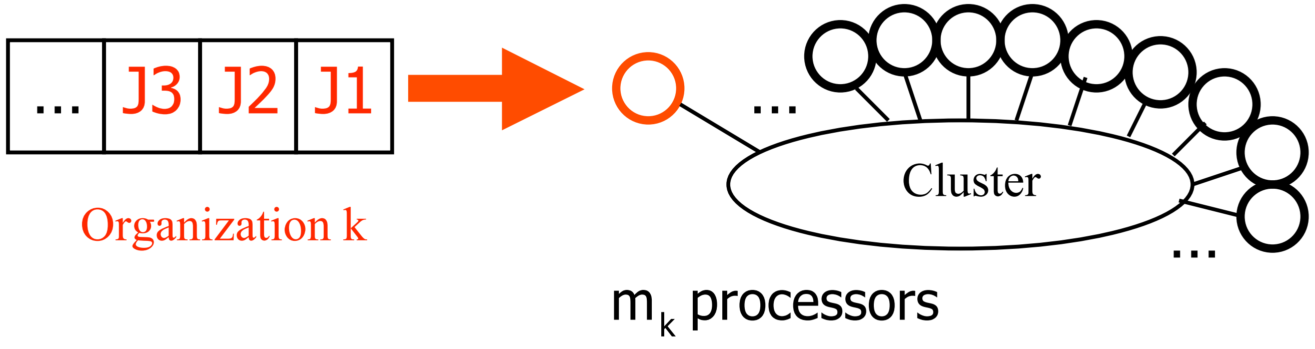
Scheduling rigid jobs: Packing algorithms (batch)

Scheduling independent rigid jobs may be solved as a 2D packing Problem (strip packing).

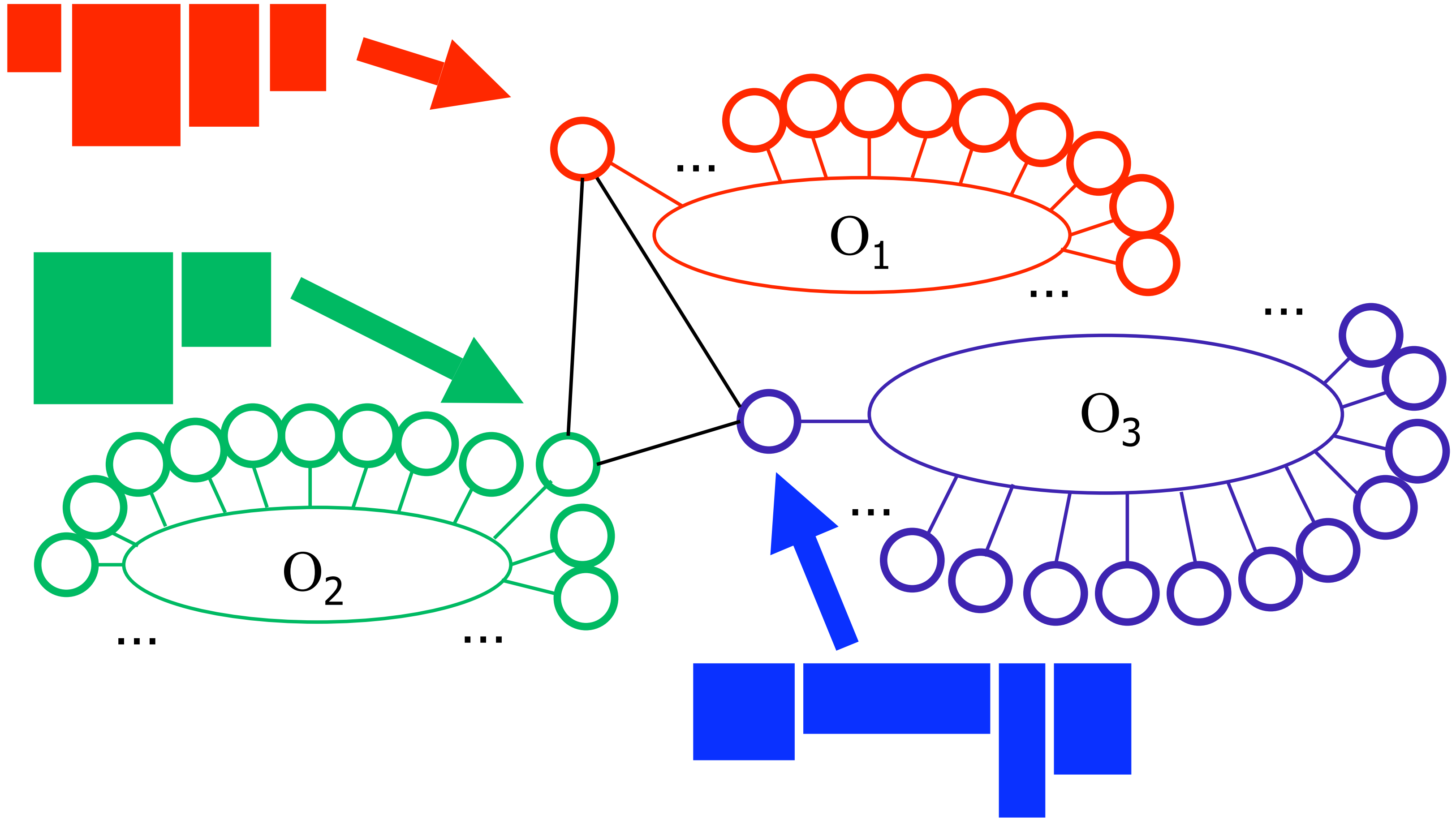


Multi-organizations

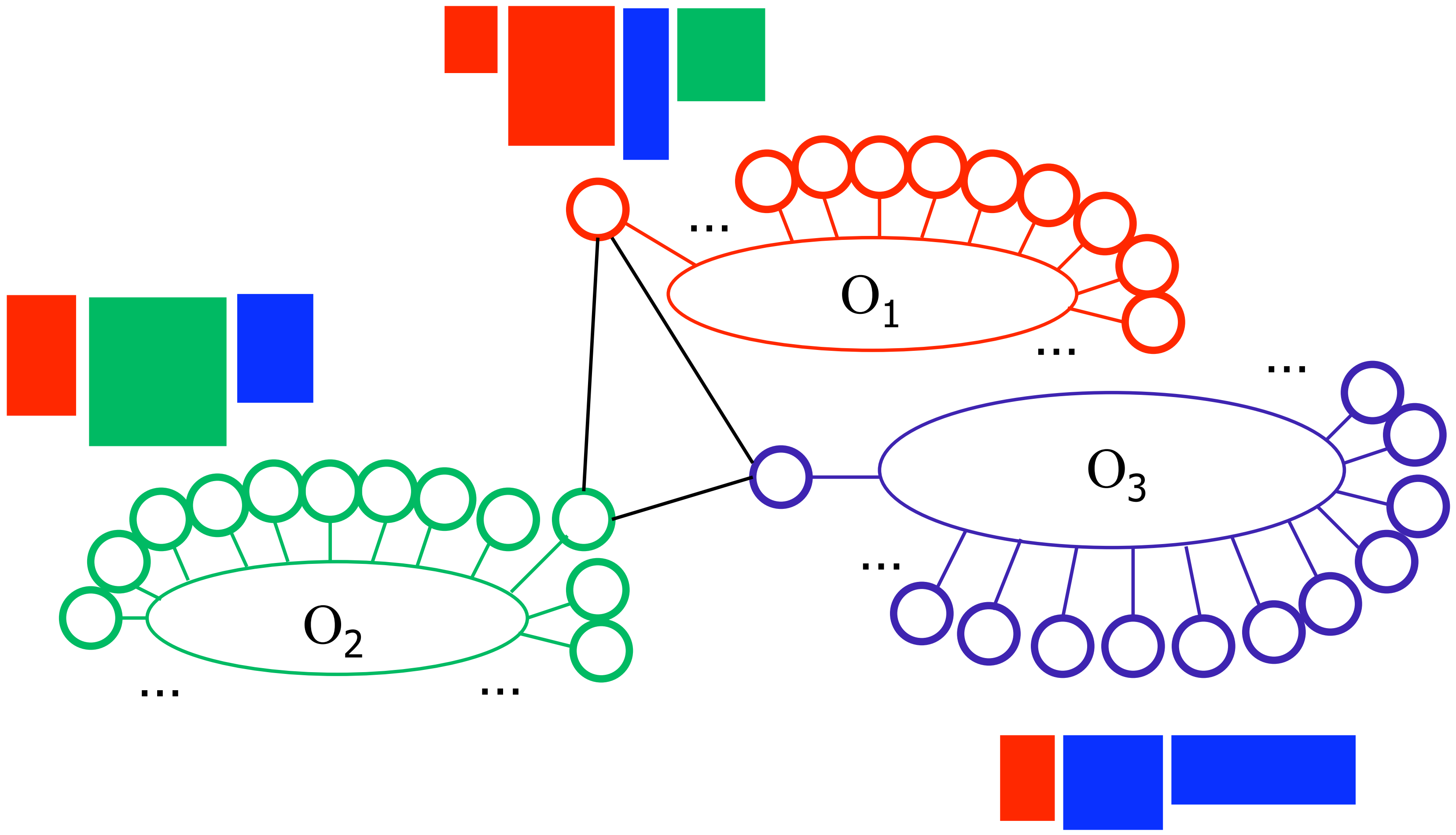
n organizations.



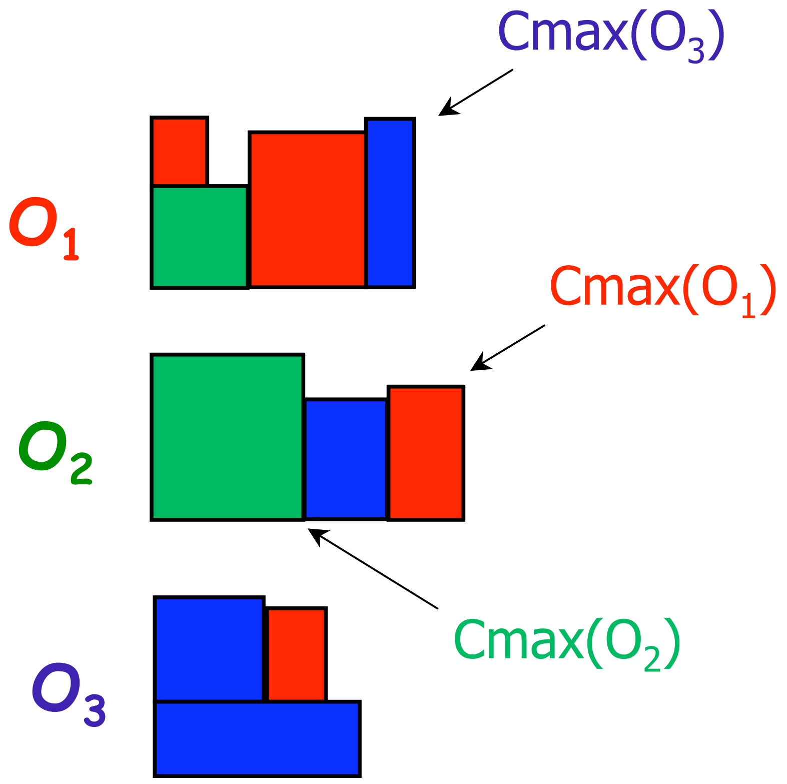
users submit their jobs locally



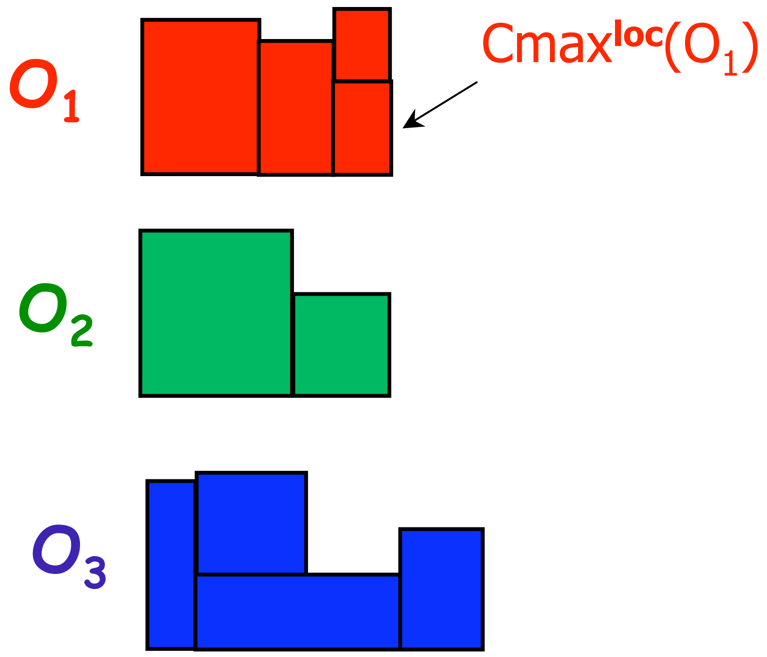
The organizations can cooperate



Constraints



Local schedules:



$C_{max}(O_k)$: maximum finishing time of jobs belonging to O_k.
Each organization aims at minimizing its own makespan.

Problem statement

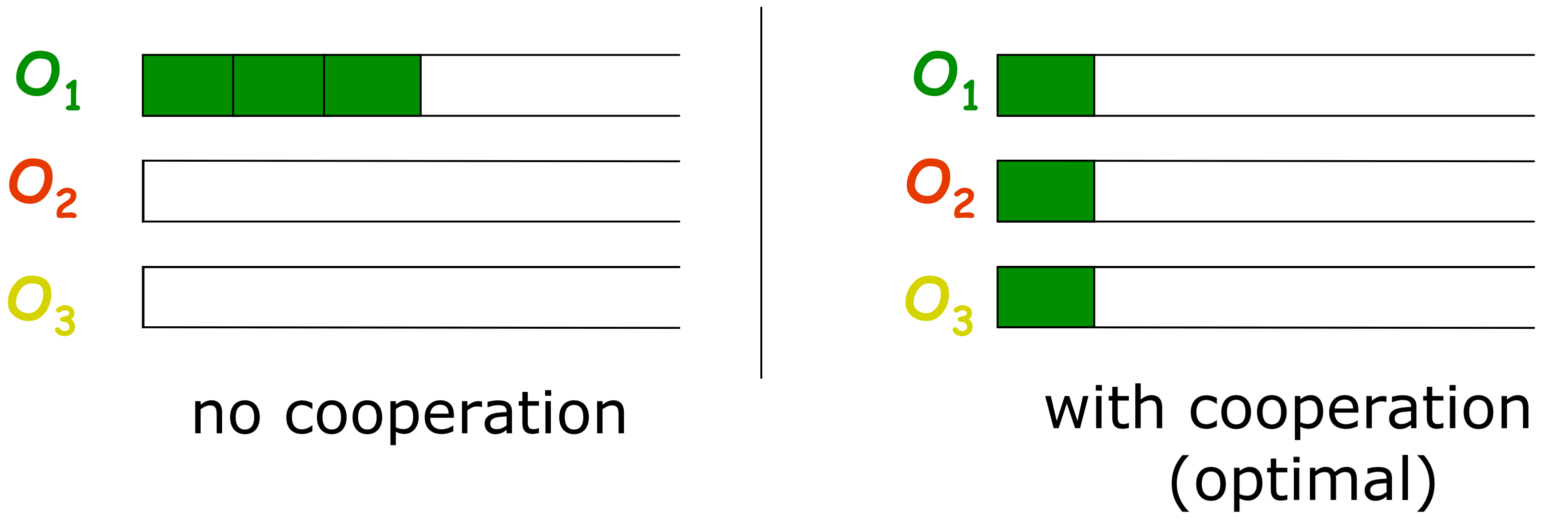
MOSP: minimization of the « global » makespan under the constraint that no local schedule is increased.

Consequence: taking the restricted instance n=1 (one organization) and m=2 with sequential jobs, the problem is the classical 2 machines problem which is NP-hard. Thus, MOSP is NP-hard.

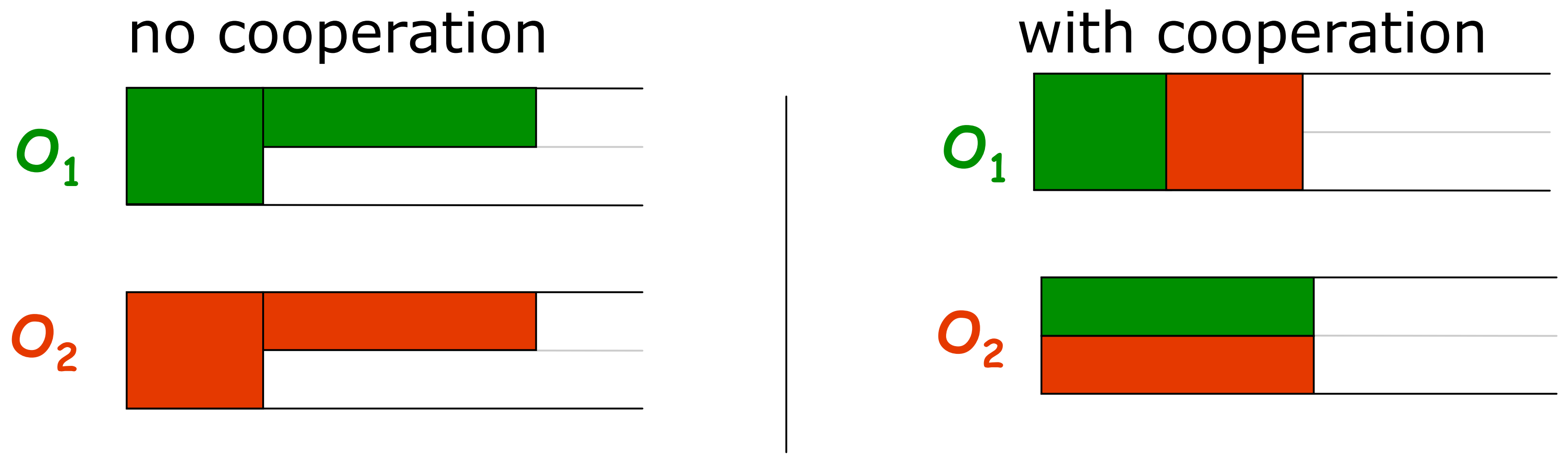
Multi-organizations

Motivation:

A non-cooperative solution is that all the organizations compute their local jobs (« my job first » policy).
However, such a solution is arbitrarly far from the global optimal (it grows to infinity with the number of organizations n).
See next example with n=3 for jobs of unit length.



More sophisticated algorithms than the simple load balancing are possible: matching certain types of jobs may lead to bilaterally profitable solutions.
However, it is a hard combitanorial problem



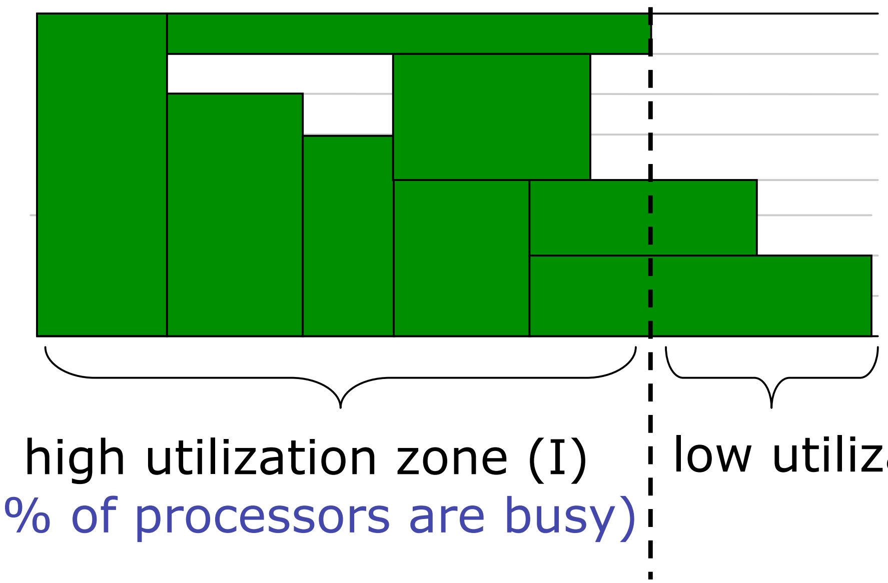
Preliminary results

- List-scheduling: (2-1/m) approximation ratio for the variant with resource constraint [Garey-Graham 1975].
- HF: Highest First schedules (sort the jobs by decreasing number of required processors). Same theoretical guaranty but perform better from the practical point of view.

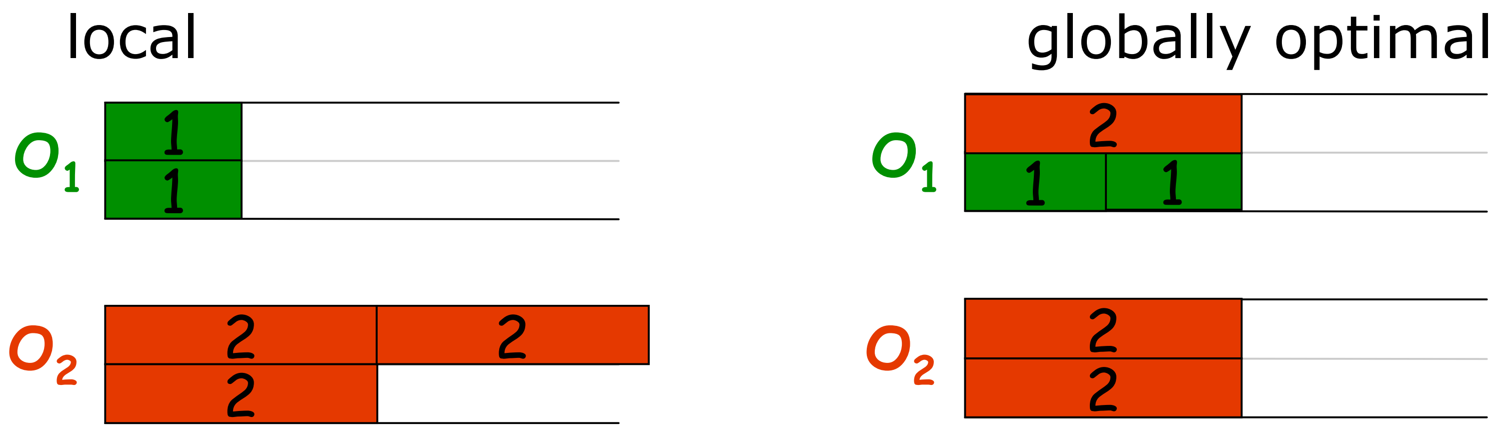
Analysis of HF (single cluster)

Proposition. All HF schedules have the same structure which consists in two consecutive zones of high (I) and low (II) utilization.

Proof. (2 steps)
By contracdiction, no high job appears after zone (II) starts


high utilization zone (I)
(more than 50% of processors are busy)
low utilization zone (II)

If we can not worsen any local makespan, the global optimum can not be reached.



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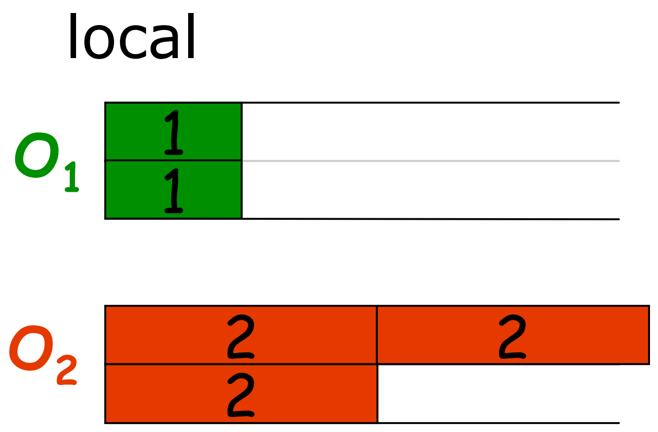
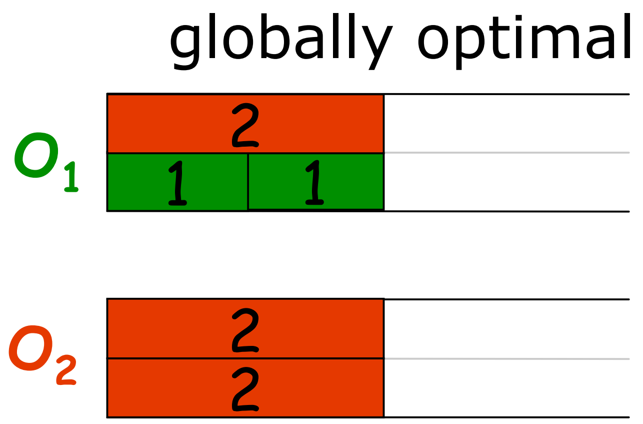
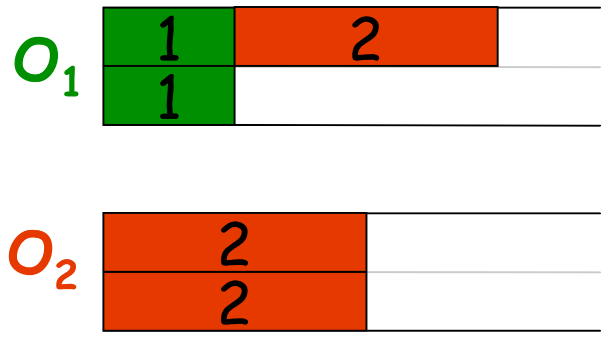


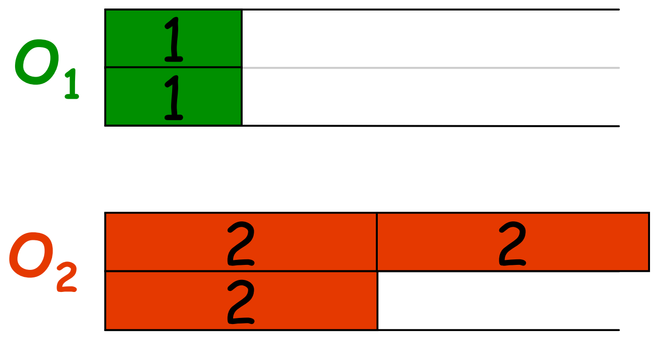


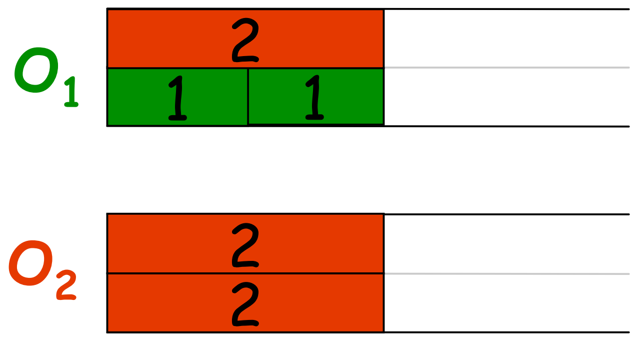
best solution that does not increase Cmax(O_1)



If we can not worsen any local makespan, the global optimum can not be reached.

Lower bound on approximation ratio greater than 3/2.





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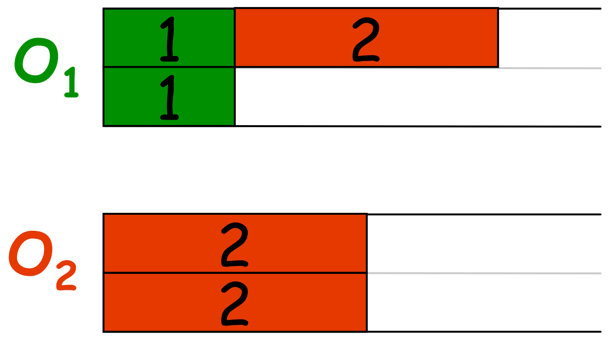


Using Game Theory?

We propose here a standard approach using Combinatorial Optimization.

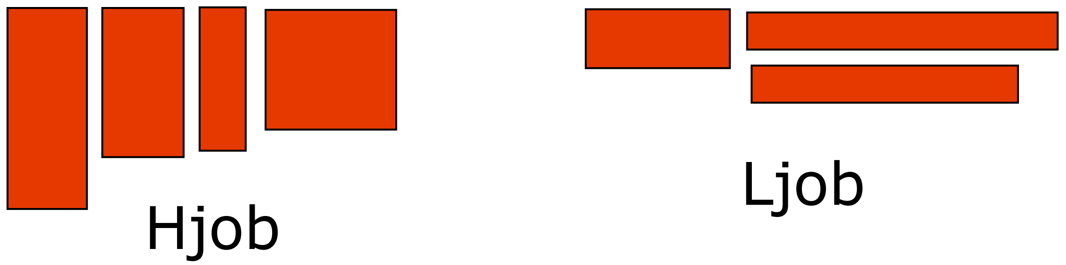
Cooperative Game Theory may also be usefull, but it assumes that players (organizations) can communicate and form coalitions. The members of the coalitions split the sum of their playoff after the end of the game.
We assume here a centralized mechanism and no communication between organizations.

Multi-Organization Load-Balancing

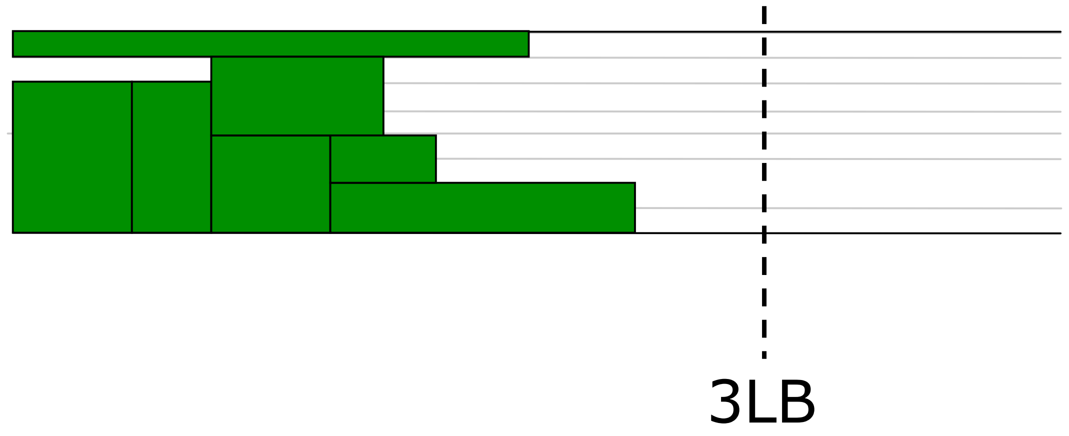
1 Each cluster is running local jobs with Highest First

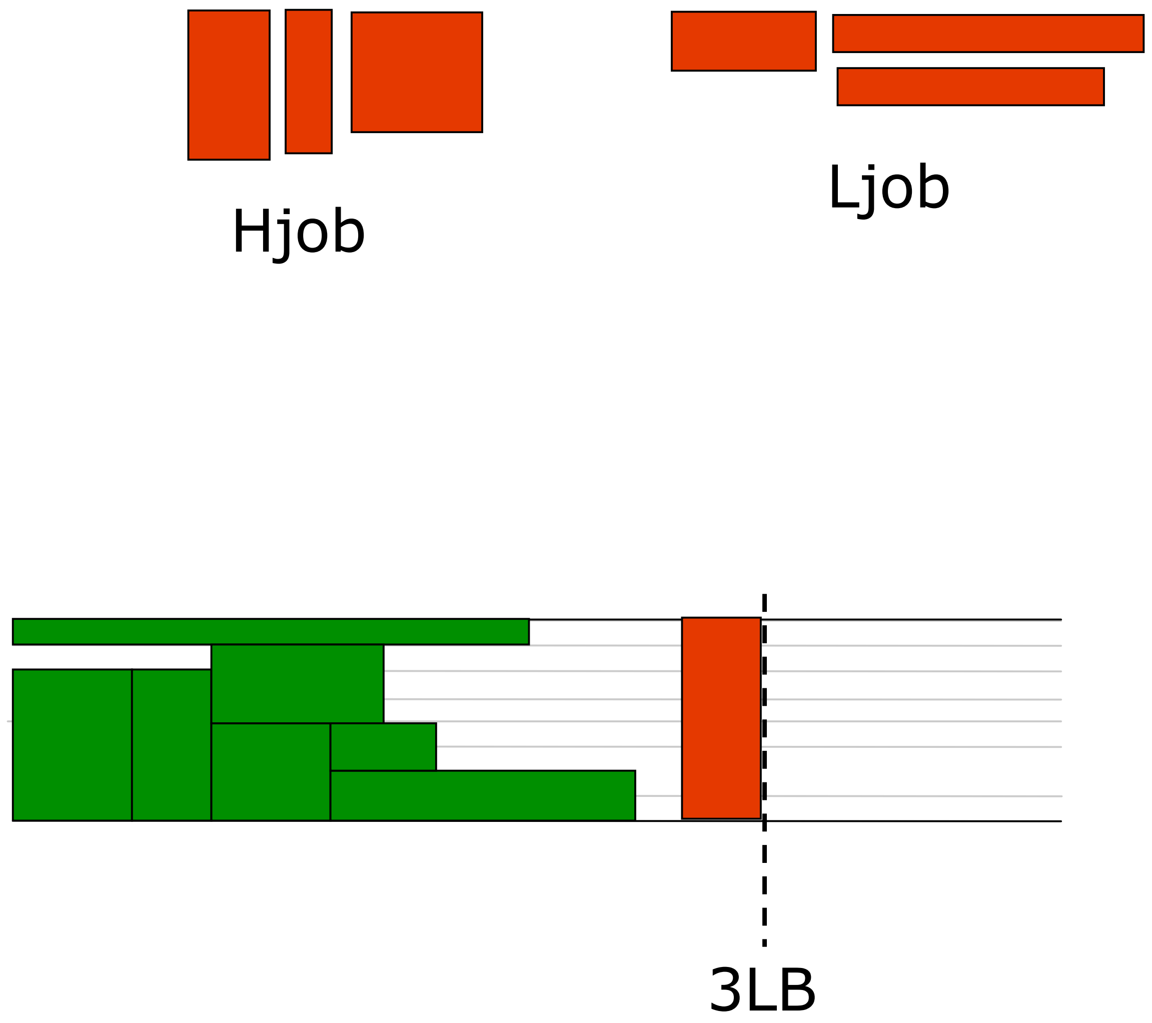
$$LB = \max(p_{max}, W/nm)$$

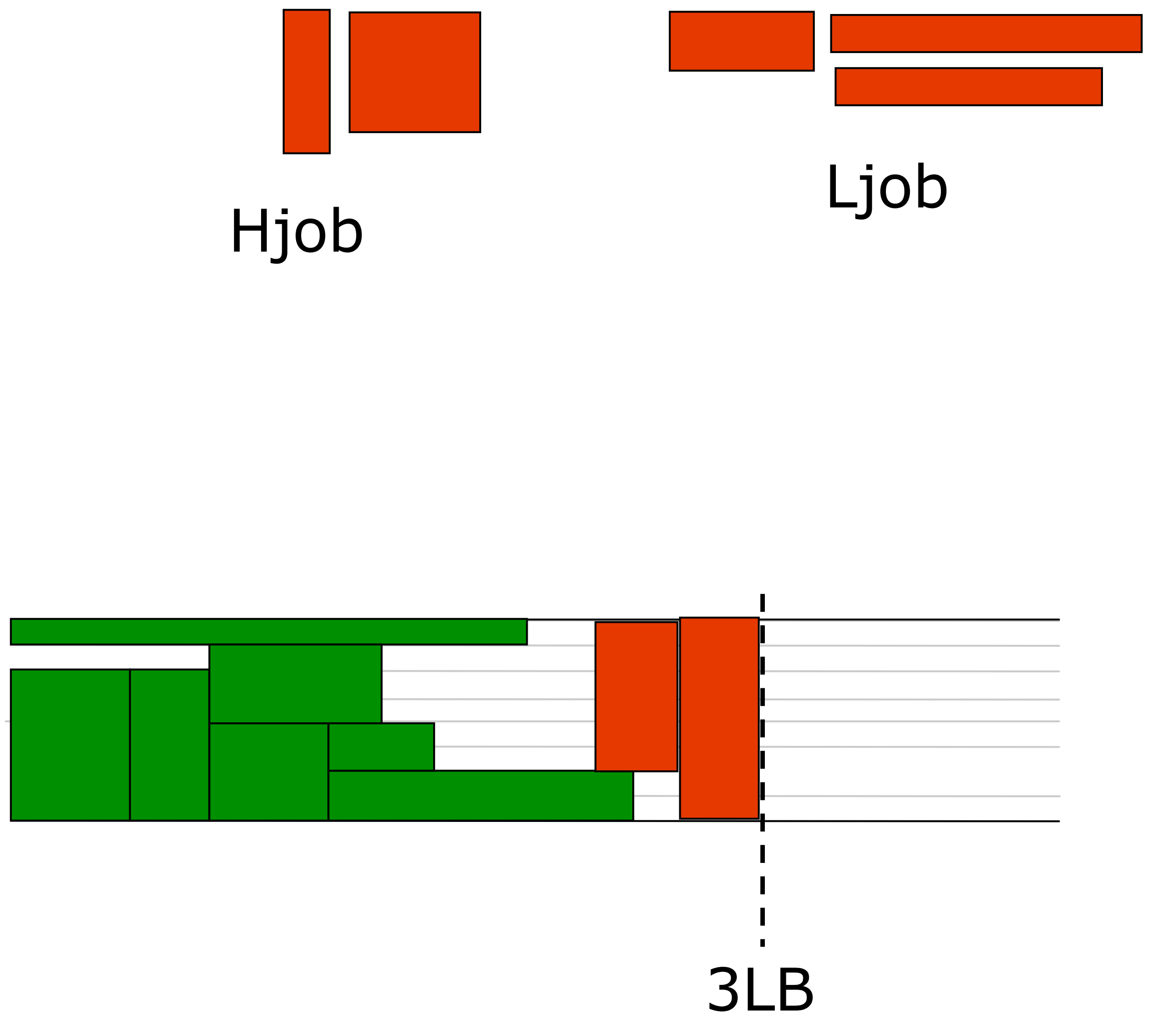
2. Unschedule all jobs that finish after 3LB.
3. Divide them into 2 sets (Ljobs and Hjobs)
4. Sort each set according to the Highest first order
5. Schedule the jobs of Hjobs backwards from 3LB on all possible clusters
6. Then, fill the gaps with Ljobs in a greedy manner

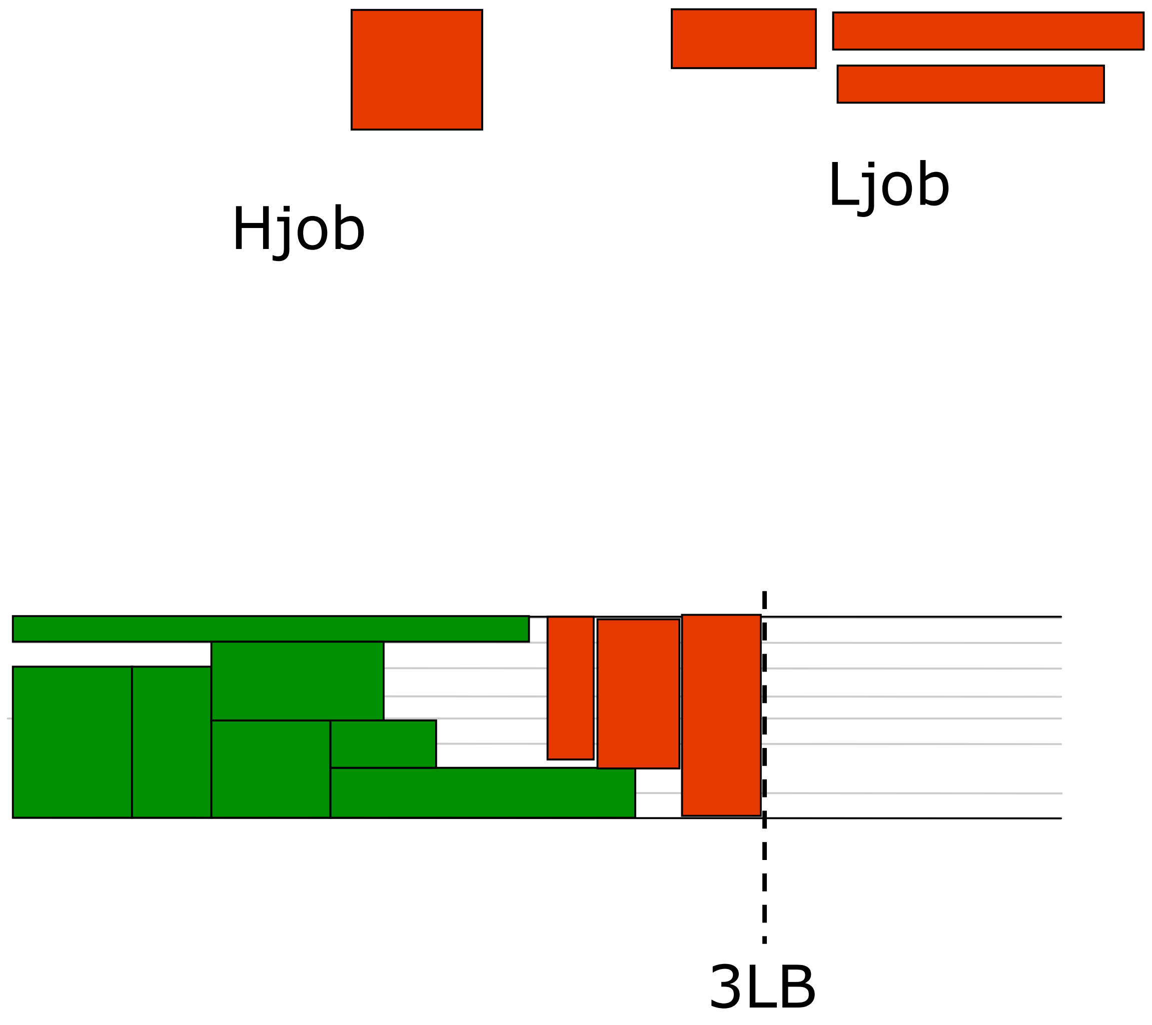


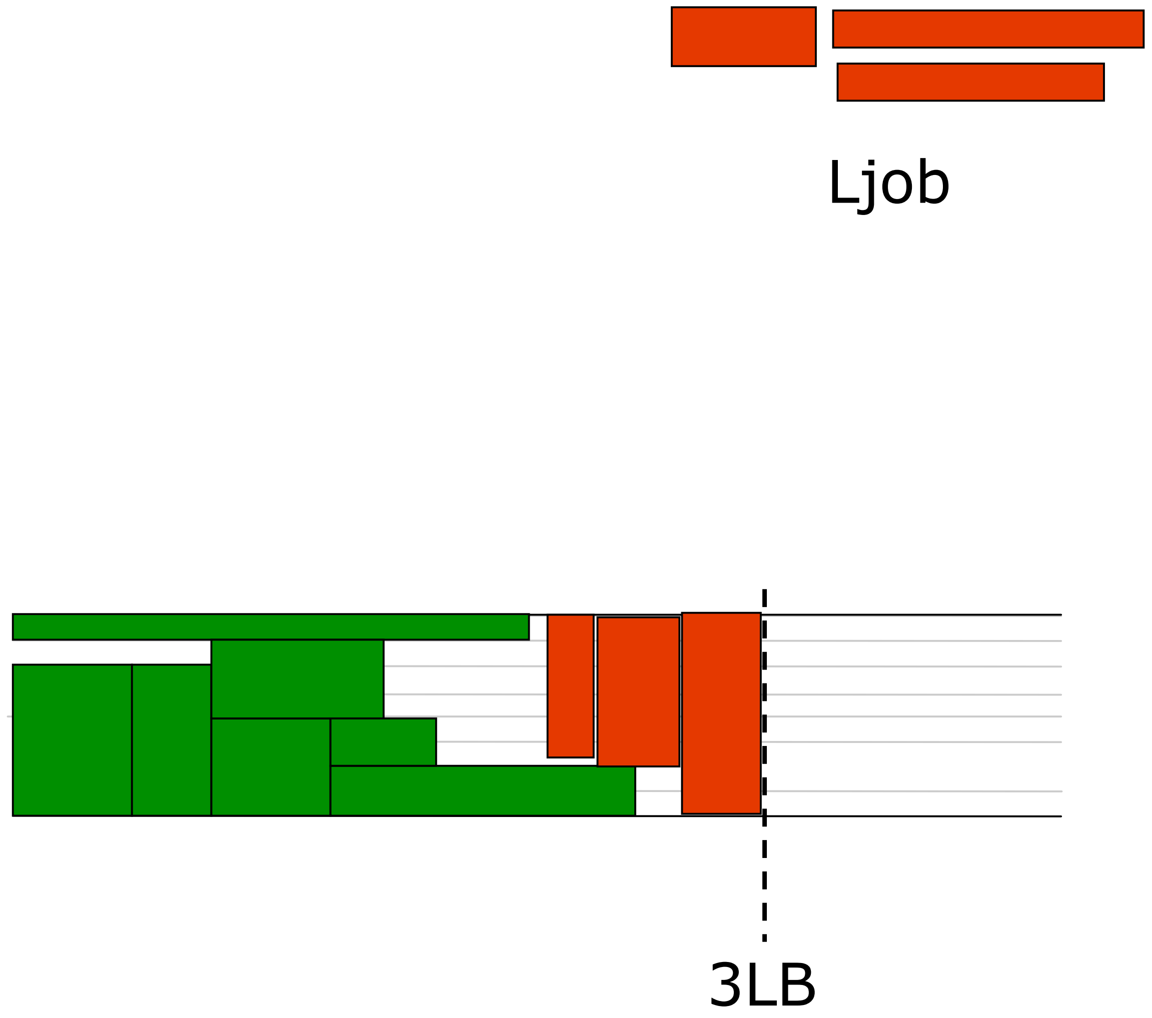
let consider a cluster whose last job finishes before 3LB

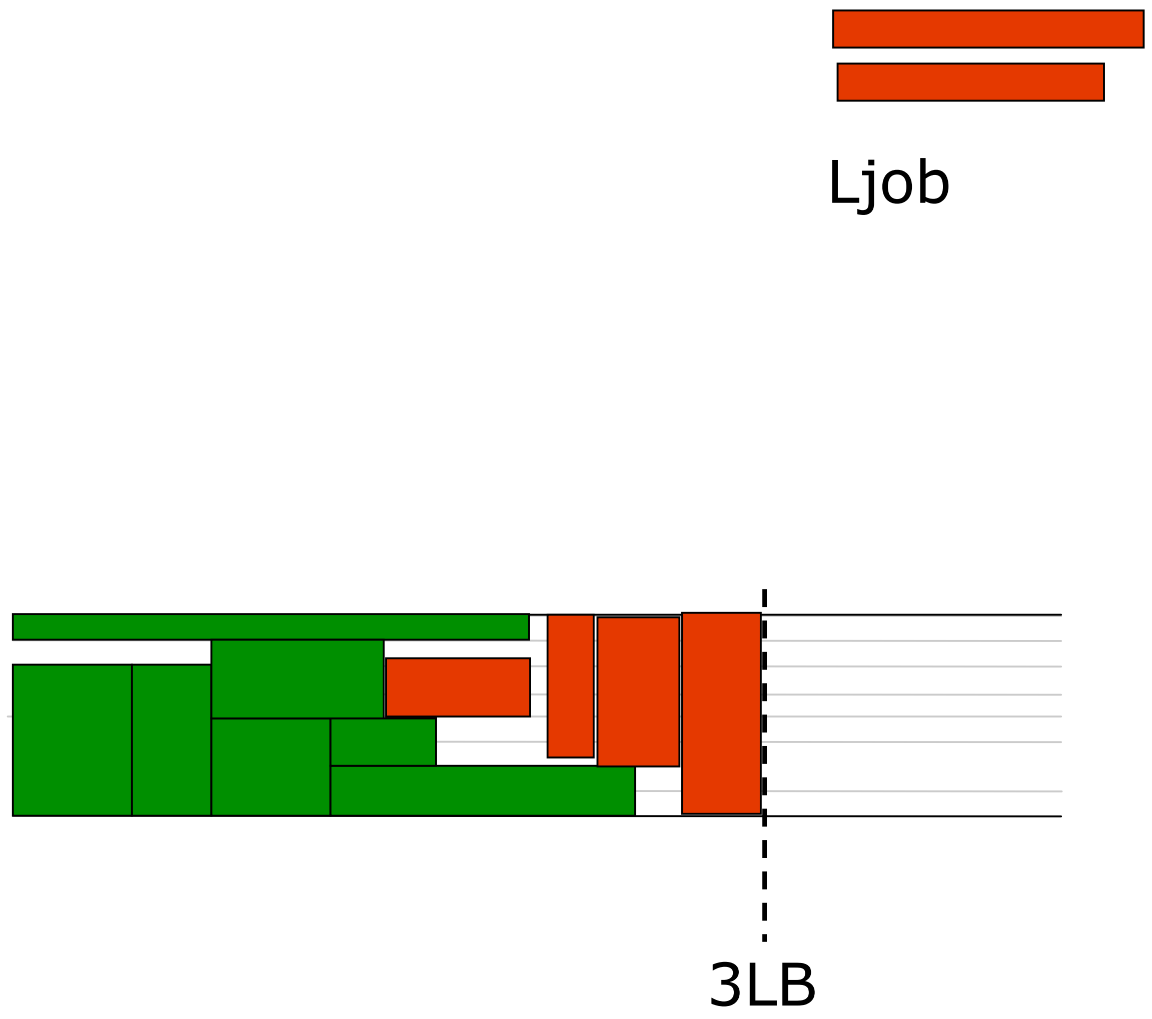



Hjob
Ljob
3LB

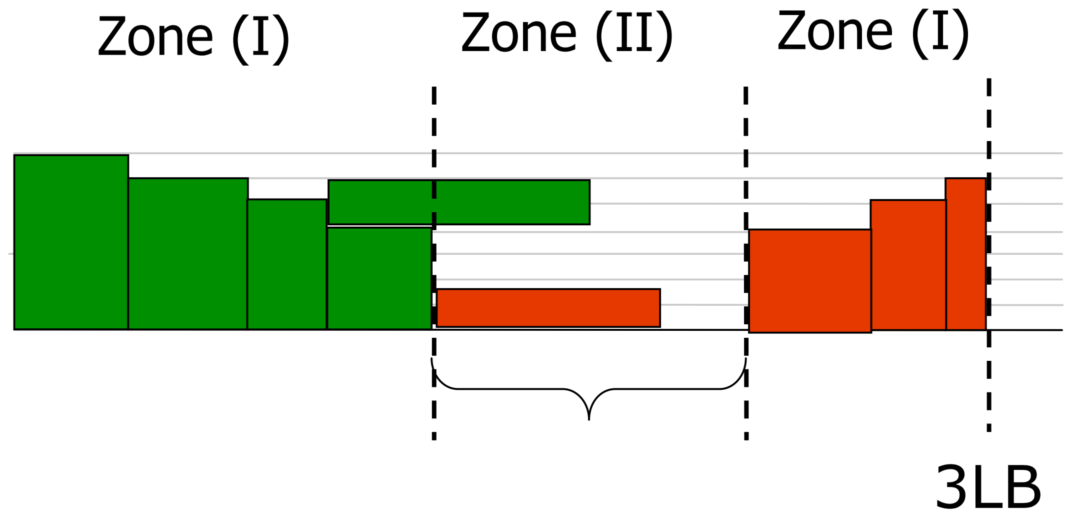

Hjob
Ljob
3LB


Hjob
Ljob
3LB


Ljob
3LB


Ljob
3LB

Feasibility (insight)



Sketch of analysis

Proof by contradiction:
let us assume that it is not feasible,
and call x the first job that does not fit in a cluster.

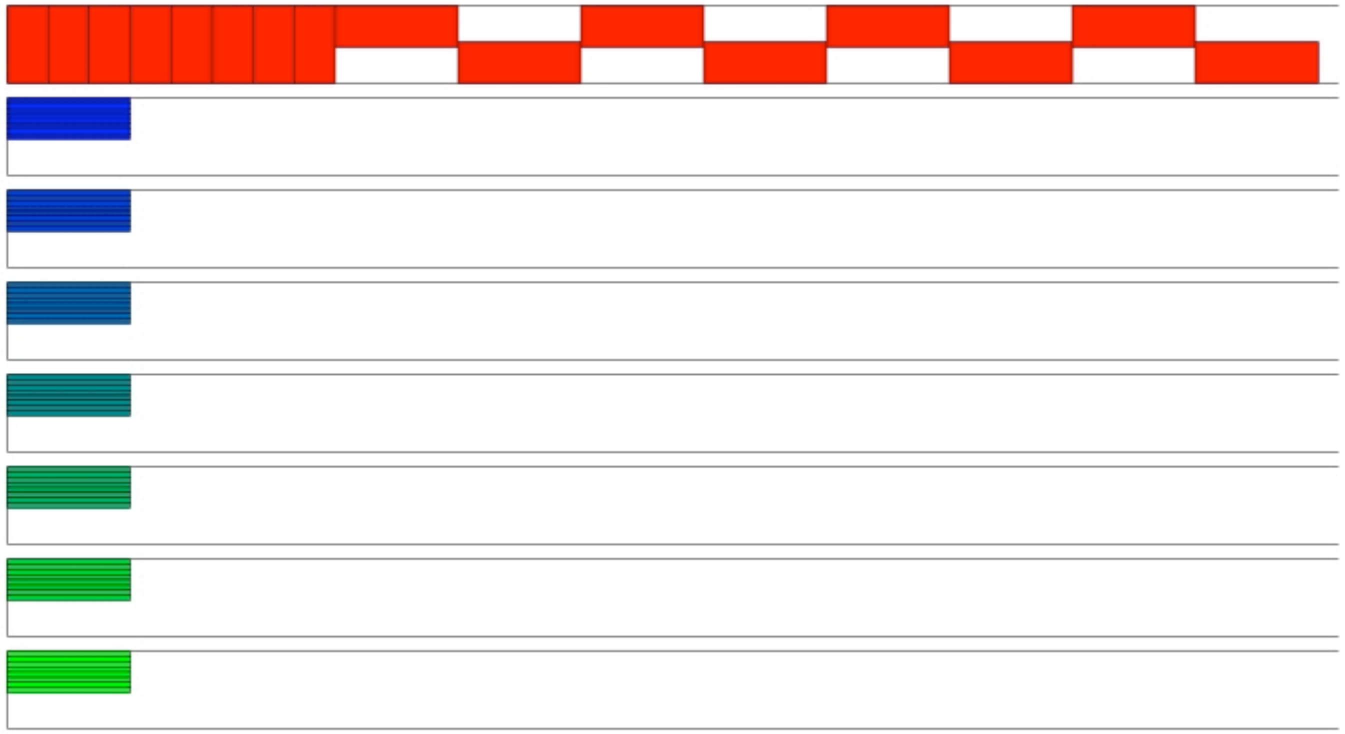
Case 1: x is a small job.
Global surface argument

Case 2: x is a high job.
Much more complicated,
see the paper for technical details

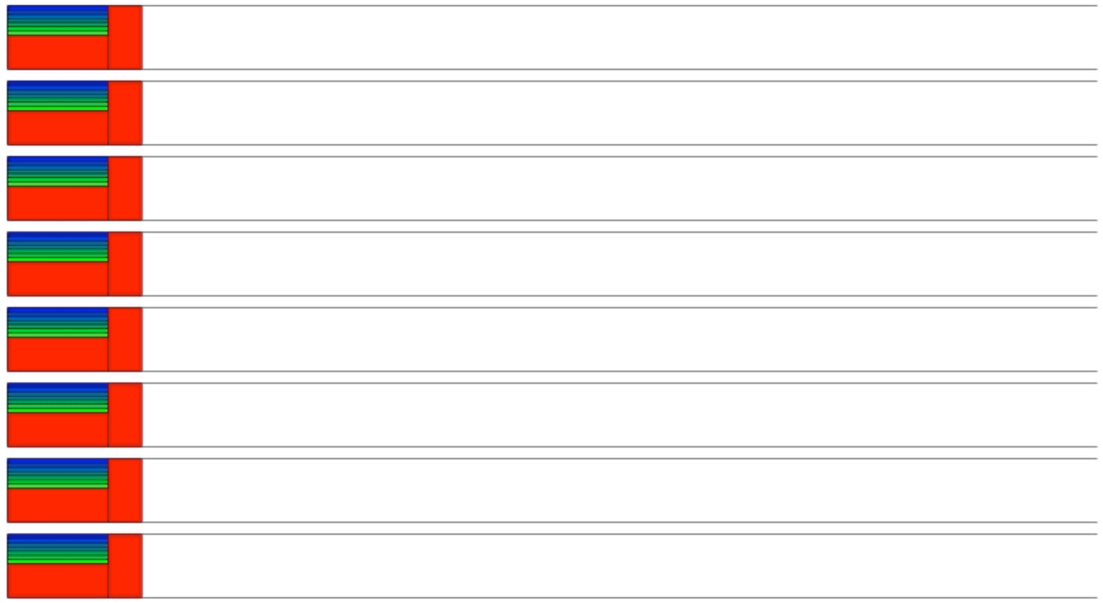
Guaranty

Proposition:
1. The previous algorithm is a 3-approximation (by construction)
2. The bound is tight (asymptotically)

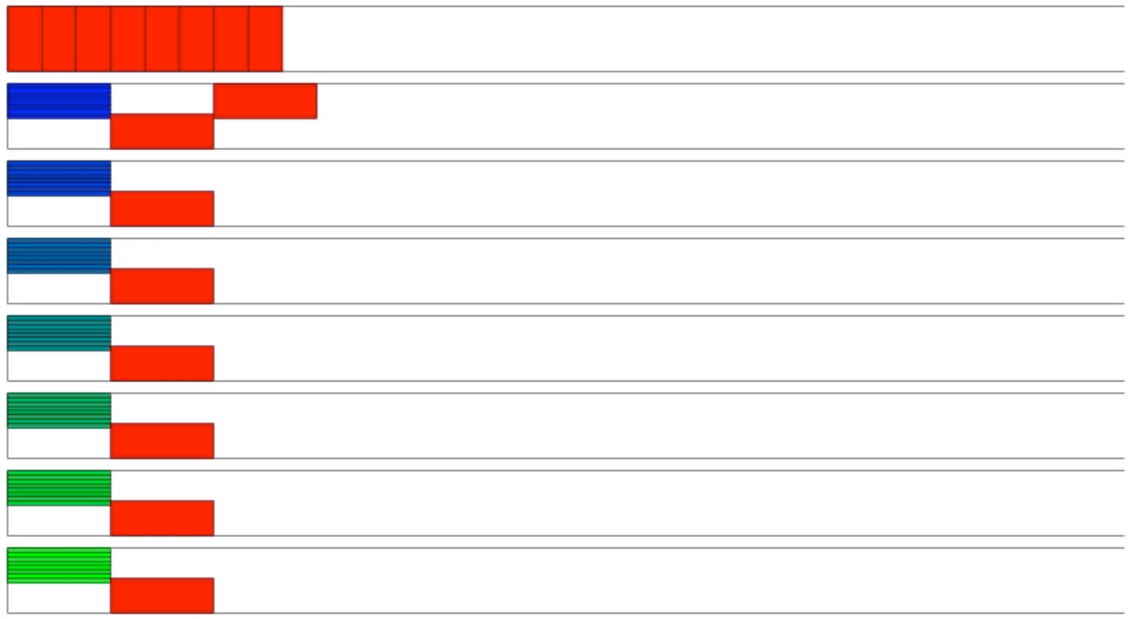
Consider the following instance:
m clusters, each with 2m-1 processors
The first organization has m short jobs requiring each the full machine (duration ε) plus m jobs of unit length requiring m processors
All the m-1 others own m sequential jobs of unit length



Local HF schedules



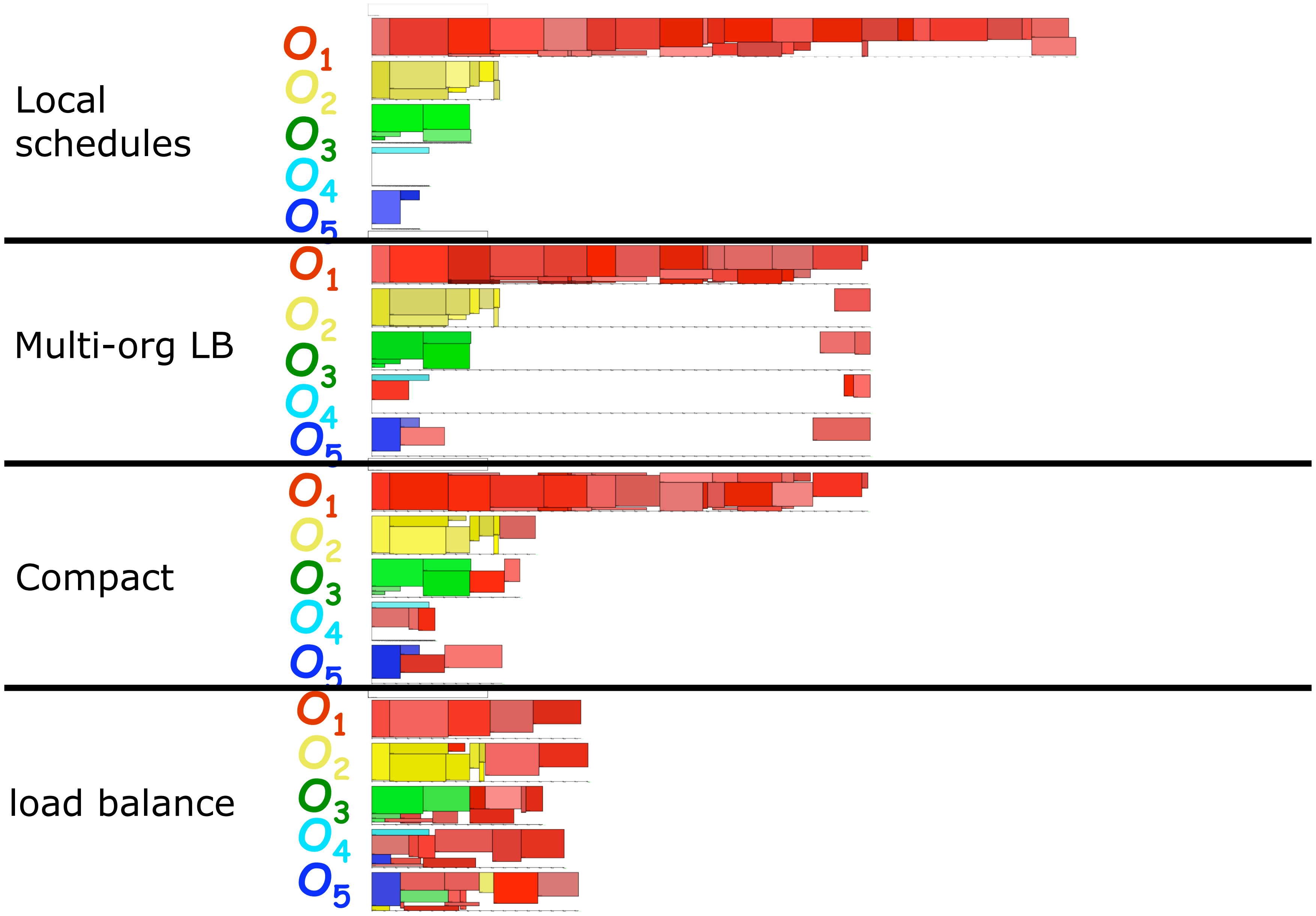
Optimal (global) schedule: Cmax* = $1+\varepsilon$



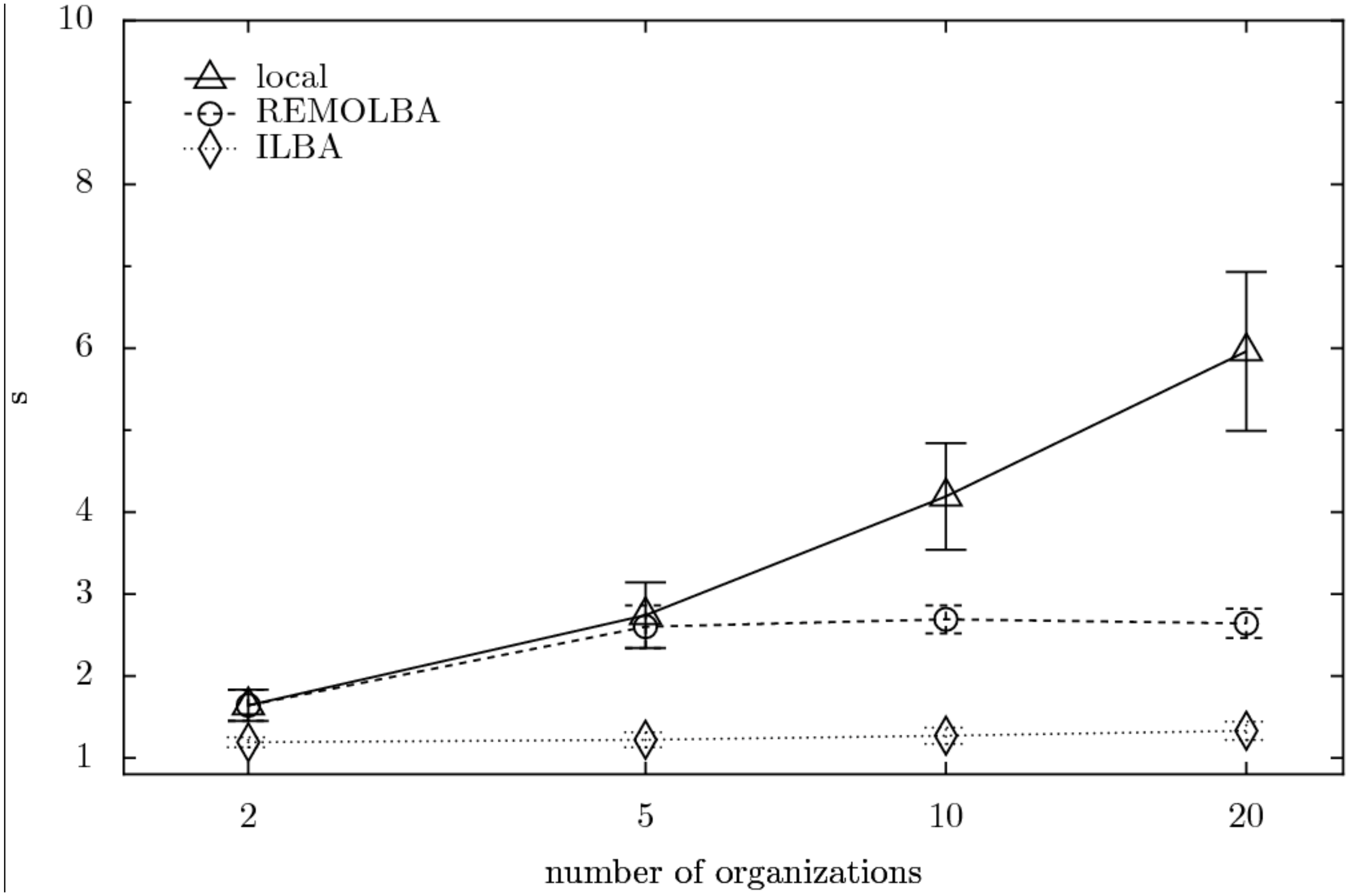
Multi-organization load-balancing: Cmax=3

Improvement

We add an extra load-balancing procedure



Some experiments



Link with Game Theory?

We propose an approach based on combinatorial optimization

Can we use Game theory?
players : organizations or users
objective: makespan, minsum, mixed

Cooperative game theory: assume that players communicate and form coalitions.

Non cooperative game theory: key concept is Nash equilibrium which is the situation where the players donot have interest to change their strategy…
stratégie : collaborer ou non; *obj. global* : min makespan

Price of stability: best Nash equilibrium over the opt. solution

Conclusion

Single unified approach based on multi-objective optimization for taking into account the users’ need or wishes.

MOSP - good guaranty for C_{max}, ΣC_i and mixed case remains to be studied
MUSP - « bad » guaranty but we can not obtain better with mow cost algorithms

Thanks for attention

Do you have any questions?