Mapping filter services on heterogeneous platforms
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Introduction

The problem:
- treatment of a data flow
- filter services with selectivity $\sigma$ and cost $c$
- precedence constraints between services
- servers with speed $s$
- one-to-one mappings

The objective:
- minimize the period
- minimize the latency
Motivation

For services of selectivity less than one

- grep
- web services
- Select-Project-Join query optimization
- ...

Related problems:

- component testing
- unsupervised systems
1. Framework
2. Period
   - General structure of optimal solutions
   - Case of homogeneous servers
   - NP-completeness of $\text{MinPeriod-Het}$
   - Integer linear program
3. Latency
   - General structure of optimal solutions
   - Polynomial algorithm on homogeneous platforms
   - NP-completeness of problem $\text{MinLatency-NoPrec-Het}$
   - Integer linear program
4. Bi-criteria problem
5. Heuristics
6. Experiments
7. Conclusion
The instances

The problems depend on:

- the criteria: `MinPeriod`, `MinLatency` or `BiCriteria`
- the platform: `Hom` or `Het`
- the dependence constraints: `NoPrec` or `Prec`
The problems depend on:

- the criteria: MinPeriod, MinLatency or BiCriteria
- the platform: Hom or Het
- the dependence constraints: NoPrec or Prec

The instances: $A = (F, G, S)$ with:

- The services: $F = \{C_1, C_2, \ldots, C_n\}$
- The precedence constraints: $G \subset F \times F$
- The servers: $S = \{S_1, S_2, \ldots, S_p\}$
The problem

Example for 3 independent services:
The plan?

\[ (C_1, S_2), (C_2, S_1), (C_3, S_3) \]

\[ (C_1, S_3), (C_2, S_2), (C_3, S_1) \]
Example

Figure: Chaining services.

\[ \mathcal{P} = \max \left( \frac{c_1}{s_1}, \frac{\sigma_1 c_2}{s_2}, \frac{\sigma_1 \sigma_2 c_3}{s_3} \right) \]

\[ \mathcal{L} = \frac{c_1}{s_1} + \frac{\sigma_1 c_2}{s_2} + \frac{\sigma_1 \sigma_2 c_3}{s_3} \]

Figure: Combining selectivities

\[ \mathcal{P} = \max \left( \frac{c_1}{s_1}, \frac{c_2}{s_2}, \frac{\sigma_1 \sigma_2 c_3}{s_3} \right) \]

\[ \mathcal{L} = \max \left( \frac{c_1}{s_1}, \frac{c_2}{s_2} \right) + \frac{\sigma_1 \sigma_2 c_3}{s_3} \]
Example

- \( c_1 = 1, \ c_2 = 4, \ c_3 = 10 \)
- \( \sigma_1 = \frac{1}{2}, \ \sigma_2 = \sigma_3 = \frac{1}{3} \)
- \( s_1 = 1, \ s_2 = 2 \) and \( s_3 = 3 \)
Example

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- \( \sigma_1 = \frac{1}{2} \), \( \sigma_2 = \sigma_3 = \frac{1}{3} \)
- \( s_1 = 1 \), \( s_2 = 2 \) and \( s_3 = 3 \)

\[
P = 1 \\
L = \frac{5}{2}
\]

**Figure**: Optimal plan for period.

\[
P = \frac{13}{6} \\
L = \frac{4}{3}
\]

**Figure**: Optimal plan for latency.
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General structure of optimal solutions

The instance: \(C_1, \ldots, C_n, S_1, \ldots, S_n\) with

- \(\sigma_1, \ldots, \sigma_p \leq 1\)
- \(\sigma_{p+1}, \ldots, \sigma_n \geq 1\)
General structure of optimal solutions

The instance: $C_1, ..., C_n, S_1, ..., S_n$ with

- $\sigma_1, ..., \sigma_p \leq 1$
- $\sigma_{p+1}, ..., \sigma_n \geq 1$

Figure: General structure
Homogeneous case without precedence constraints

The instance: \( C_1, \ldots, C_n \) with

- \( c_1 \leq c_2 \leq \ldots \leq c_p \)
- \( \sigma_1, \ldots, \sigma_p < 1 \)
- \( \sigma_{p+1}, \ldots, \sigma_n \geq 1 \)

The matching: \( C_1 \rightarrow C_2 \rightarrow \ldots \rightarrow C_p \)
Homogeneous case with precedence constraints

Computing the optimal subgraph for $C$ in the graph $G$:
Let $D = \max_i \{\log \sigma_i\}$.
We construct a network flow graph $W$ with:
- a source $s$
- a node $f_i$ by service in $G$
- a sink node $t$
- an edge $s \rightarrow f_i$ with capacity $+\infty$ if $C_i$ is ancestor of $C$ in $G$, $D$ else
- an edge $f_i \rightarrow f_j$ of capacity $+\infty$ if $C_j$ is an ancestor of $C_i$ in $G$
- an edge $f_i \rightarrow t$ with capacity $D + \log \sigma_i$

The set of services on the side of $s$ in a min-cut is the optimal subset of predecessors for latency.
Computing the optimal subgraph for $C$ in the graph $G$:

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The set of services on the side of $s$ in a min-cut is the optimal subset of predecessors for latency.

Optimal algorithm: at each step place the available service with minimal possible period.
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Proof: NP-completeness of **MinPeriod-Het**

**Problem (RN3DM)**

*Given an integer vector* \( A = (A[1], \ldots, A[n]) \) *of size* \( n \), *does there exist two permutations* \( \lambda_1 \) *and* \( \lambda_2 \) *of* \( \{1, 2, \ldots, n\} \) *such that*

\[
\forall 1 \leq i \leq n, \quad \lambda_1(i) + \lambda_2(i) = A[i]
\]
Proof: NP-completeness of MinPeriod-Het

Problem (RN3DM)

Given an integer vector $A = (A[1], \ldots, A[n])$ of size $n$, does there exist two permutations $\lambda_1$ and $\lambda_2$ of $\{1, 2, \ldots, n\}$ such that

$$\forall 1 \leq i \leq n, \quad \lambda_1(i) + \lambda_2(i) = A[i]$$

The associated instance:

- $c_i = 2^{A[i]}$
- $\sigma_i = 1/2$
- $s_i = 2^i$
- $P = 2$
Proof: NP-completeness of \textsc{MinPeriod-Het}

Problem (\textsc{RN3DM})

Given an integer vector $A = (A[1], \ldots, A[n])$ of size $n$, does there exist two permutations $\lambda_1$ and $\lambda_2$ of $\{1, 2, \ldots, n\}$ such that

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The associated instance:

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- $P = 2$

$$\forall 1 \leq i \leq n, \quad \lambda_1(i) + \lambda_2(i) \geq A[i]$$

$$\iff \forall 1 \leq i \leq n, \quad \left(\frac{1}{2}\right)^{\lambda_1(i)-1} \times \frac{2^{A[i]}}{2^{\lambda_2(i)}} \leq 2$$
Inapproximability of \texttt{MinPeriod-Het}

**Proposition**

For any $K > 0$, there exists no $K$-approximation algorithm for \texttt{MinPeriod-NoPrec-Het}, unless $P=NP$. 
Inapproximability of \texttt{MinPeriod-Het}

### Proposition

For any $K > 0$, there exists no $K$-approximation algorithm for \texttt{MinPeriod-NoPrec-Het}, unless $P=NP$.

**Reduction from RN3DM:**

- $c_i = K^{A[i]-1}$
- $\sigma_i = 1/K$
- $s_i = K^i$
- $P = 1$
The variables:

- $t_{i,u} = 1$ if service $C_i$ is assigned to server $S_u$
- $s_{i,j} = 1$ if service $C_i$ is an ancestor of $C_j$
- $M$ is the logarithm of the optimal period
Integer linear program

The variables:
- \( t_{i,u} = 1 \) if service \( C_i \) is assigned to server \( S_u \)
- \( s_{i,j} = 1 \) if service \( C_i \) is an ancestor of \( C_j \)
- \( M \) is the logarithm of the optimal period

The constraints:
- \( \forall i, \sum_u t_{i,u} = 1 \)
- \( \forall u, \sum_i t_{i,u} = 1 \)
- \( \forall i, j, k, s_{i,j} + s_{j,k} - 1 \leq s_{i,k} \)
- \( \forall i, s_{i,i} = 0 \)
- \( \forall i, \log c_i - \sum_u t_{i,u} \log s_u + \sum_k s_{k,i} \log \sigma_k \leq M \)

The objective function: Minimize \( M \)
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Structure of the optimal plan

Proposition

Let $C_1, ..., C_n, S_1, ..., S_n$ be an instance of $\text{MinLatency}$. Then, the optimal latency is obtained with a plan $G$ such that, for any $v_1 = (C_{i_1}, S_{u_1})$, $v_2 = (C_{i_2}, S_{u_2})$,

1. If $d_{i_1}(G) = d_{i_2}(G)$, they have the same predecessors and the same successors in $G$.
2. If $d_{i_1}(G) > d_{i_2}(G)$ and $\sigma_{i_2} \leq 1$, then $c_{i_1}/s_{u_1} < c_{i_2}/s_{u_2}$.
3. All nodes with a service of selectivity $\sigma_i > 1$ are leaves ($d_i(G) = 0$).
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Algorithm without dependence constraint

**Data:** $n$ services of cost $c_1 \leq \cdots \leq c_n$ and of selectivities $\sigma_1, \ldots, \sigma_n \leq 1$

**Result:** a plan $G$ optimizing the latency

$G$ is the graph reduced to node $C_1$;

```plaintext
for $i = 2$ to $n$ do
  for $j = 0$ to $i - 1$ do
    Compute the completion time $t_j$ of $C_i$ in $G$ with predecessors $C_1, \ldots, C_j$;
  end
  Choose $j$ such that $t_j = \min_k \{t_k\}$;
  Add the node $C_i$ and the edges $C_1 \rightarrow C_i, \ldots, C_j \rightarrow C_i$ to $G$;
end
```

**Algorithm 1:** Optimal algorithm for $\text{MINLATENCY-NOPREC-HOM}$. 
$G$ is the graph reduced to the node $C$ of minimal cost with no predecessor in $G$;

$\textbf{for } i = 2 \textbf{ to } n \textbf{ do}$

Let $S$ be the set of services not yet in $G$ and such that their set of predecessors in $G$ is included in $G$;

$\textbf{for } C \in S \textbf{ do}$

$\textbf{for } C' \in G \textbf{ do}$

Compute the set $S'$ minimizing the product of selectivities among services of latency less than $L_G(C')$, and including all predecessors of $C$ in $G$;

end

Let $S_C$ be the set that minimizes the latency of $C$ in $G$ and $L_C$ be this latency;

end

Choose a service $C$ such that $L_C = \min\{L_{C'}, C' \in S\}$;

Add to $G$ the node $C$, and $\forall C' \in S_C$, the edge $C' \rightarrow C$;

$\textbf{end}$

$\textbf{Algorithm 2: Optimal algorithm for } \text{MinLatency-Prec-Hom.}$
$C_1$
$C_1$
Example

$C_1$ \rightarrow $C_2$
Example

$C_1 \rightarrow C_2$
Example

\[ C_1 \rightarrow C_2 \rightarrow C_3 \]
Example
Example
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Proof: NP-completeness of $\text{MinLatency-Het}$

Lemma

Let $C_1, \ldots, C_n, S_1, \ldots, S_n$ be an instance such that $\forall i$, $c_i$ and $s_i$ are integer power of 2 and $\sigma_i \leq \frac{1}{2}$. Then the optimal latency is obtained with a plan $G$ such that

1. Proposition 2 is verified;

2. for all nodes $(C_{i_1}, S_{u_1})$ and $(C_{i_2}, S_{u_2})$ with $d_{i_1}(G) = d_{i_2}(G)$, we have
   \[
   \frac{c_{i_1}}{s_{u_1}} = \frac{c_{i_2}}{s_{u_2}}.
   \]
Lemma

Let $C_1, \ldots, C_n, S_1, \ldots, S_n$ be an instance such that $\forall i$, $c_i$ and $s_i$ are integer power of 2 and $\sigma_i \leq \frac{1}{2}$. Then the optimal latency is obtained with a plan $G$ such that

1. Proposition 2 is verified;
2. for all nodes $(C_{i_1}, S_{u_1})$ and $(C_{i_2}, S_{u_2})$ with $d_{i_1}(G) = d_{i_2}(G)$, we have
   \[
   \frac{c_{i_1}}{s_{u_1}} = \frac{c_{i_2}}{s_{u_2}}.
   \]

- $c_i = 2^{A[i] \times n + (i-1)}$
- $\sigma_i = \left(\frac{1}{2}\right)^n$
- $s_i = 2^n \times (i+1)$
- $L = 2^n - 1$
Proposition

For any $K > 0$, there exists no $K$-approximation algorithm for \textsc{MinLatency-NoPrec-Het}, unless $P=NP$. 
Proposition

For any $K > 0$, there exists no $K$-approximation algorithm for \textsc{MinLatency-NoPrec-Het}, unless $P=NP$.

Reduction from \textsc{RN3DM}

- $c_i = K^{A[i] \times n + (i-1)}$
- $\sigma_i = \left(\frac{1}{K}\right)^n$
- $s_i = K^{n \times (i+1)}$
- $L = \frac{K^n - 1}{K - 1}$
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The variables:
- $z(i, u, e) = 1$ if the service $C_i$ is associated to the server $S_u$ and its set of predecessors is $e \subset C$.
- $t(i)$ is the completion time of $C_i$
- $M$ is the optimal latency
The variables:

- \( z(i, u, e) = 1 \) if the service \( C_i \) is associated to the server \( S_u \) and its set of predecessors is \( e \subset C \).

- \( t(i) \) is the completion time of \( C_i \)

- \( M \) is the optimal latency

The constraints:

- \( \forall u \in S, \sum_{i \in C} \sum_{e \subset C} z(i, u, e) = 1 \)

- \( \forall i \in C, \sum_{u \in S} \sum_{e \subset C} z(i, u, e) = 1 \)
The variables:
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- \( t(i) \) is the completion time of \( C_i \)
- \( M \) is the optimal latency

The constraints:
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- \( \forall i \in C, \sum_{u \in S} \sum_{e \subset C} z(i, u, e) = 1 \)
- \( \forall i, i' \in C, \forall u, u' \in S, \forall e, e' \subset C, e \nsubseteq e', i \in e', z(i, u, e) + z(i', u', e') \leq 1 \)
- \( \forall u \in S, \forall e \subset C, \forall i \in e, z(i, u, e) = 0 \)
The variables:

- $z(i, u, e) = 1$ if the service $C_i$ is associated to the server $S_u$ and its set of predecessors is $e \subset C$.
- $t(i)$ is the completion time of $C_i$
- $M$ is the optimal latency

The constraints:

- $\forall u \in S, \sum_{i \in C} \sum_{e \subset C} z(i, u, e) = 1$
- $\forall i \in C, \sum_{u \in S} \sum_{e \subset C} z(i, u, e) = 1$
- $\forall i, i' \in C, \forall u, u' \in S, \forall e, e' \subset C, e \not\subset e', i \in e', \quad z(i, u, e) + z(i', u', e') \leq 1$
- $\forall u \in S, \forall e \subset C, \forall i \in e, \quad z(i, u, e) = 0$
- $\forall i \in C, \forall e \subset C, \forall k \in e, \quad t(i) \geq \sum_{u \in S} z(i, u, e) \left( \frac{c_i}{s_u} \times \prod_{C_j \in e \sigma_j} + t(k) \right)$
- $\forall i \in C, \quad t(i) \geq \sum_u z(i, u, e) \frac{c_i}{s_u} \times \prod_{C_j \in e \sigma_j}$
- $\forall i \in C, \quad t(i) \leq M$

The objective function: Minimize $M$
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Data: $n$ services of cost $c_1 \leq \cdots \leq c_n$ and of selectivities $\sigma_1, \ldots, \sigma_n \leq 1$ and a maximum throughput $K$

Result: a plan $G$ optimizing the latency with a throughput less than $K$

$G$ is the graph reduced to node $C_1$;

**for** $i = 2$ **to** $n$ **do**

**for** $j = 0$ **to** $i - 1$ **do**

Compute the completion time $t_j$ of $C_i$ in $G$ with predecessors $C_1, \ldots, C_j$;

**end**

Let $S = \{ k \mid c_i \prod_{0 \leq k < i} \sigma_k \leq K \}$;

Choose $j$ such that $t_j = \min_{k \in S} \{ t_k \}$;

Add the node $c_i$ and the edges $C_1 \rightarrow C_i, \ldots, C_j \rightarrow C_i$ to $G$;

**end**

**Algorithm 3:** Optimal algorithm for latency with a fixed throughput.
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**sigma-inc** We place services on a chain in increasing order of $\sigma$.

**short service/fast server** We associate the service with shortest cost to the server with fastest speed.

**long service/fast server** We associate the service with largest cost with the server with fastest speed.

**opt-homo** We randomly associate services to servers.

**greedy min** This simple heuristic consists in running successively the four previous heuristics on the problem instance, and returning as a result the best of the four solutions.

**random** This last heuristic is fully random: we randomly associate services and servers, and we randomly place these pairs on a linear chain.
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4. **Bi-criteria problem**

5. **Heuristics**

6. **Experiments**

7. **Conclusion**
The instances:

- independent services
- the cost of services: $1 \leq c \leq 100$
- the selectivities: $0, 01 \leq \sigma \leq 1$
- the speed of servers: $1 \leq s \leq 100$
Figure: Experiment 1: general experiment.
Figure: Experiment 1: general experiment.
Figure: Experiment 2: with small selectivity.
Figure: Experiment 3: with high selectivity.
Figure: Experiment 4: with low speed.
Figure: Experiment 5: with low heterogeneity.
Figure: Experiment 1: Computing times.
References


Conclusion

The results:

- \textbf{MinLatency-Hom} is polynomial
- \textbf{MinPeriod-Het} is NP-complete
- \textbf{MinLatency-Het} is NP-complete
- \textbf{BiCriteria-Hom} is polynomial

The experiments on \textbf{MinPeriod-NoPrec-Het}:

- heuristics close to the optimal for small instances
- better performance than random

Future work:

- model with communication costs