Offline and online master-worker scheduling of concurrent bags-of-tasks on heterogeneous platforms

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joint work with Anne BENOIT, Jean-François PINEAU, Yves ROBERT and Frédéric VIVIEN

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Object of the Study

- Bags-of-tasks application
  - independent tasks
  - large number of similar tasks
  - models embarrassingly parallel applications
  - argues for the use of wide distributed platforms

- Online scheduling
  - applications arrive at different time (release dates)
  - no knowledge on the future
  - no global makespan, try to lower the suffering of each user

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Building on our previous results

- Large number of tasks ⇒ steady-state scheduling
  - designed for large applications
  - suited for heterogeneous platforms, multiple applications

(*Centralized versus distributed schedulers for multiple bag-of-task applications*, IPDPS'06)

  - optimal platform utilization: throughput maximization
  - neglect transient phases (initialization/clean-up)

- Online scheduling ⇒ maximum stretch minimization
  - other metrics not suited

(*Minimizing the stretch when scheduling flows of biological requests*, SPAA '06)

  - stretch is a kind of *price for sharing resources*
  - minimize the maximum stretch among applications: give a guarantee on each application slowdown

NB: maximize throughput and minimize max-stretch could seem contradictory

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Simple idea to bring things together

- Suppose we want to reach the maximum stretch $\mathcal{S}$
- For a given application, we can compute its makespan "if it was alone": MS
- This gives a deadline:

$$\text{deadline} = \text{release date} + \mathcal{S} \times MS$$

- Each application has now a release date and a deadline.
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With a single bag-of-task application

Several bag-of-task applications: offline case

Discussion on models

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Simulations and Experiments

Conclusion

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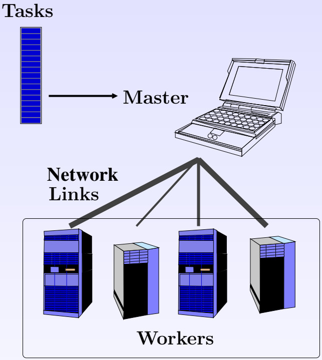
Single bag-of-task application – context

- Master-Slave platform (heterogeneous):



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- Computing optimal makespan: already difficult problem
- Steady-state relaxation to get a lower bound

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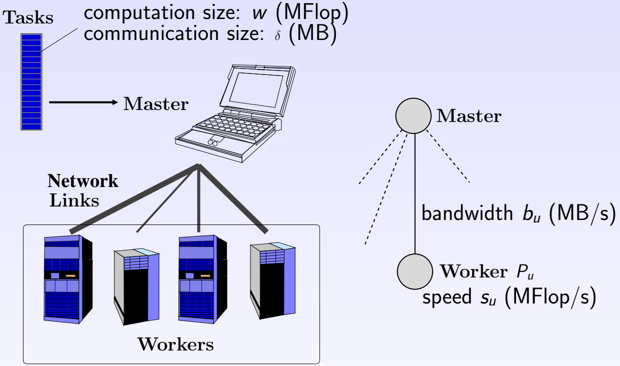

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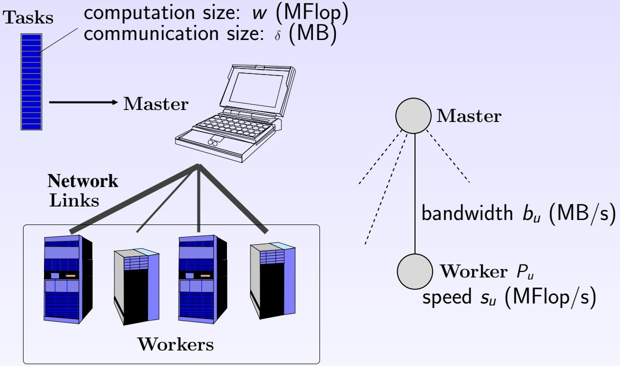

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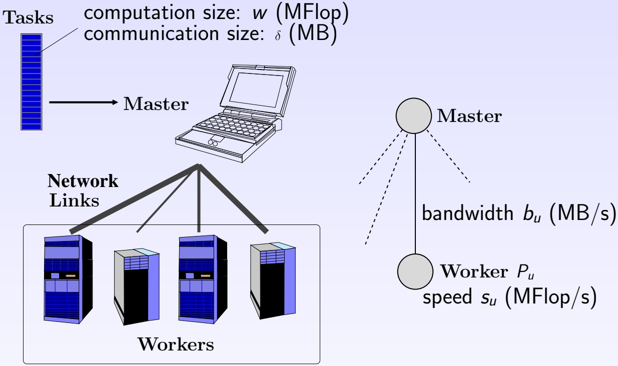

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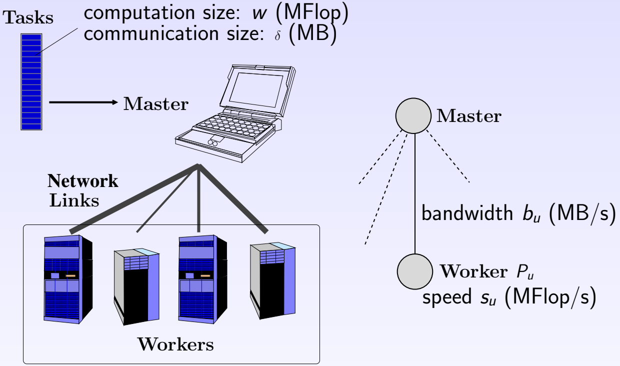

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Single bag-of-task application – steady-state

Motivations:

- Assume the number of tasks is huge
- Forget about makespan (meaningless)
- Concentrate on throughput (fluid framework)

How it works:

- Consider average values:
  "master sends 5.3 tasks per second to worker 3"
- Write constraints on these variables
- Optimize total throughput under these constraints
  (with the help of linear programming)
- Reconstruct near-optimal schedule from average values
  (we skip this step for now)

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Single bag-of-task application – linear program

$$\begin{cases} \textsc{Maximize } \rho = \sum_{u=1}^{p} \rho_u \\ \text{subject to} \\ \rho_u \frac{w}{s_u} \leq 1 \\ \rho_u \frac{\delta}{b_u} \leq 1 \\ \sum_{u=1}^{p} \rho_u \frac{\delta}{\mathcal{B}} \leq 1 \end{cases}$$

ρ_u: throughput of worker P_u
ρ: Total throughput

Analytical solution

$$\rho = \min\left\{\frac{\mathcal{B}}{\delta}, \sum_{u=1}^{p} \min\left\{\frac{s_u}{w}, \frac{b_u}{w}\right\}\right\}.$$

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Offline multi-application – framework

For each application k (task of sizes $w^{(k)}$, $\delta^{(k)}$), we have:

- a release date
- the optimal throughput (alone): $\rho^{*(k)}$

$\rightsquigarrow$ a bound on the makespan alone:

$$MS^{(k)} \leq \frac{\text{number of tasks}}{\text{optimal throughput}} = \frac{\Pi^{(k)}}{\rho^{*(k)}}$$

- not only a lower bound, rather an approximation...

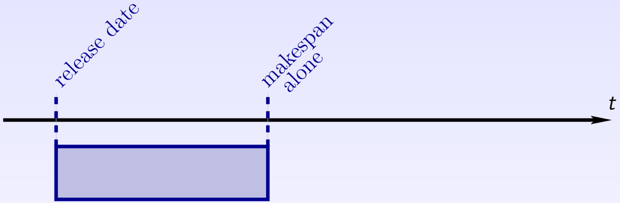
We try to reach stretch $\mathcal{S}$:

- deadline:

$$\text{deadline}^{(k)} = \text{release date}^{(k)} + \mathcal{S} \times \frac{\Pi^{(k)}}{\rho^{*(k)}}$$

Time-intervals for target stretch

If we try to reach stretch $\mathcal{S} = 2$:



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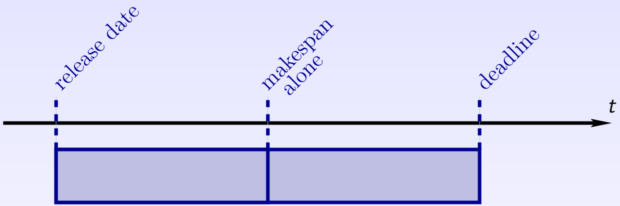

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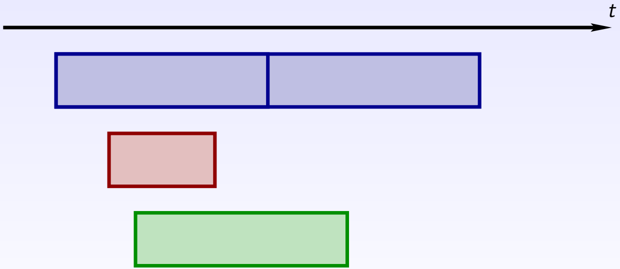

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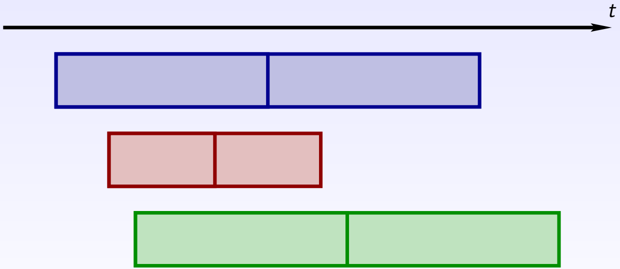

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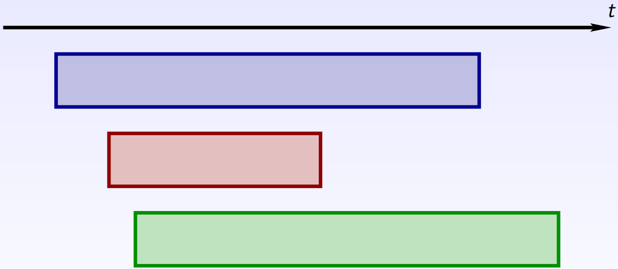

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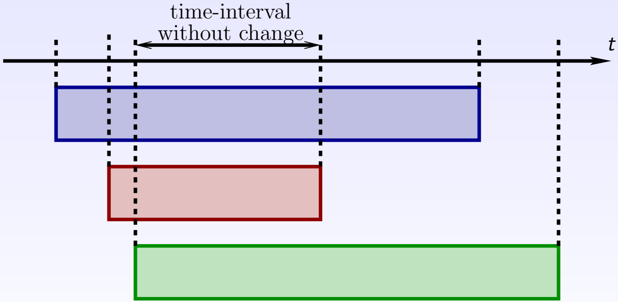


Resolution for a target stretch $\mathcal{S}$

New variables:

- communication throughput $\rho^{(k)}_{M\to u}(t_j, t_{j+1})$
- computation throughput $\rho^{(k)}_{u}(t_j, t_{j+1})$
- state of buffers: $B^{(k)}_{u}(t_j)$
  (number of non-executed tasks at time t_j)

New constraints:

- Complex (but straightforward) conservation laws between throughputs and buffer state ▸ details
- Assert that all tasks of an application are treated.
- Resource limitations

Set of linear constraints, defining a convex $K(\mathcal{S})$.

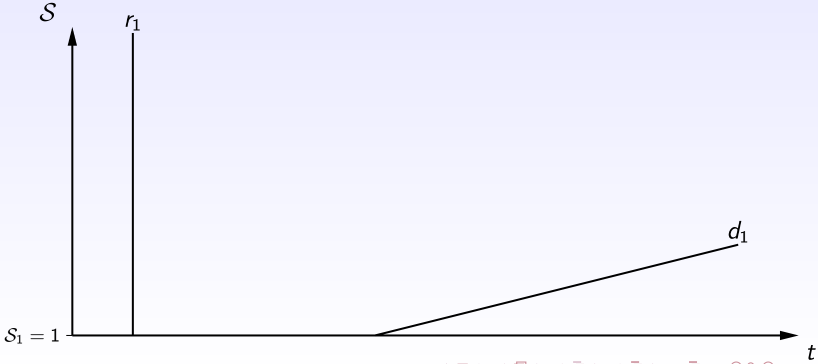
$$K(\mathcal{S}) \text{ non-empty} \Leftrightarrow \mathcal{S} \text{ feasible}$$

Binary search of optimal stretch

We have a toolbox to know if a given stretch is feasible. Search of the optimal (minimum) stretch:

- Basic binary search (with precision ϵ), or
- Involved search among stretch-intervals:

$$d^{(k)}(\mathcal{S}) = r^{(k)} + \mathcal{S} \times MS^{*(k)}.$$



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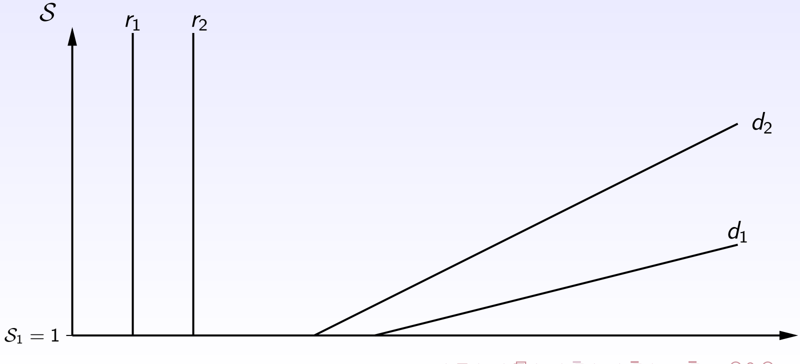

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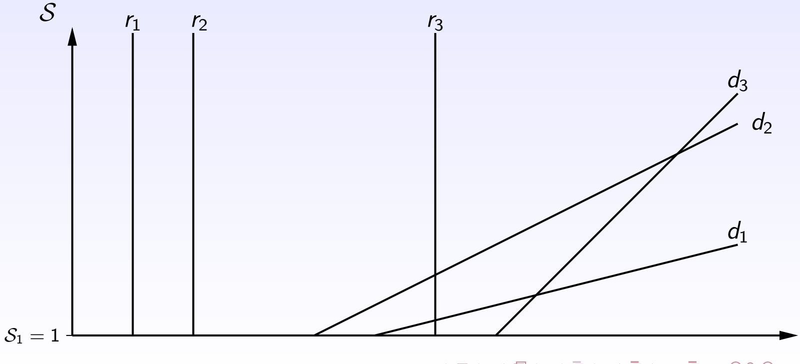

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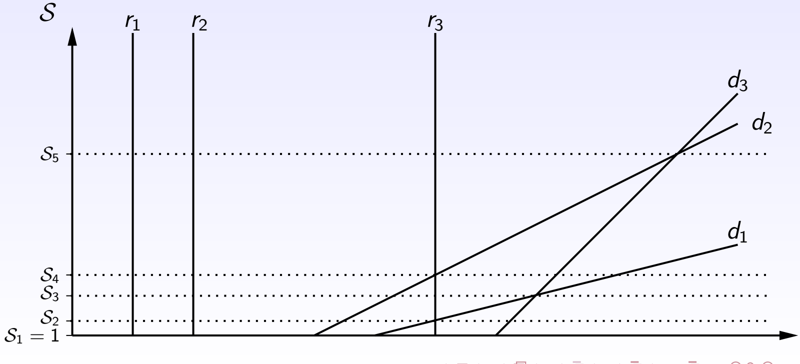

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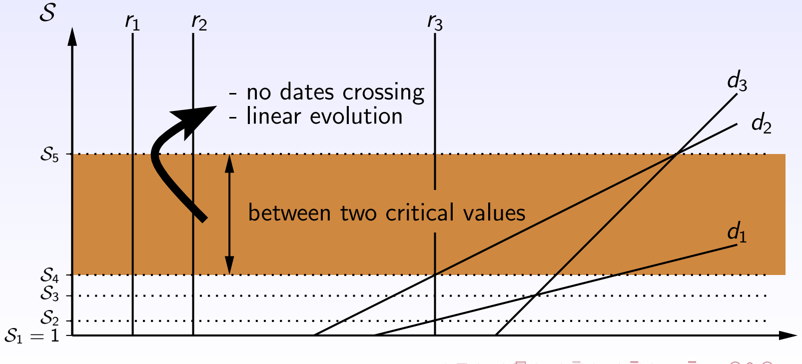

Binary search between stretch-interval

- Consider a stretch-interval between two critical values $[\mathcal{S}_a; \mathcal{S}_b]$
- Deadlines have a linear evolution
- Everything is linear !? Not really:
  when computing what receives a buffer during a time-interval:

  $T_{\text{end}}, T_{\text{start}}$: linear function in $\mathcal{S}$
  $\leadsto$ quadratic constrains ☹
- Switch from *throughput* to *amount* variables:

$$
\begin{aligned}
A^{(k)}_{M\to u}(t_j, t_{j+1}) &= \rho^{(k)}_{M\to u}(t_j, t_{j+1}) \times (t_{j+1} - t_j) \\
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- My favorite over-classical one-port model ?
  (a processor sends/receives one message at a time, and can overlap the communications by computations)
- No! no schedule reconstructed from the linear programs ☹
- Solution of a linear program : fluid throughput $\rho_u^{(k)}$, assumes
  - time-sharing for communication and computation
  - "Synchronous Start" for communication and computation
- Nice model for scheduling, but far from reality:
  - no data dependency (!)
  - Concurrent applications
  - Perfect time-sharing for computation and communication (!)
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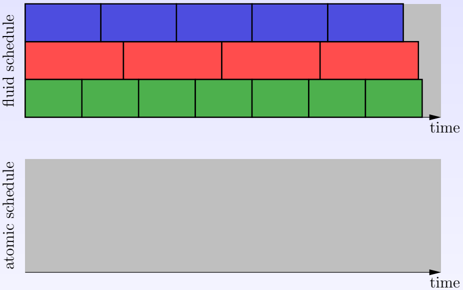
One-dimensional load-balancing

- General fluid schedule with rate α_k for application k
- task of application k takes time t_k at full speed



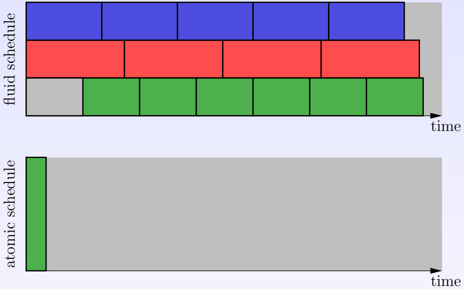
At each step, choose application which minimize

$$(n_k + 1) \times \frac{t_k}{\alpha_k}$$

n_k: number of task from application k already scheduled

One-dimensional load-balancing

- General fluid schedule with rate α_k for application k
- task of application k takes time t_k at full speed



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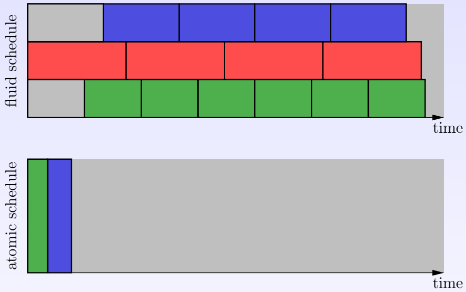

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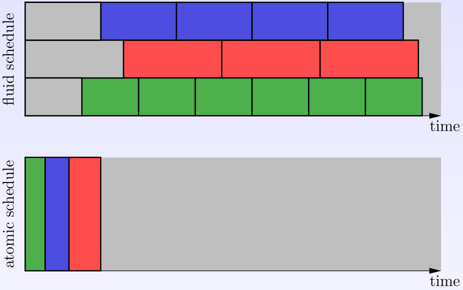

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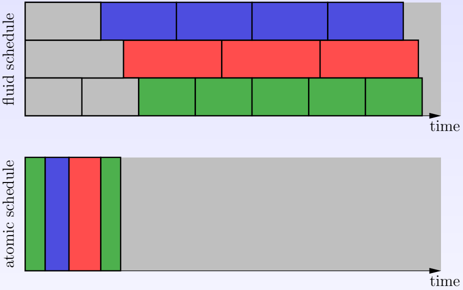

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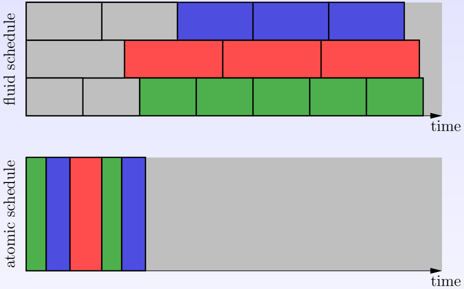

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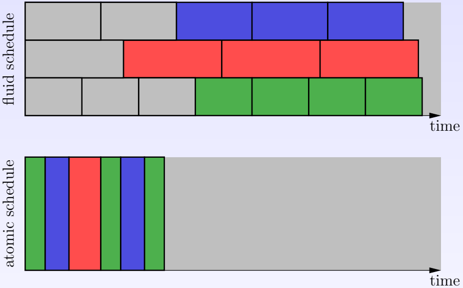

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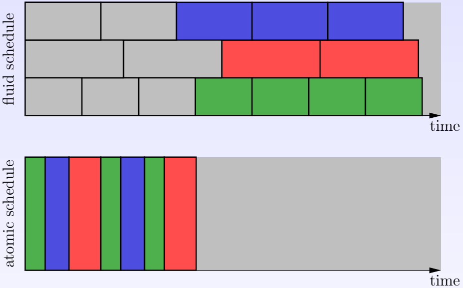

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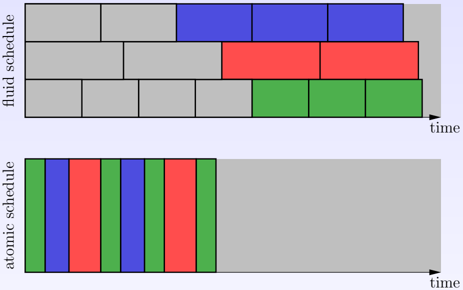

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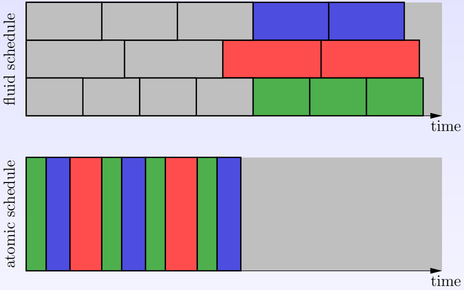

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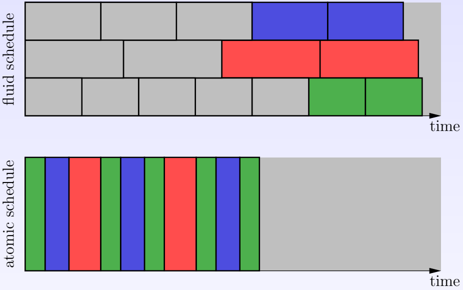

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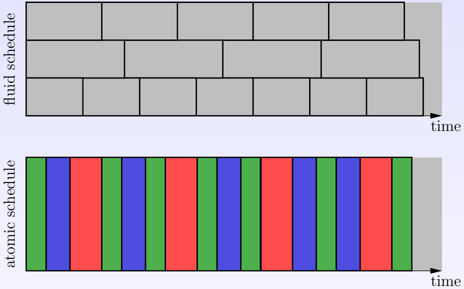

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Properties of 1D schedules

Lemma (1D).

In the 1D schedule, a task does not terminate later than in the fluid schedule.

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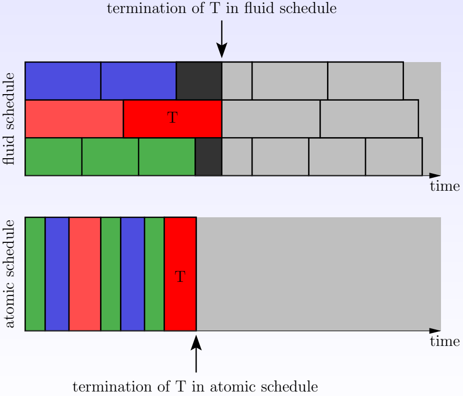

Properties of 1D schedules

Lemma (1D).

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Construction of 1D-inv schedule from a fluid schedule (M: Makespan):

1. Reverse the time: $t \rightsquigarrow M - t$
2. Apply 1D algorithm
3. Reverse the time one more time

Lemma (1D-inv).

In the 1D-inv schedule, a task does not start earlier than in the fluid schedule, and 1D-inv has a makespan $\leq M$.

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Lemma (1D-inv).

In the 1D-inv schedule, a task does not start earlier than in the fluid schedule, and 1D-inv has a makespan $\leq M$.

Back to the one-port model

From a fluid schedule (of communications and computations):

1. Round every quantities down to integer values
2. Shift all computations by one task (to cope with dependencies)
3. Apply 1D algorithm to communications
   $\longrightarrow$ communications finish in time
4. Apply 1D-inv algorithm to computations
   $\longrightarrow$ computations do not start in advance

Results:

- We guarantee that data dependencies are satisfied
- Some tasks may be forgotten: at most a fixed number
- Take some time at the end of an application to process the missing tasks

Back to the one-port model

Asymptotic optimality: when the granularity of the application gets smaller (lots of small tasks), the one-port makespan gets closer to the fluid makespan.

- Construction of an atomic schedule for performance guarantee
- In practice:
  - 1D schedule for communications
  - Earliest Deadline First for computations

Outline

Framework

With a single bag-of-task application

Several bag-of-task applications: offline case

Discussion on models

Several bag-of-task applications: online case

Simulations and Experiments

Conclusion

Online multi-application – framework

- No available information about future submission
- Information for application k available at release date $r^{(k)}$

Adaptation:

- Consider only available information (already submitted applications)
- Restart offline algorithm at each release date (with updated information)

- online heuristic named **CBS3M**-online
- we also test the offline algorithm: **CBS3M**-offline

Online multi-application – framework

Classical heuristics to prioritize applications:

- First In First Out (**FIFO**)
- Shortest Processing Time (**SPT**)
- Shortest Remaining Processing Time (**SRPT**)
- Shortest Weighted Remaining Processing Time (**SWRPT**)

Previous heuristics do not mix applications,

- Master-Worker Multi-Application (**MWMA**)
  (previous work, designed for simultaneous submissions)

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Simulations and Experiments – settings

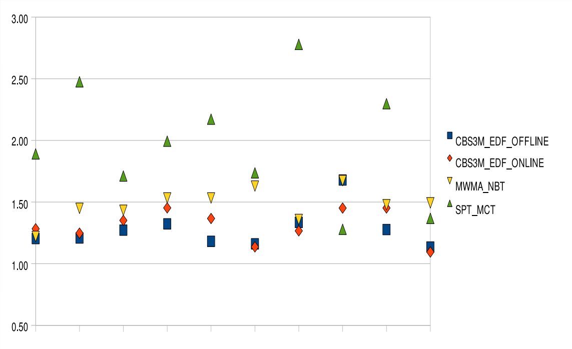
Experiments:

- GDSDMI cluster (8 workers)
- MPI communications
- Artificially slow-down communication and/or computations to emulate heterogeneity

Simulation:

- SimGrid simulator
- Two scenarios:
    1. simulate MPI experiments
    2. extensive simulations with larger applications

MPI experiments results

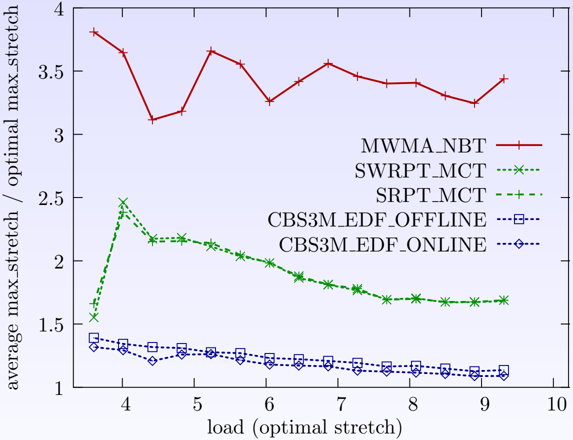


MPI experiments vs simulations

Comparison of relative max-stretch:

- average difference around 16%
- standard deviation of 14% (maximum of 72%).

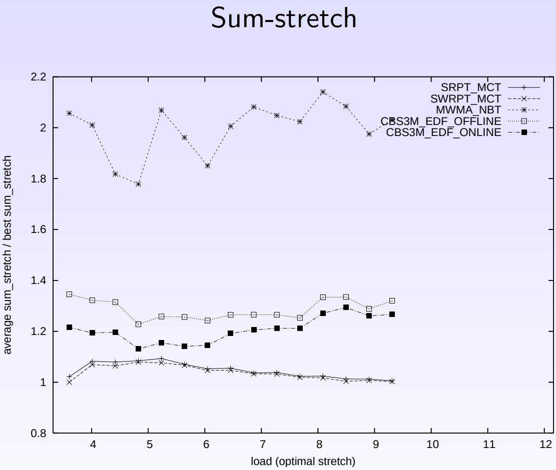
Simulations results – graph



Simulations results – table

Algorithm	minimum	average	(± stddev)	maximum	(fraction of best result)
FIFO_RR	4.550	16.689	(± 7.897)	62.6	(the best in 0.0 %)
FIFO_MCT	1.857	6.912	(± 2.404)	17.9	(the best in 0.0 %)
FIFO_DD	4.550	16.689	(± 7.897)	62.6	(the best in 0.0 %)
SPT_RR	1.348	4.274	(± 1.771)	13.8	(the best in 0.0 %)
SPT_MCT	1.007	1.928	(± 0.610)	5.99	(the best in 1.3 %)
SPT_DD	1.348	4.274	(± 1.771)	13.8	(the best in 0.0 %)
SRPT_RR	1.348	4.121	(± 1.737)	13.8	(the best in 0.0 %)
SRPT_MCT	1.007	1.861	(± 0.601)	6.87	(the best in 2.2 %)
SRPT_DD	1.348	4.121	(± 1.737)	13.8	(the best in 0.0 %)
SWRPT_RR	1.344	4.119	(± 1.739)	13.8	(the best in 0.0 %)
SWRPT_MCT	1.007	1.857	(± 0.601)	6.87	(the best in 1.9 %)
SWRPT_DD	1.344	4.119	(± 1.739)	13.8	(the best in 0.0 %)
MWMA_NBT	1.477	3.433	(± 1.044)	8.49	(the best in 0.0 %)
MWMA_MS	2.435	8.619	(± 2.420)	20.4	(the best in 0.0 %)
CBS3M_FIFO_ONLINE	1.003	1.322	(± 0.208)	2.83	(the best in 6.9 %)
CBS3M_EDF_ONLINE	**1.003**	**1.163**	**(± 0.118)**	**1.93**	**(the best in 64.0 %)**
CBS3M_FIFO_OFFLINE	1.022	1.379	(± 0.276)	3.74	(the best in 3.8 %)
CBS3M_EDF_OFFLINE	**1.011**	**1.213**	**(± 0.125)**	**2.06**	**(the best in 26.2 %)**

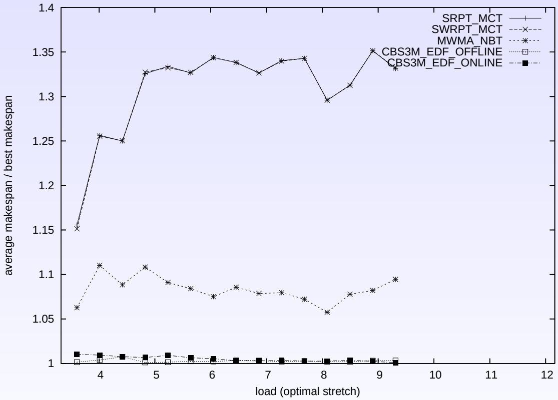
Simulations results – other metrics



- best strategy: **SWRPT** (known to be optimal)
- CBSSM within 30-40%

Simulations results – other metrics

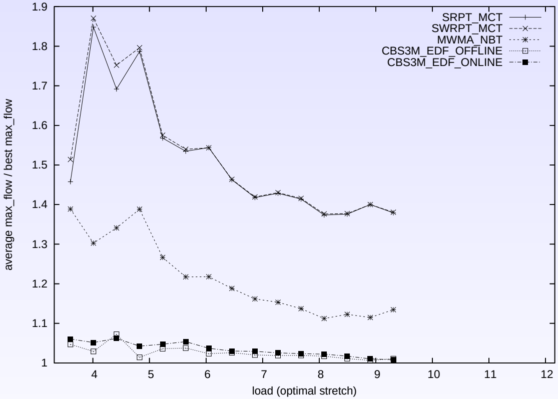
Makespan



- best strategy: **CBS3M**

Simulations results – other metrics

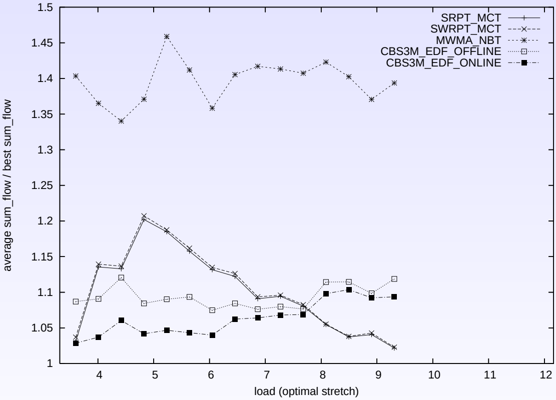
Max-flow



- best strategy: **CBS3M**

Simulations results – other metrics

Sum-flow



- best strategy: **CBS3M/ SWRPT**

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Conclusion

- Key points:
  - Realistic platform model
  - Optimal offline algorithm
  - Efficient online algorithm based on offline study
- Extensions:
  - Extend the simulation to larger platform
  - Bi-criteria

This work will be presented in APDCM (workshop of IPDPS'08).

Positive values

- **Non-negative throughputs.**

$$\forall 1 \leq u \leq p, \forall 1 \leq k \leq n, \forall 1 \leq j \leq 2n-1,$$
$$\rho_{M\to u}^{(k)}(t_j, t_{j+1}) \geq 0 \text{ and } \rho_u^{(k)}(t_j, t_{j+1}) \geq 0. \quad (1)$$

- **Non-negative buffers.**

$$\forall\ 1 \leq k \leq n, \forall 1 \leq u \leq p, \forall 1 \leq j \leq 2n,$$
$$B_u^{(k)}(t_j) \geq 0. \quad (2)$$

Physical constraints

- **Bounded link capacity.**

$$\forall 1 \leq j \leq 2n-1, \forall 1 \leq u \leq p,$$

$$\sum_{k=1}^{n} \rho_{M\to u}^{(k)}(t_j, t_{j+1}) \frac{\delta^{(k)}}{b_u} \leq 1. \quad (3)$$

- **Limited sending capacity of master.**

$$\forall 1 \leq j \leq 2n-1,$$

$$\sum_{u=1}^{p} \sum_{k=1}^{n} \rho_{M\to u}^{(k)}(t_j, t_{j+1}) \frac{\delta^{(k)}}{\mathcal{B}} \leq 1. \quad (4)$$

- **Bounded computing capacity.**

$$\forall 1 \leq j \leq 2n-1, \forall 1 \leq u \leq p,$$

$$\sum_{k=1}^{n} \rho_{u}^{(k)}(t_j, t_{j+1}) \frac{w^{(k)}}{s_u^{(k)}} \leq 1. \quad (5)$$

Buffer constraints

- **Buffer initialization.**

$$\forall\ 1 \leq k \leq n, \forall 1 \leq u \leq p, \qquad B_u^{(k)}(r^{(k)}) = 0. \quad (6)$$

- **Emptying Buffer.**

$$\forall\ 1 \leq k \leq n, \forall 1 \leq u \leq p, \qquad B_u^{(k)}(d^{(k)}) = 0. \quad (7)$$

- **Bounded size**

$$\forall 1 \leq u \leq p, \forall 1 \leq j \leq 2n, \qquad \sum_{k=1}^{n} B_u^{(k)}(t_j)\delta^{(k)} \leq M_u. \quad (8)$$

Tasks constraints

- **Task conservation.**

$$\forall\ 1 \le k \le n, \forall 1 \le j \le 2n-1, \forall 1 \le u \le p,$$
$$B_u^{(k)}(t_{j+1}) = B_u^{(k)}(t_j) + \left(\rho_{M\to u}^{(k)}(t_j, t_{j+1}) - \rho_u^{(k)}(t_j, t_{j+1})\right) \times \left(t_{j+1} - t_j\right). \tag{9}$$

- **Total number of tasks.**

$$\forall\ 1 \le k \le n,$$
$$\sum_{\substack{1 \le j \le 2n-1 \\ t_j\ \ge\ r^{(k)} \\ t_{j+1}\ \le\ d^{(k)}}} \sum_{u=1}^{p} \rho_{M\to u}^{(k)}(t_j, t_{j+1}) \times (t_{j+1} - t_j) = \Pi^{(k)}. \tag{10}$$

Polyhedron

$$\begin{cases} \text{find } \rho_{M \to u}^{(k)}(t_j, t_{j+1}), \rho_u^{(k)}(t_j, t_{j+1}), \\ \forall k, u, j \text{ such that } 1 \leq k \leq n, 1 \leq u \leq p, 1 \leq j \leq 2n-1 \\ \text{under the constraints } (1), (2), (3), (4), (5), (6), (7), (8), (9) \text{ and } (10) \end{cases} \quad (K)$$

A given max-stretch $\mathcal{S}'$ is achievable if and only if the Polyhedron (K) is not empty

In practice, we add a fictitious linear objective function. Back

New constraints

- **Bounded link capacity.**

$$\forall 1 \leq j \leq 2n-1, \forall 1 \leq u \leq p,$$
$$\sum_{k=1}^{n} A^{(k)}_{M \to u}(t_j, t_{j+1}) \frac{\delta^{(k)}}{b_u} \leq \ (\alpha_{j+1} - \alpha_j)\mathcal{S} + (\beta_{j+1} - \beta_j)$$

Back

New constraints

- **Bounded link capacity.**
- **Limited sending capacity of master.**

$$\forall 1 \leq j \leq 2n-1,$$

$$\sum_{u=1}^{p}\sum_{k=1}^{n} A_{M \to u}^{(k)}(t_j, t_{j+1})\delta^{(k)} \leq \mathcal{B} \times \left((\alpha_{j+1} - \alpha_j)\mathcal{S} + (\beta_{j+1} - \beta_j)\right)$$

Back

New constraints

- **Bounded link capacity.**
- **Limited sending capacity of master.**
- **Bounded computing capacity.**

$$\forall 1 \leq j \leq 2n-1, \forall 1 \leq u \leq p,$$
$$\sum_{k=1}^{n} A_u^{(k)}(t_j, t_{j+1}) \frac{w^{(k)}}{s_u^{(k)}} \;\leq\; (\alpha_{j+1} - \alpha_j)\mathcal{S} + (\beta_{j+1} - \beta_j)$$

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$$\forall\ 1 \le k \le n,$$

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◂ Back

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- **Limited sending capacity of master.**
- **Bounded computing capacity.**
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- **Task conservation.**

$$\forall\ 1 \le k \le n, \forall 1 \le j \le 2n-1, \forall 1 \le u \le p,$$
$$B_u^{(k)}(t_{j+1}) = B_u^{(k)}(t_j) + A_{M \to u}^{(k)}(t_j, t_{j+1}) - A_u^{(k)}(t_j, t_{j+1})$$

Back

New constraints

- Bounded link capacity.
- Limited sending capacity of master.
- Bounded computing capacity.
- Total number of tasks.
- Task conservation.
- Non-negative buffer.
- Buffer initialization.
- Emptying Buffer.

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New constraints

- **Bounded link capacity.**
- **Limited sending capacity of master.**
- **Bounded computing capacity.**
- **Total number of tasks.**
- **Task conservation.**
- **Non-negative buffer.**
- **Buffer initialization.**
- **Emptying Buffer.**
- **Bounded stretch**

$$\mathcal{S}_a \leq \mathcal{S} \leq \mathcal{S}_b \tag{11}$$

Back