Toward a Fully Decentralized Algorithm for Multiple Bag-of-tasks Application Scheduling on Grids

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Aussois Workshop

Outline

Motivation

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Designing a *Fair* and Distributed scheduling algorithm for this framework.

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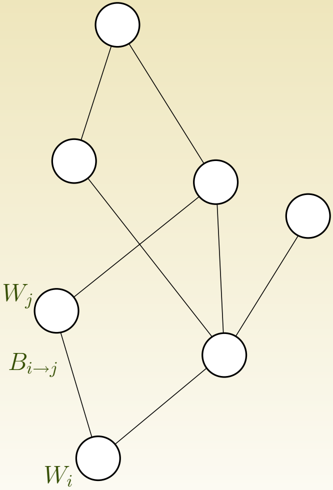
Platform Model



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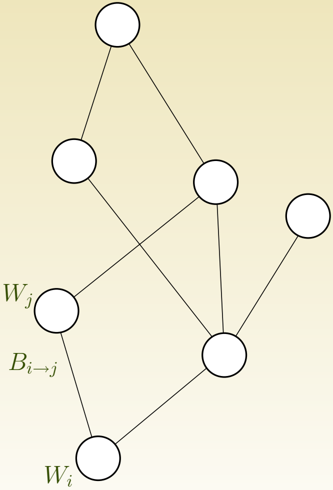

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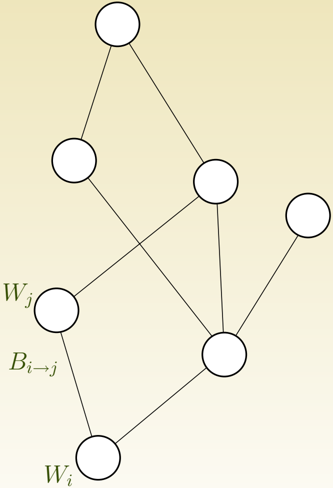

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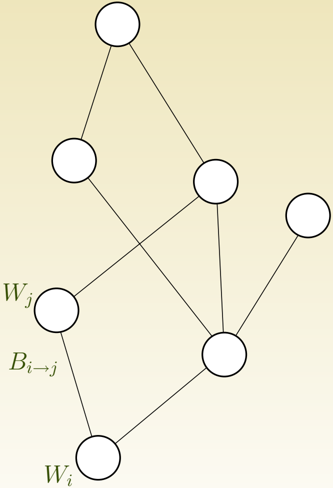

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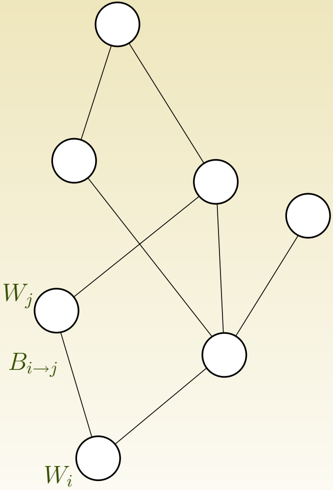

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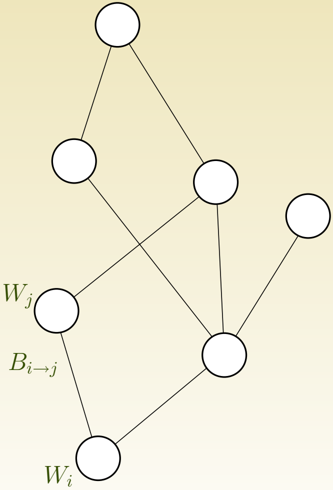

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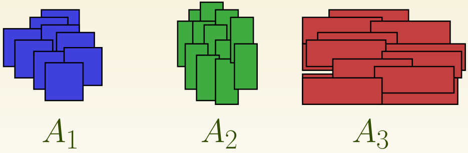
- Multi-port communication model.

Application Model

Multiple applications:

- A set A of K applications $A_1, \ldots, A_K$.



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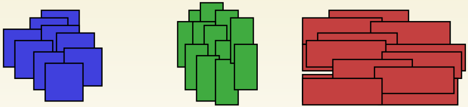

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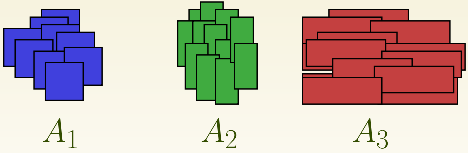
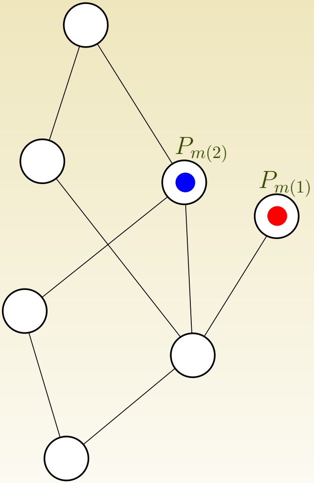
- Different communication and computation demands for different applications.



Hierarchical Deployment



- Each application originates from a master node $P_{m(k)}$ that initially holds all the input data necessary for each application A_k.

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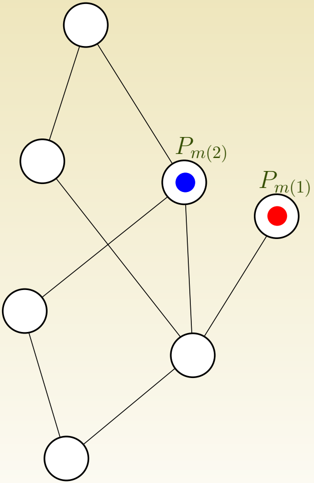

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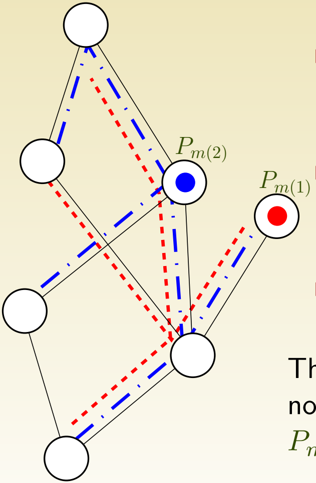

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- Communication are only required outwards from the master nodes: the amount of data returned by the worker is negligible.
- Each application A_k is deployed on the platform as a tree.

Therefore if an application k wants to use a node P_n, all its data will use a single path from $P_{m(k)}$ to P_n denoted by $(P_{m(k)} \rightsquigarrow P_n)$.

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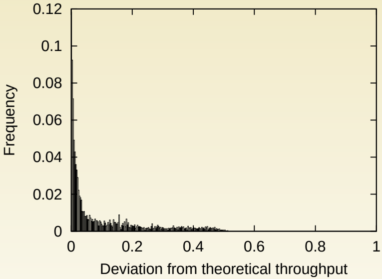
Theorem 1.

From "feasible" $\varrho_{n,k}$, it is possible to build an optimal periodic infinite schedule (i.r. whose steady-state rates are exactly the $\varrho_{n,k}$). Such a schedule is asymptotically optimal for the makespan.

The rates $\varrho_{n,k}$ are sufficient to help simple demand-driven scheduling algorithms.

- Dispatch incoming tasks of type k to the queues (n,k) with "proportion" $\varrho_{n,k}$.
- Request tasks from your father when incomming queue sizes get below a fixed threshold.



$$\text{Deviation} = \frac{\varrho_k^{(th)} - \varrho_k^{(exp)}}{\varrho_k^{(th)}}$$

$\rightsquigarrow$ We can focus on finding the $\varrho_{n,k}$.

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1. Framework
2. Lagrangian Optimization
3. Simulations: Early Results

Lagrangian Optimization: Basics

- Designed to solve non linear optimization problems:
  - Let $\alpha \rightarrow f(\alpha)$ be a function to maximize.
  - Let $(C_i(\alpha) \geqslant 0)_{i\in[1..n]}$ be a set of n constraints.
  - We wish to solve:

$$(P) \begin{cases} \text{maximize } f(\alpha) \\ \forall i \in [1..n], C_i(\alpha) \geqslant 0, \text{ and } \alpha \geqslant 0 \end{cases}$$

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- The dual functional: $d(\lambda) = \max_{\alpha\geqslant 0} \mathcal{L}(\alpha,\lambda)$.
- Under some weak hypothesis, solving (P) is equivalent to solve the dual problem:

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So what?..

- Two coupled problems with simple constraints.
- The structure of constraints is transposed to (D) and a gradient descent algorithm is a natural way to solve these two problems.
- This technique has been used successfully for network resource sharing [Kelly.98], TCP analysis [Low.03], flow control in multi-path network [Hang.et.al.03], ...

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Trying to use Lagrangian optimization

- What does the Lagrangian function look like ?

$$\mathcal{L}(\alpha, \lambda, \mu) = \sum_{k \in K} \log \left(\sum_i \varrho_{i,k} \right) + \sum_i \lambda_i \left(W_i - \sum_k \varrho_{i,k} w_k \right) + \sum_{(P_i \to P_j)} \mu_{i,j} \left(B_{i,j} - \sum_k \sum_{\substack{n \text{ such that} \\ (P_i \to P_j) \in (P_{m(k)} \leadsto P_n)}} \varrho_{n,k} b_k \right)$$

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- Remember, we want to compute $\min_{\lambda,\mu\geqslant 0} \max_{\varrho\geqslant 0} \mathcal{L}(\alpha,\lambda,\mu)$. We can solve this problem by simply doing a "alternate" gradient descent (I'm skipping a few details here to keep it simple and just present the general idea):

$$\begin{cases} \varrho_{i,k} & \leftarrow \varrho_{i,k} + \gamma \frac{\partial \mathcal{L}}{\partial \varrho_{i,k}} \\ \lambda_i & \leftarrow \lambda_i - \gamma \frac{\partial \mathcal{L}}{\partial \lambda_i} \\ \mu_{i,j} & \leftarrow \mu_{i,j} - \gamma \frac{\partial \mathcal{L}}{\partial \mu_{i,j}} \end{cases}$$

Toward a Distributed Algorithm...

- $\varrho_{i,k}$ is "private" to the agent of application k running on node i.
- λ_i is attached to node i and $\mu_{i,j}$ is attached to $(P_i \to P_j)$. λ_i and $\mu_{i,j}$ are called shadow variables or shadow prices. They can naturally thought of as the *price to pay to use the corresponding resource*.
- A gradient descent algorithm on the primal-dual problem can thus be seen as a bargain between applications and resources.
- We need to find an efficient way to implement this bargain, i.e., to compute the update. To this end, the following quantities are useful and easy to compute via recursive propagation:

$$\begin{cases} \sigma_k^n = \displaystyle\sum_{p \text{ such that } n \in (P_{m(k)} \rightsquigarrow P_p)} \varrho_{p,k} & \left\{ \begin{array}{l} \text{aggregate throughput} \\ \text{of a subtree.} \end{array} \right. \\ \eta_k^n = \displaystyle\sum_{(P_i \to P_j) \in (P_{m(k)} \rightsquigarrow P_n)} \mu_{i,j} & \left\{ \begin{array}{l} \text{aggregate communication} \\ \text{price} \end{array} \right. \end{cases}$$

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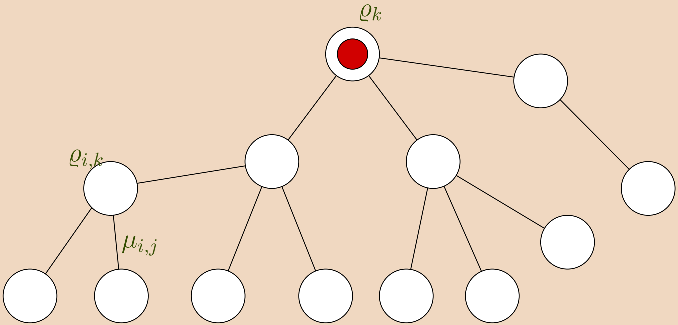

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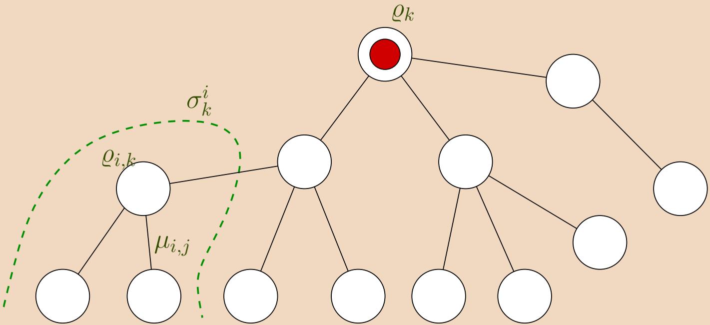

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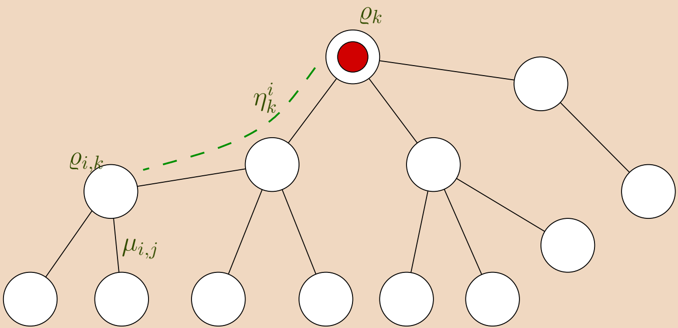

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Toward a Distributed Algorithm...

Prices and rates can thus be propagated and aggregated to perform the following updates:

$$p_k^i(t+1) \leftarrow b_k \eta_k^i(t) + w_k \lambda_i(t)$$

$$\varrho_k(t+1) \leftarrow \sigma_k^{m(k)}(t+1)$$

$$\varrho_{i,k}(t+1) \leftarrow \left[\varrho_{i,k}(t) + \gamma_\varrho(U_k'(\varrho_k(t)) - p_k^i(t))\right]^+$$

$$\lambda_i(t+1) \leftarrow \left[\lambda_i(t) + \gamma_\lambda \left(\sum_k w_k \varrho_{i,k} - W_i\right)\right]^+$$

$$\mu_{i,j}(t+1) \leftarrow \left[\mu_{i,j}(t) + \gamma_\mu \left(\sum_k b_k \sigma_k^i - B_{i,j}\right)\right]^+$$

- This algorithm is *fully distributed* and converges to the *optimal* solution provided a good choice of γ_ϱ, γ_λ and γ_μ is done.
- This algorithm *seamlessly adapts* to application/node arrival and to load variations.

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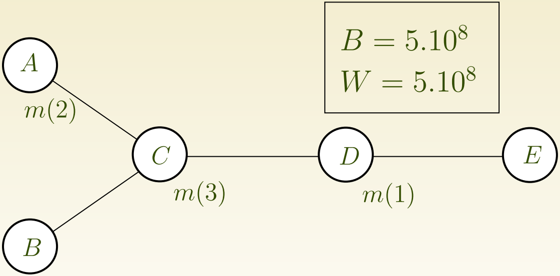
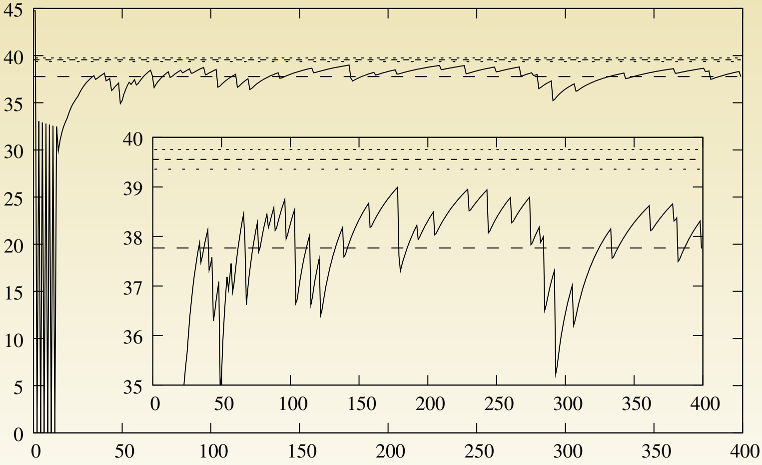
- Checking the correctness of the results using semi-definite programming.
- Very simple platform and applications:

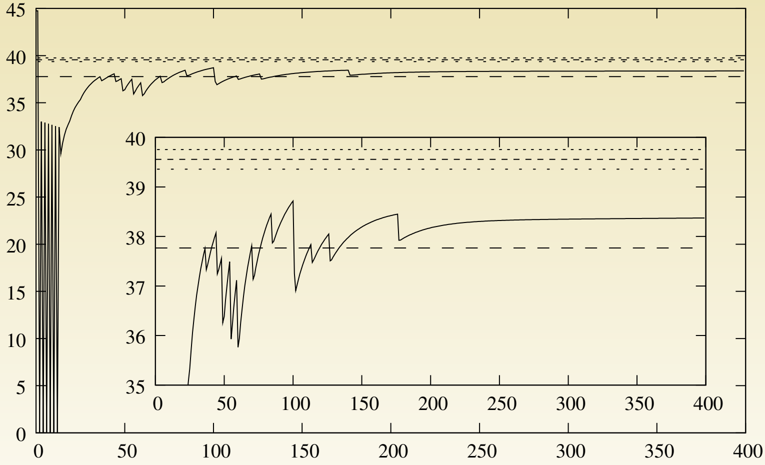


We used three kinds of applications of respective (b, w): $(1000, 5000)$, $(2000, 800)$, and $(1500, 1500)$.

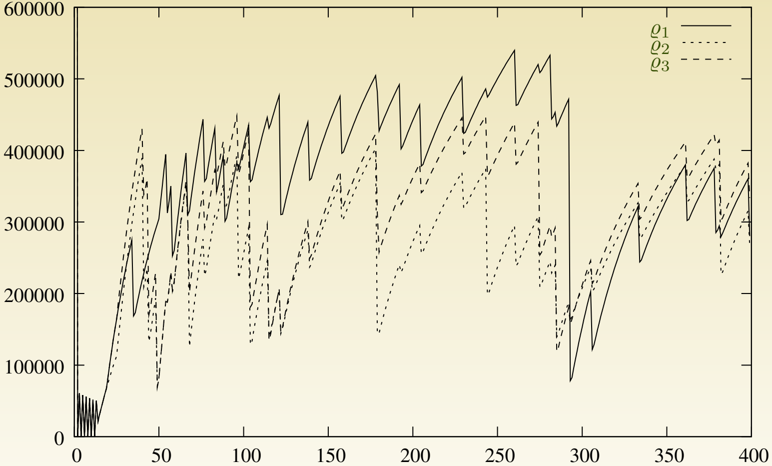
Basic Version of the Algorithm



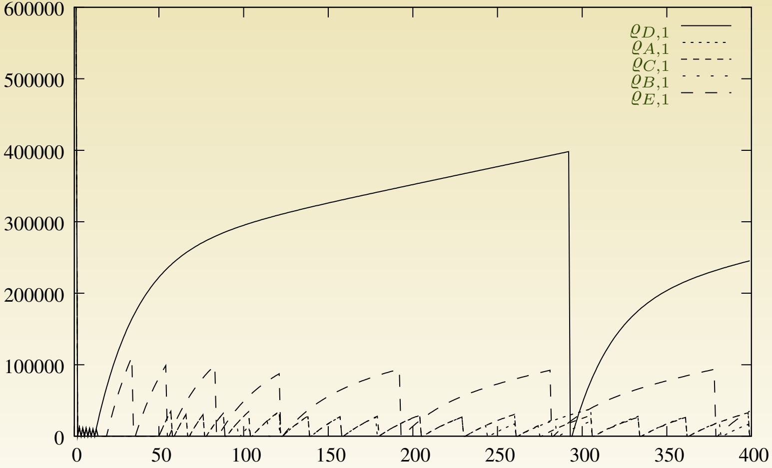
Objective function $\sum_k \log \varrho_k$: numerical instabilities and global inefficiencies.



Objective function $\sum_k \log \varrho_k$: using a smaller steps $\gamma_\varrho \rightsquigarrow$ no more instability but slow convergence.

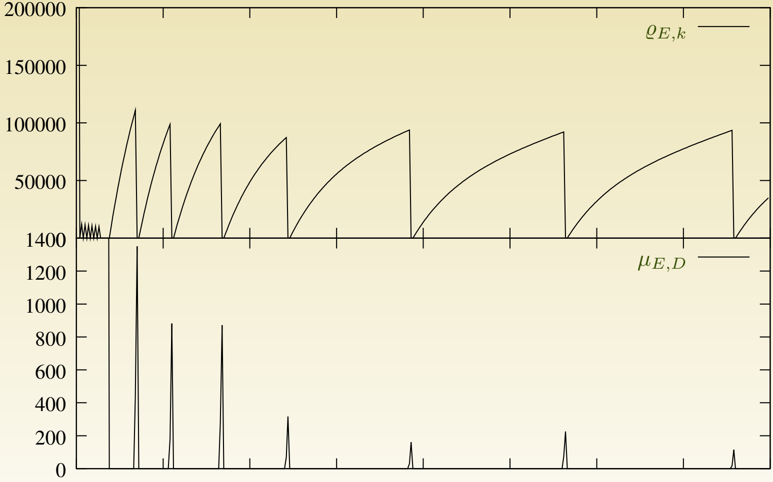


Throughput of each of the three applications: between two iterations, a decrease or increase of magnitude five or more can happen!



Detailing the rates for application 1.

Basic Version of the Algorithm



Correlation between the rate of an application on a given node and the price it experiences.

Scaling!

The original update equation for ϱ is:

$$\varrho_{i,k}(t+1) \leftarrow \left[\varrho_{i,k}(t) + \gamma_\varrho \left(\frac{1}{\varrho_k(t)} - p_k^i(t)\right)\right]^+$$

A small value of ϱ leads to huge updates and thus to severe oscillations.

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$$\varrho_{i,k}(t+1) \leftarrow \left[\varrho_{i,k}(t) + \gamma_\varrho \left(1 - \varrho_k(t).p_k^i(t)\right)\right]^+ .$$

Unfortunately, it does not help (the previous experiments actually use this normalized update). It merely avoids division by 0 but is insufficient to damp oscillations.

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Updating ϱ has an impact on the prices λ and μ, which in turn impact on the ϱ's update. The second update of ϱ should have the same order of magnitude (or be smaller) as the first one to avoid numerical instabilities that prevent convergence of the algorithm.

Scaling Again!

Assume that we have reached the equilibrium. Then increase λ_i by $\Delta\lambda_i$. Then:

$$\Delta\varrho_{i,k} = -\gamma_\varrho^{(2)} w_k \Delta\lambda_i \varrho_k.$$

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In turn, such a variation incurs a variation of λ_i:

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Thus, the solution of our gradient is stable only if

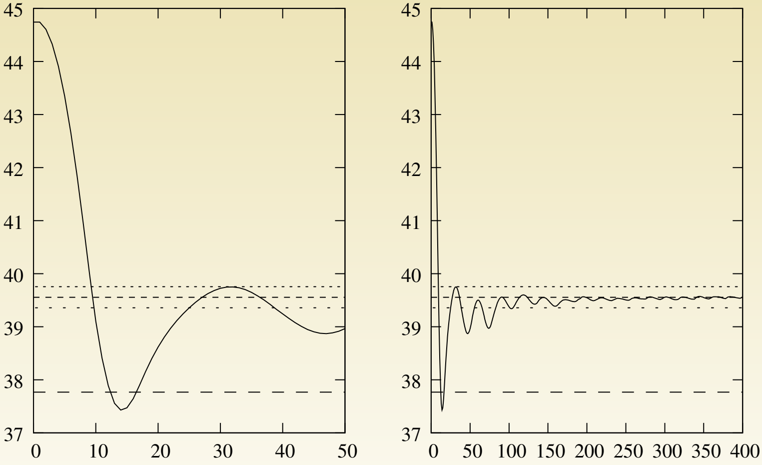
$$\sum_k \gamma_\lambda . \gamma_\varrho^{(2)} w_k^2 \varrho_k < 1.$$

Therefore, λ's update should be replaced by

$$\lambda_i(t+1) \leftarrow \left[\lambda_i(t) + \gamma_\lambda \frac{\sum_k w_k \varrho_{i,k} - W_i}{\sum_k w_k^2 \varrho_k} \right]^+$$

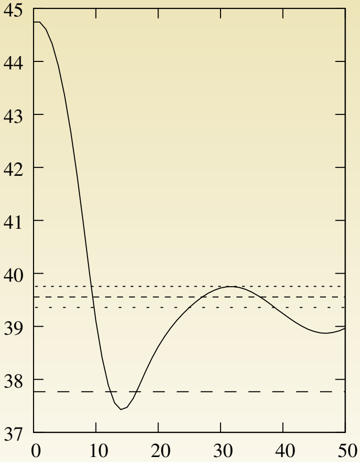
It doesn't hurt and similar scaling can be done for the μ's.

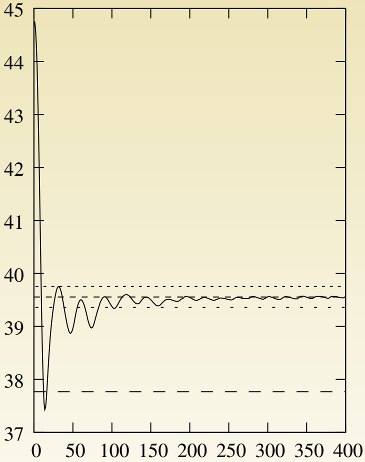
Scaled Version of the Algorithm



The oscillations, due to a really badly chosen initialization value quickly vanish (left graph).

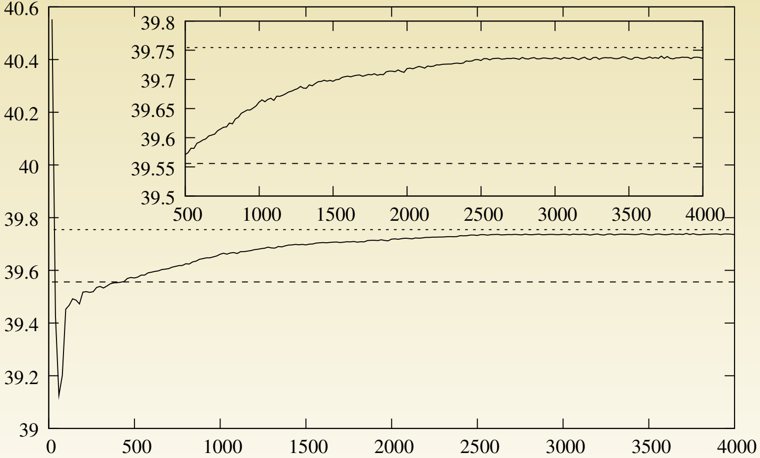
Scaled Version of the Algorithm





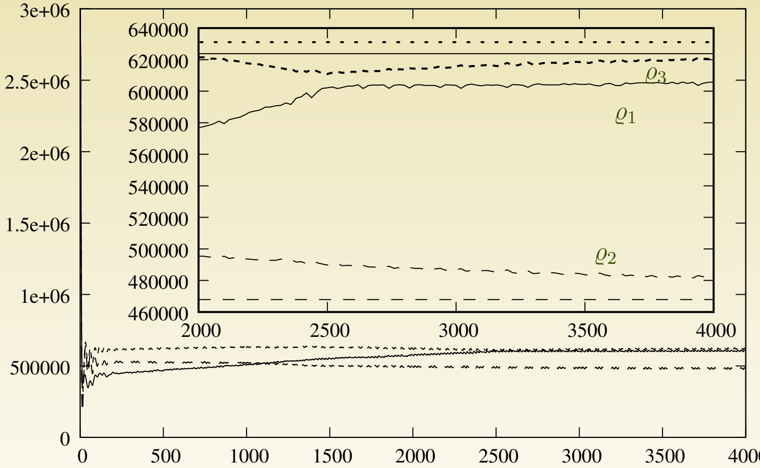
The algorithm almost instantly reaches a decent value (5% of the optimal value after 17 iterations), and relatively quickly to a good value (1% of the optimal value after 83 iterations) (right plot).

Scaled Version of the Algorithm



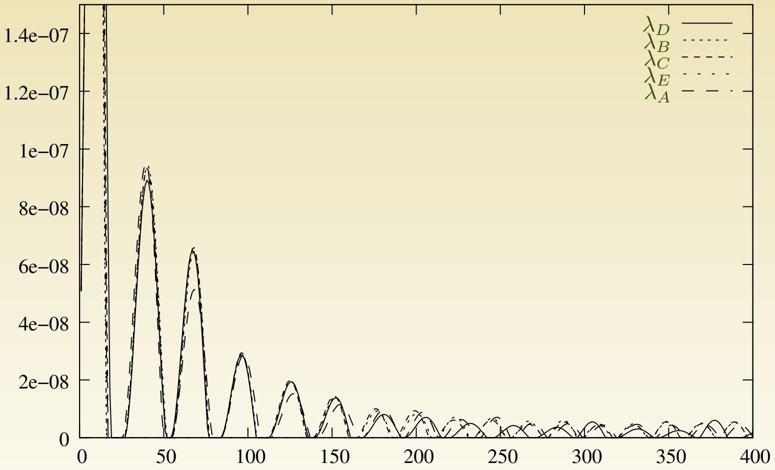
High number of iterations: after 498 iterations, the performance remains higher than 99.5% of the optimal and still further increase with the number of iterations.

Scaled Version of the Algorithm



Convergence of ϱ_i with $i = 1..3$: no more oscillations occur. The throughput of each application slowly converges to their "optimal" values.

Scaled Version of the Algorithm



Prices evolve smoothly. As the number of iterations increase, they converge to their optimal value while remaining positive, meaning that the resources they refer to is neither under utilized nor overloaded.

Conclusion

- Not enough time to present related work but this approach is very inspired by Low's work [Hang.et.al.03] on flow control in multi-path network.
- The setting (BoT applications, grids) is different though and new problem arise.
- The resulting algorithms are different (few sources and many sinks here).
- The convergence issue is mainly due to the fact that the resource usage is not homogeneous (each application has its own w_k and b_k). The previous scaling is effective and easy to implement.

Future Work

- There may be situations where the previous scaling may not be sufficient though. When the optimal throughput of the applications do not have the same order of magnitude, it may be necessary for each application to have its own step size $\gamma_{\varrho}^{(2)}$. We may need to find auto-scaling for the ϱ's update as well.
- The present convergence study is rather limited in term of scalability...
- We target grid or desktop-grid-like platforms. What if the number of application has the same order of magnitude as the number of participants in the system (like in a peer-to-peer system)? Would the steady-state approach still make sense (completion-based metrics like stretch...)?
- We rely on steady-state. How does such a system react to high churn?

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