Loris Marchal

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Algorithms for pipelined Multicast

- Groupe de Travail Graal, 2 mars 2004 -

Introduction

- multicast = broadcast to a strict subset of targets in the platform nodes
- lots of studies of multicast:
  - Steiner trees (minimize the cost of a single multicast tree, NP hard problem)
  - for a wide variety of particular architectures and technologies (wormhole-routed, wireless, ad-hoc, optical netwworks)
- focus on pipelined multicast:
  maximize the throughput of a series of multicast
- same framework as in previous work for other collective communications:

  $$\text{scatter, reduce, broadcast} \Rightarrow \begin{cases} \text{optimal throughput,} \\ \text{asymptotically optimal algorithms} \end{cases}$$

- surprisingly, multicast turned out to be more challenging

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1. Framework and Model
2. Some theoretical results: Multicast is hard !
3. Heuristics based on linear programming
4. Tree-based heuristics
5. Experimental results
6. Further on complexity study ?

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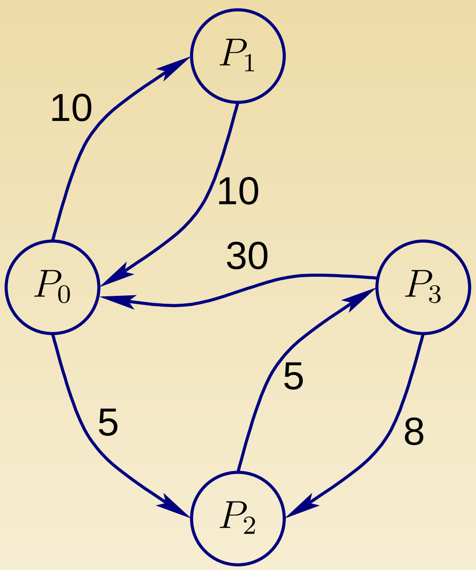
Framework and Model

Framework of the platform

- $G = (V, E, c)$
- Let $P_1, P_2, \ldots, P_n$ be the n processors
- P_{source}: processor initiating the multicast
- $\mathcal{P}_{\text{target}}$: set of target processors
- $(j, k) \in E$: communication link between P_i and P_j
- $c(j, k)$: time to transfer one unit message from P_j to P_k
- one-port for incoming communications
- one-port for outgoing communications



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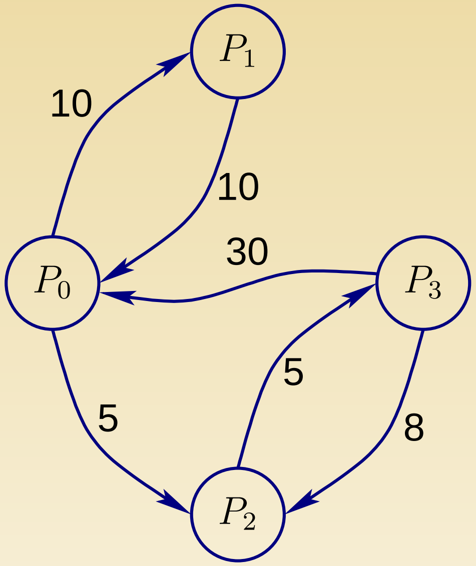

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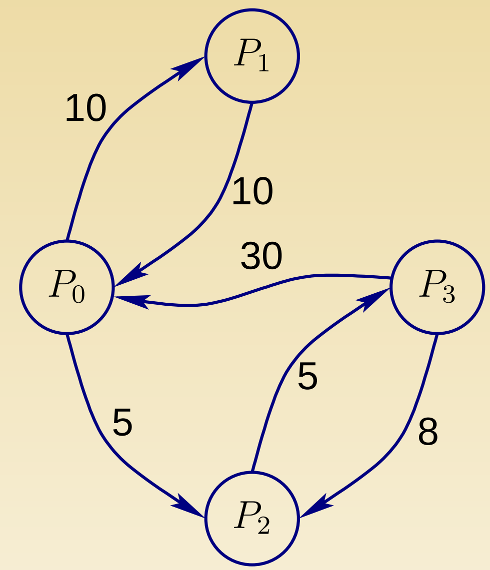

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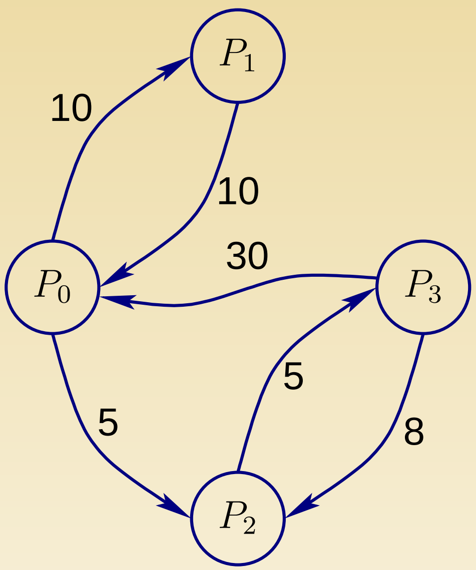

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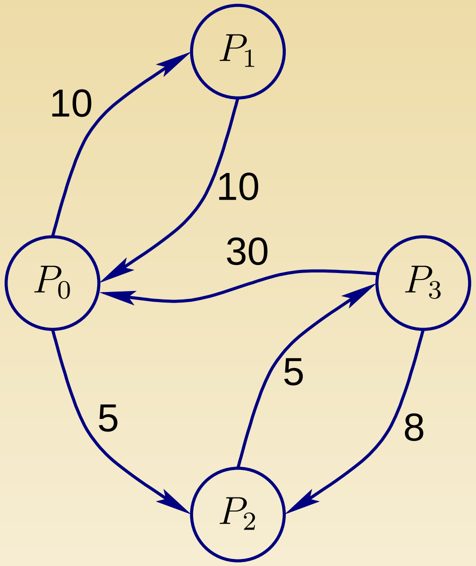

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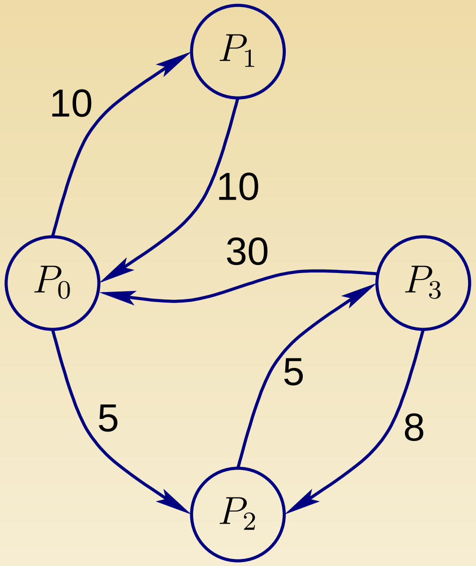

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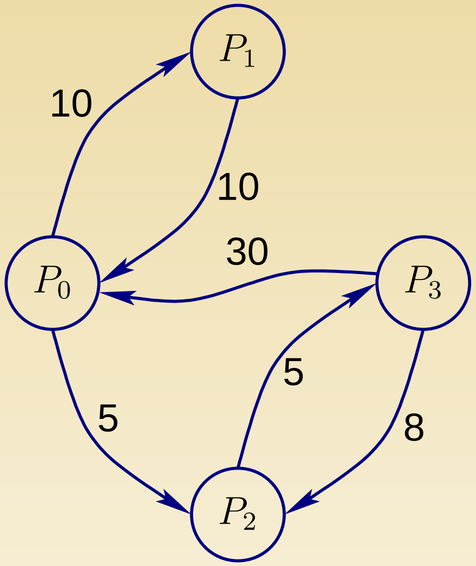

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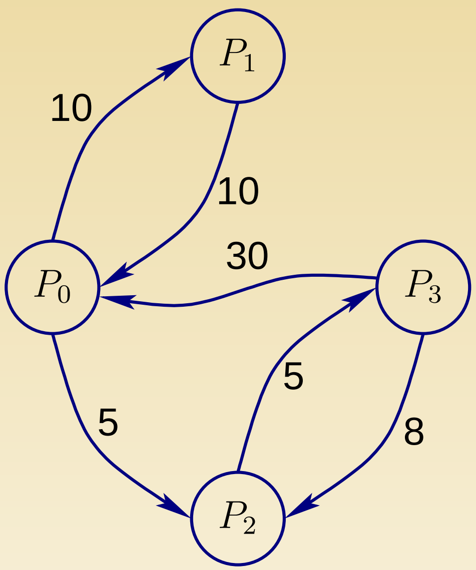

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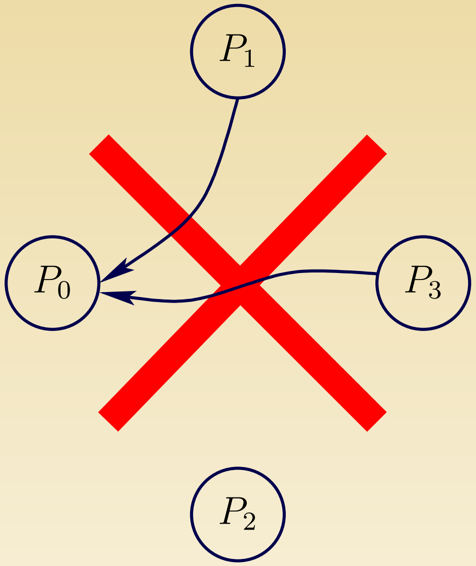

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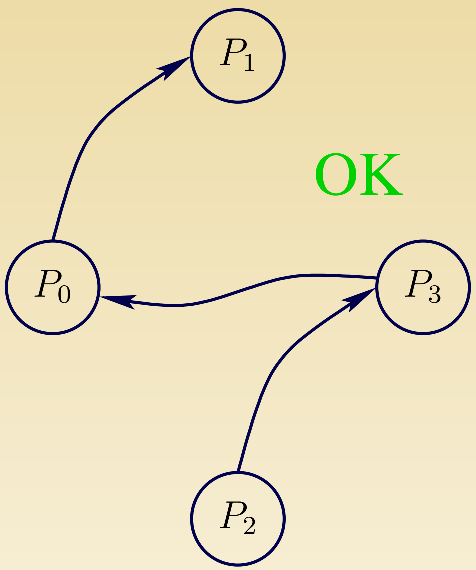

Steady-state approach

- Focus on the average quantities, over one time-unit:
  - $t_{i,j}$ average occupation time of edge $P_i \to P_j$
  - $n_{i,j}$ average number of messages going through edge $P_i \to P_j$
  - $x_{i,j}^k$ average number of messages targeting P_k going through $P_i \to P_j$
- Some relations between these quantities:
  - clearly, $0 \leq t_{i,j} \leq 1$
  - a node P_i has a limited sending capacity (one port):
    $$\sum_{j \text{ such that } (i,j)\in E} t_{i,j} \leq 1$$
  - a node P_i has a limited receiving capacity (one port):
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  - link between number of messages and occupation time:
    $$t_{i,j} = n_{i,j} \cdot c_{i,j}$$

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If we consider the message sent to the target node P_k:

- P_{source} sends TP such messages:

$$\sum_{j/(\text{source},j)\in E} x^k_{\text{source},j} = TP$$

- P_k receives TP such messages:

$$\sum_{j/(j,k)\in E} x^k_{j,k} = TP$$

- On another node P_i, these messages are conserved:

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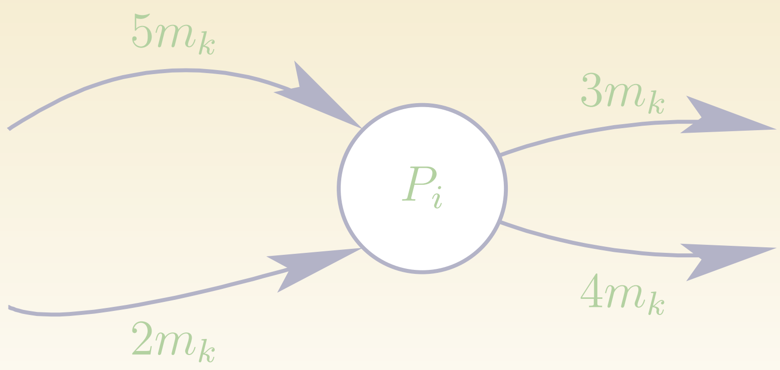

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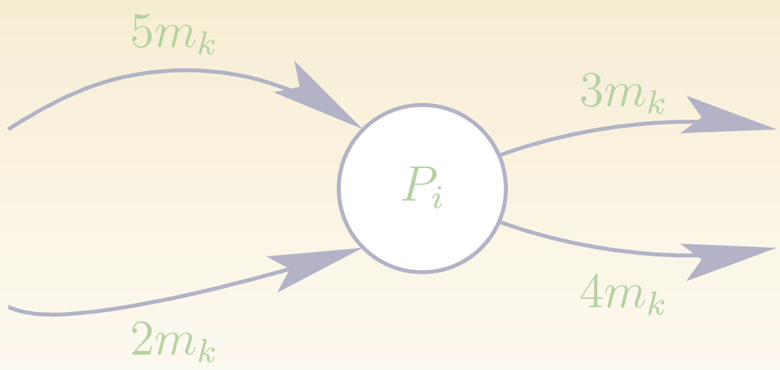

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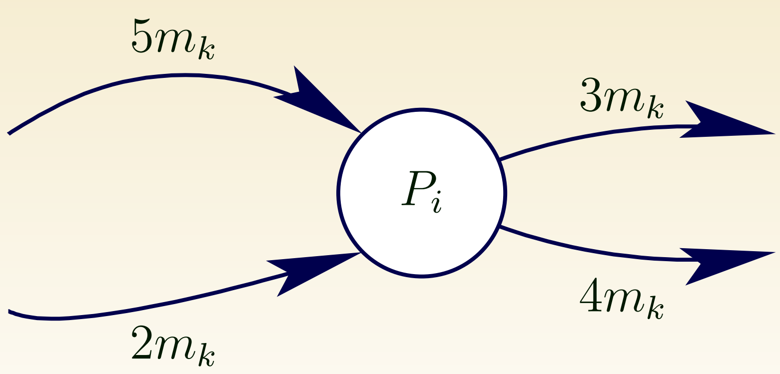

Introduction of linear programming

- Objective function: maximize throughput TP
- Which relation between $x_{i,j}^k$ and $n_{i,j}$?

1. Pessimistic view: $n_{i,j} = \sum_k x_{i,j}^k$
   - may be too pessimistic since $x_{i,j}^{k_1}$ and $x_{i,j}^{k_2}$ denotes the same messages
   - as if the source sends differents messages to each target
   - provides a lower bound on the throughput
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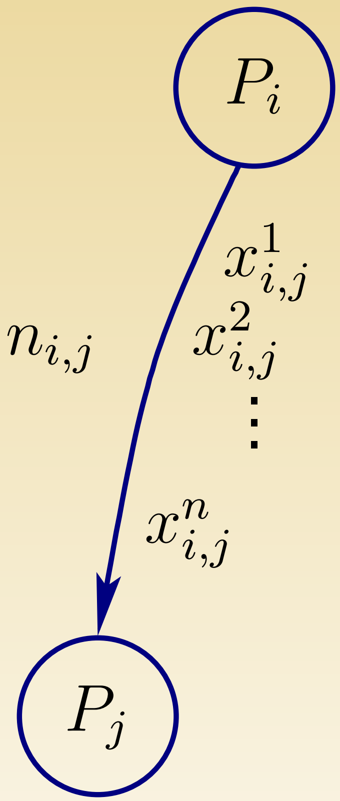

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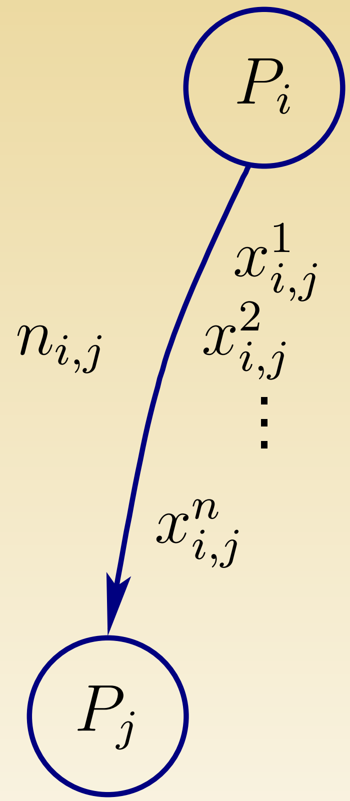

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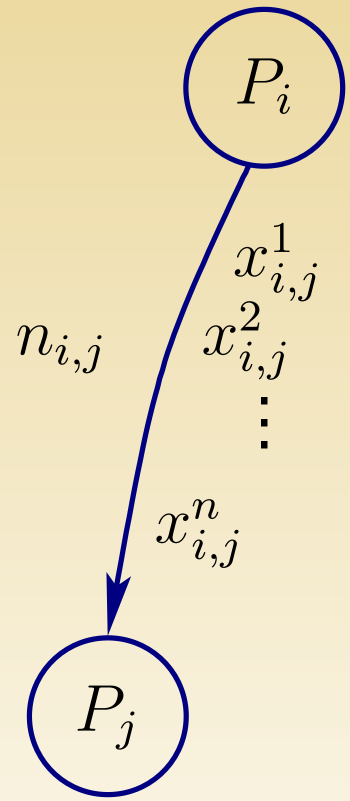

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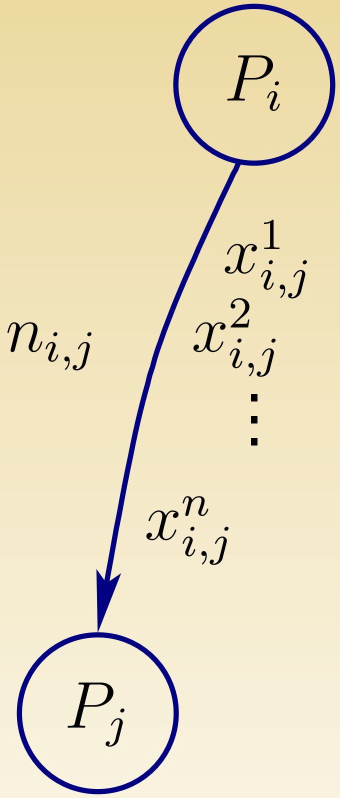
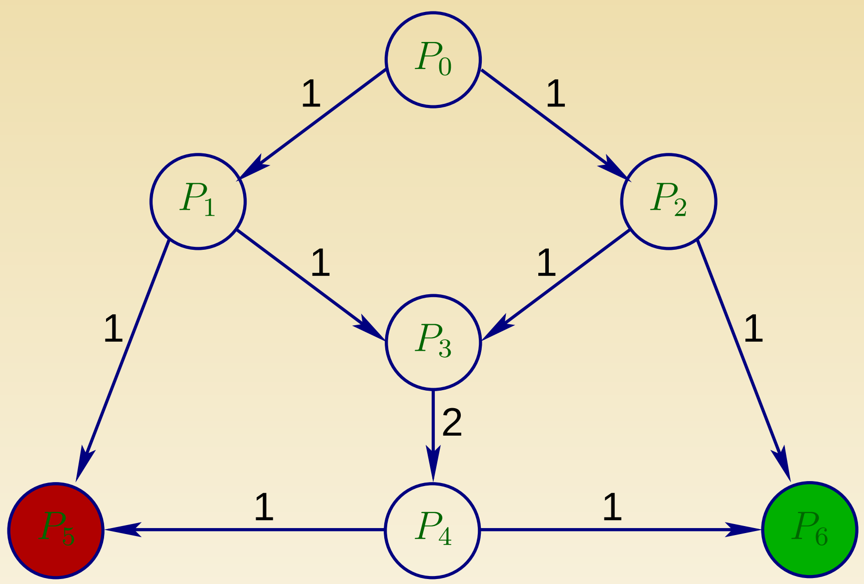


Why solution 2 is not always feasible

Consider the following platform, where the multicast set consists in the colored nodes:



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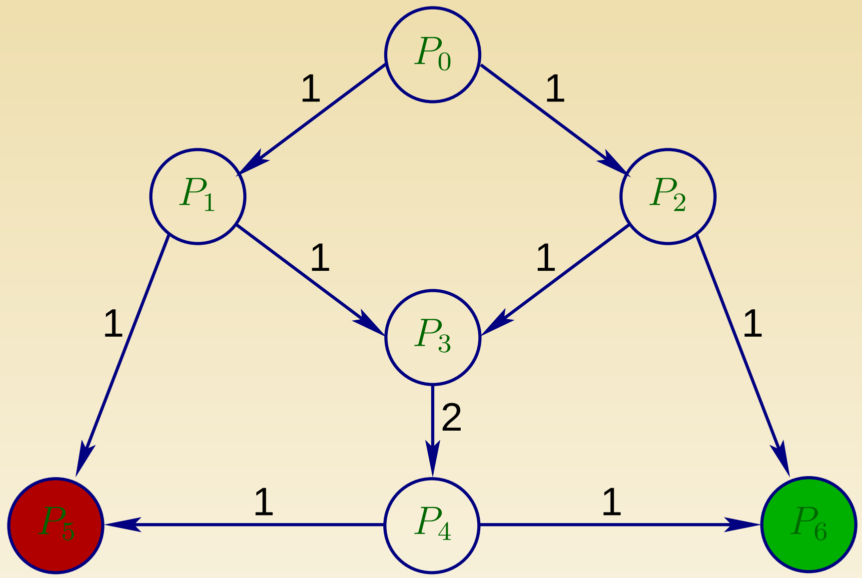

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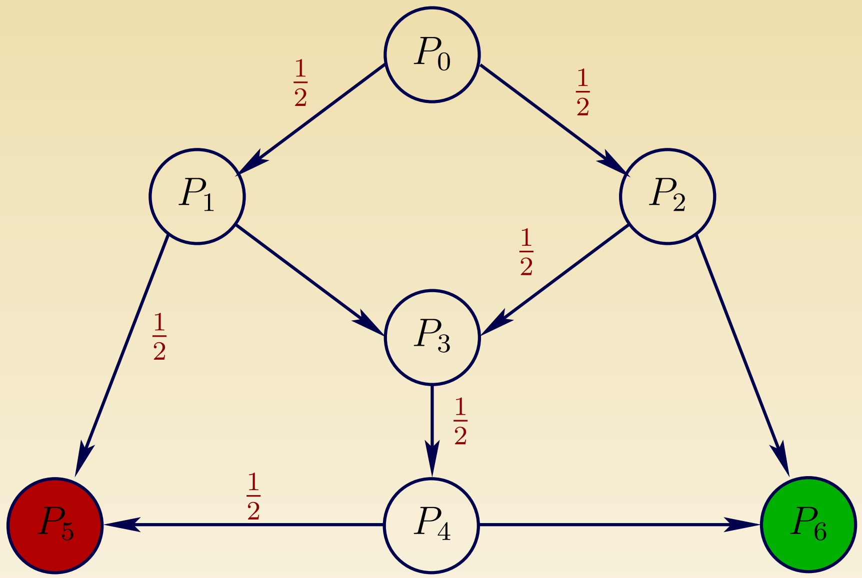

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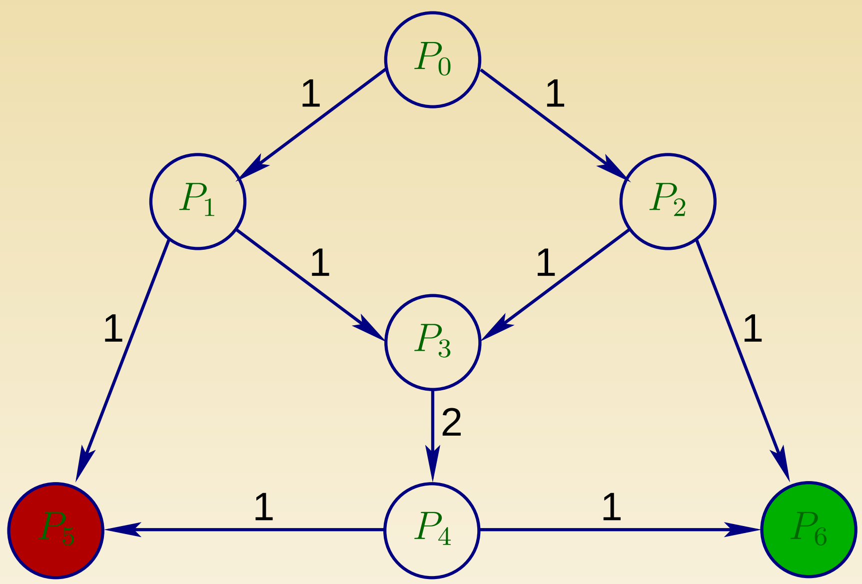

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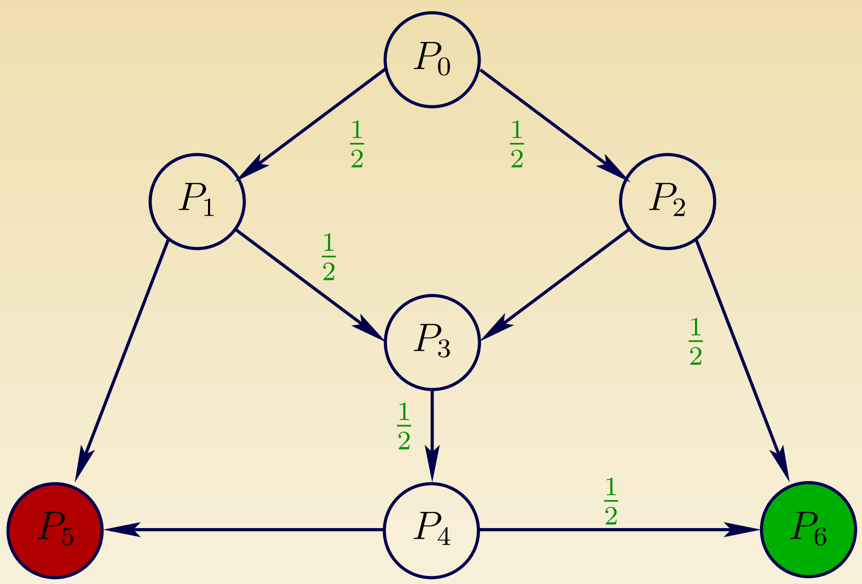

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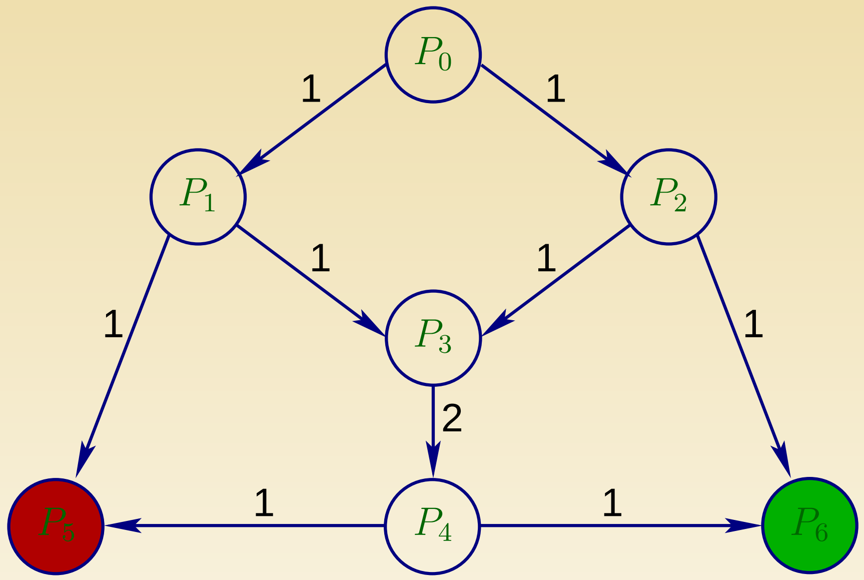

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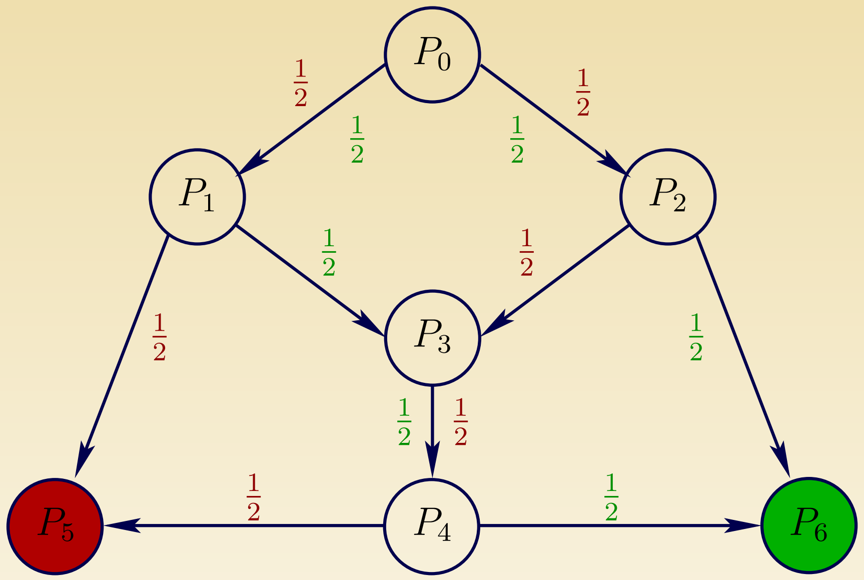
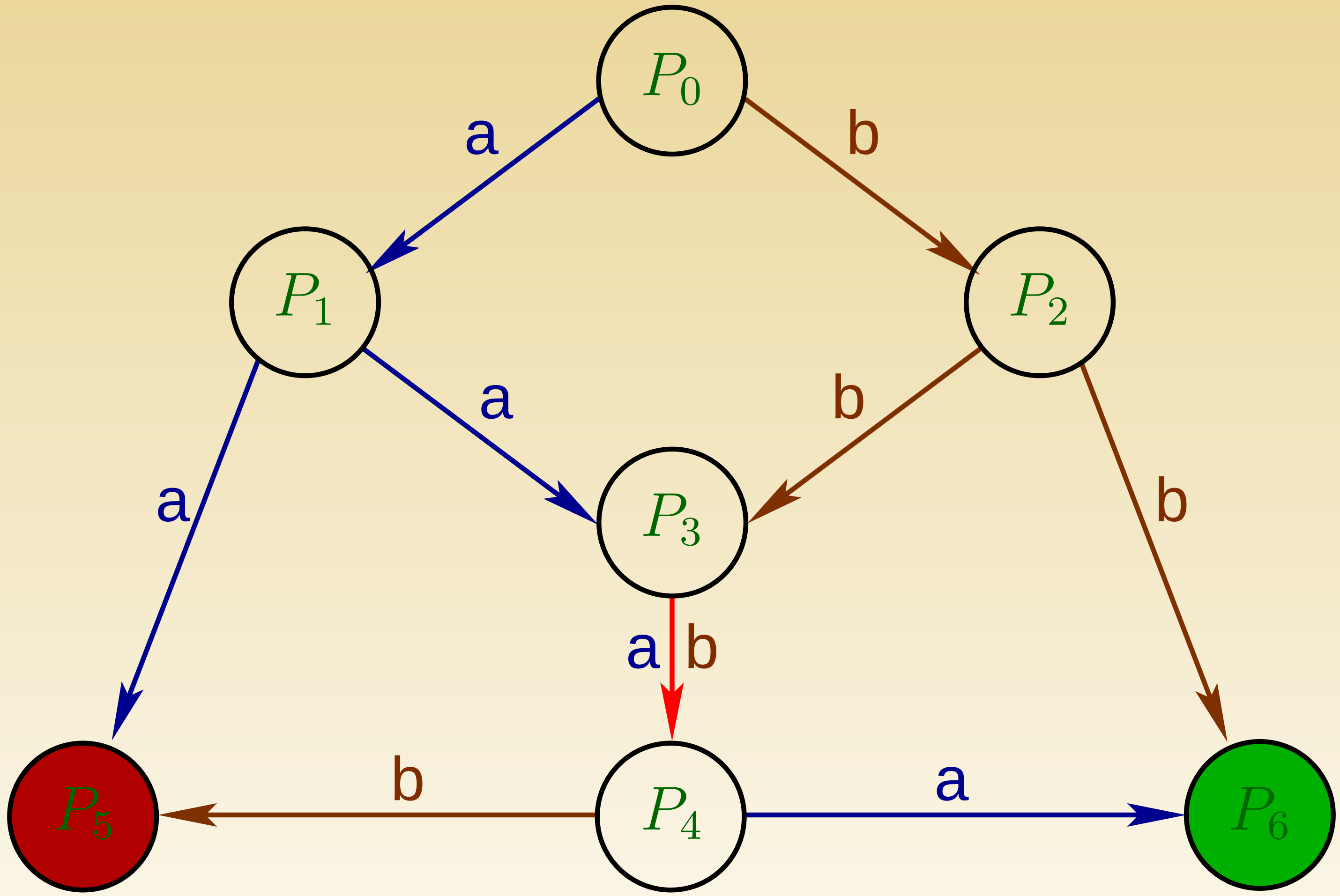


Why solution 2 is not always feasible

Nevertheless, the obtained throughput is not feasible:



Theoretical results

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- Neither linear programs can compute the true optimal throughput
- **theorem :** computing the best throughput for a multicast operation on a given platform is NP-hard
- **definition** multicast tree:
  a tree, rooted in P_{source}, spanning all the nodes of $\mathcal{P}_{\text{target}}$, and made up valid edges from E

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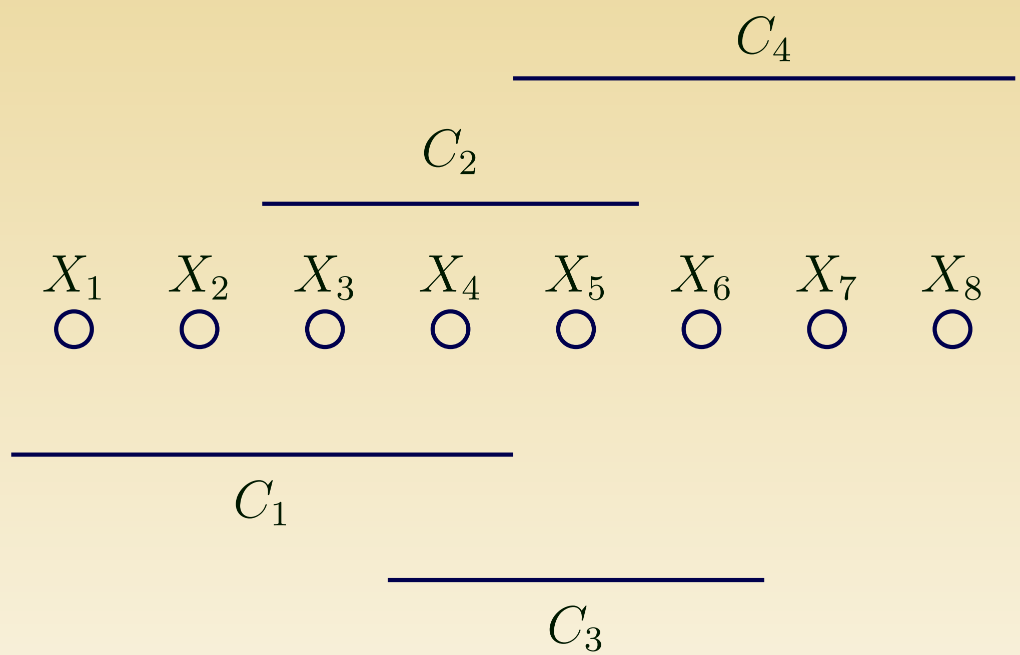
- reduction from MINIMUM-SET-COVER:
  C is a collection of subsets of X, a B is a bound
  does C contain a cover of X of size at most B ?



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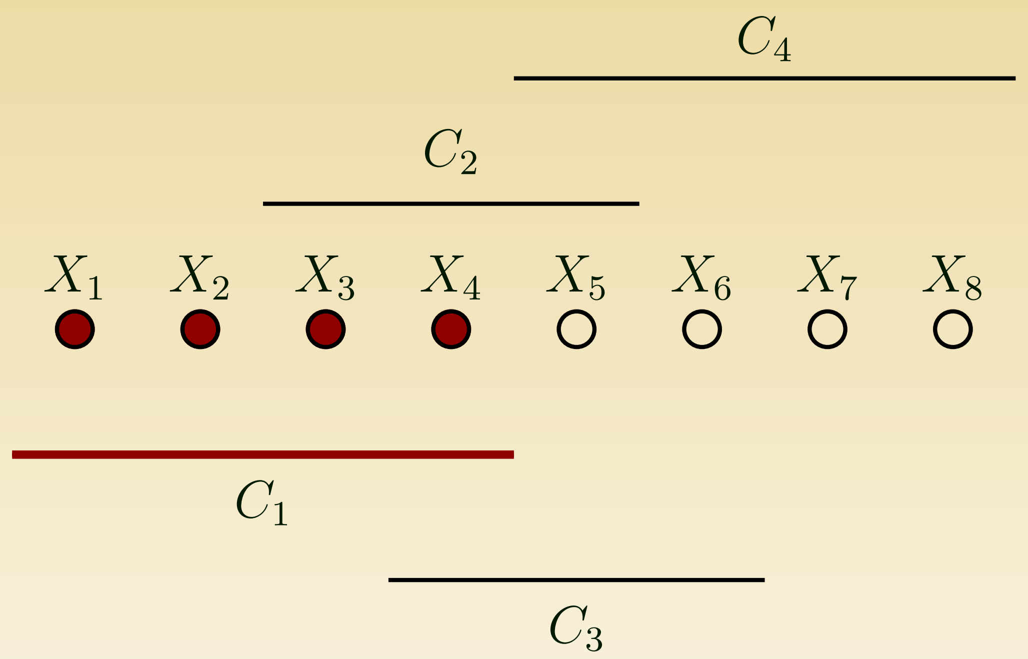

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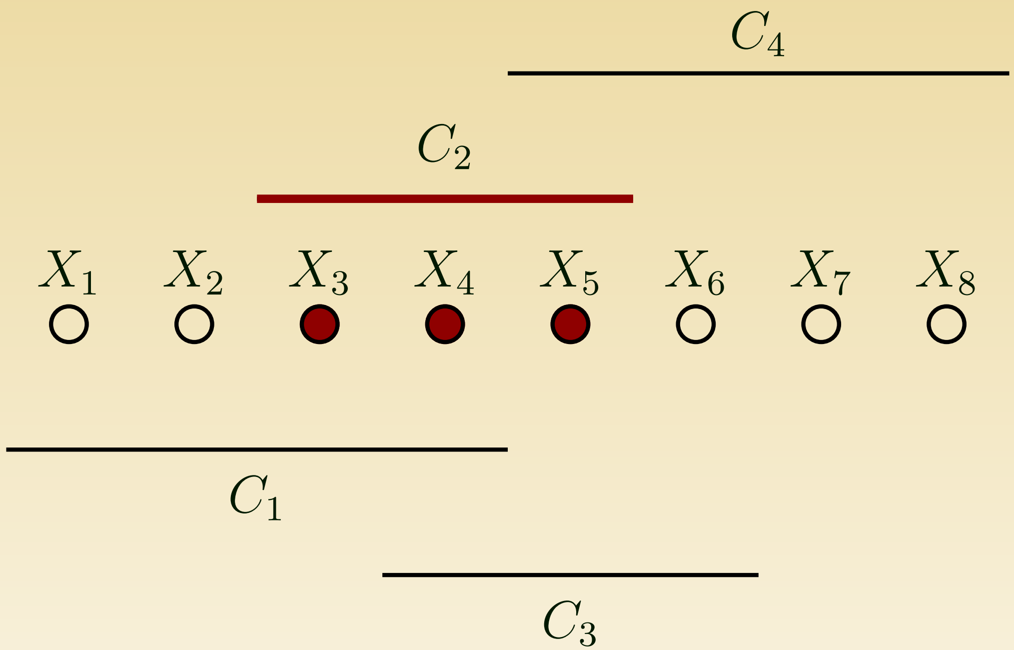

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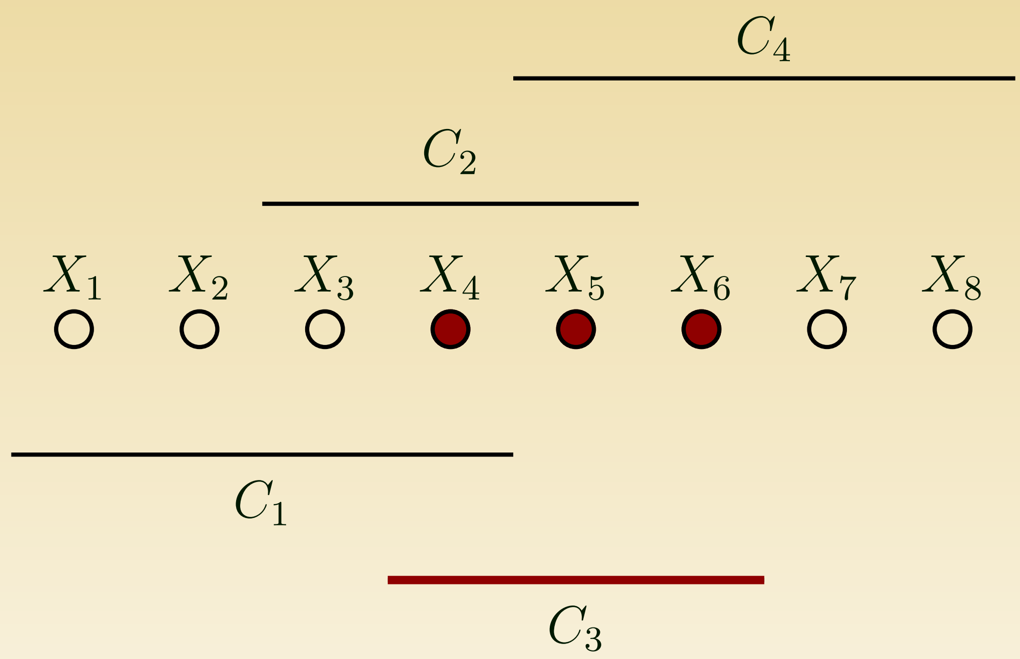

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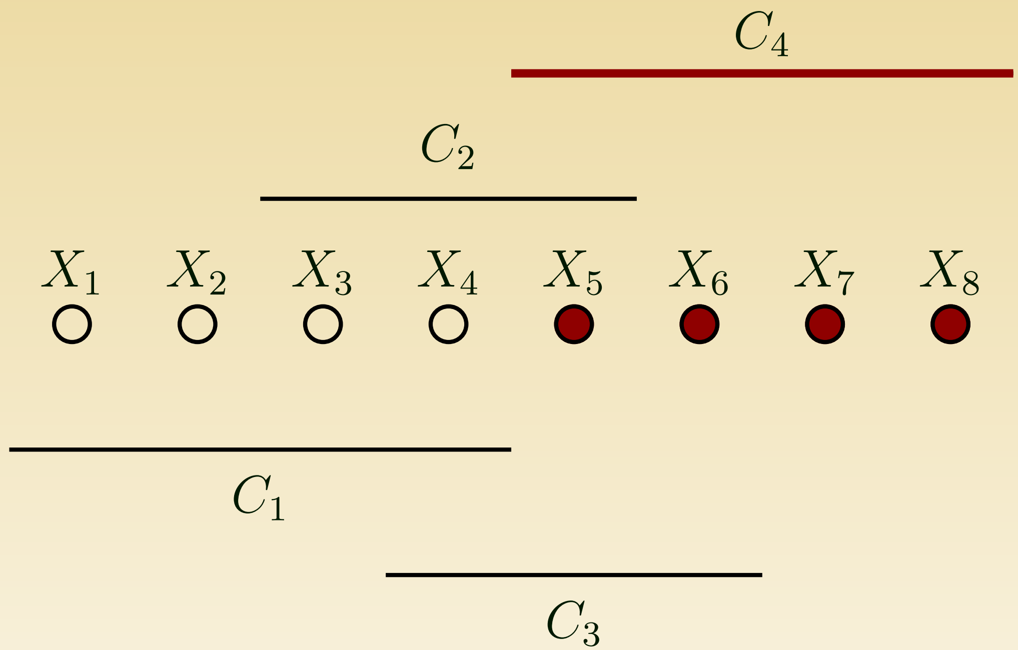

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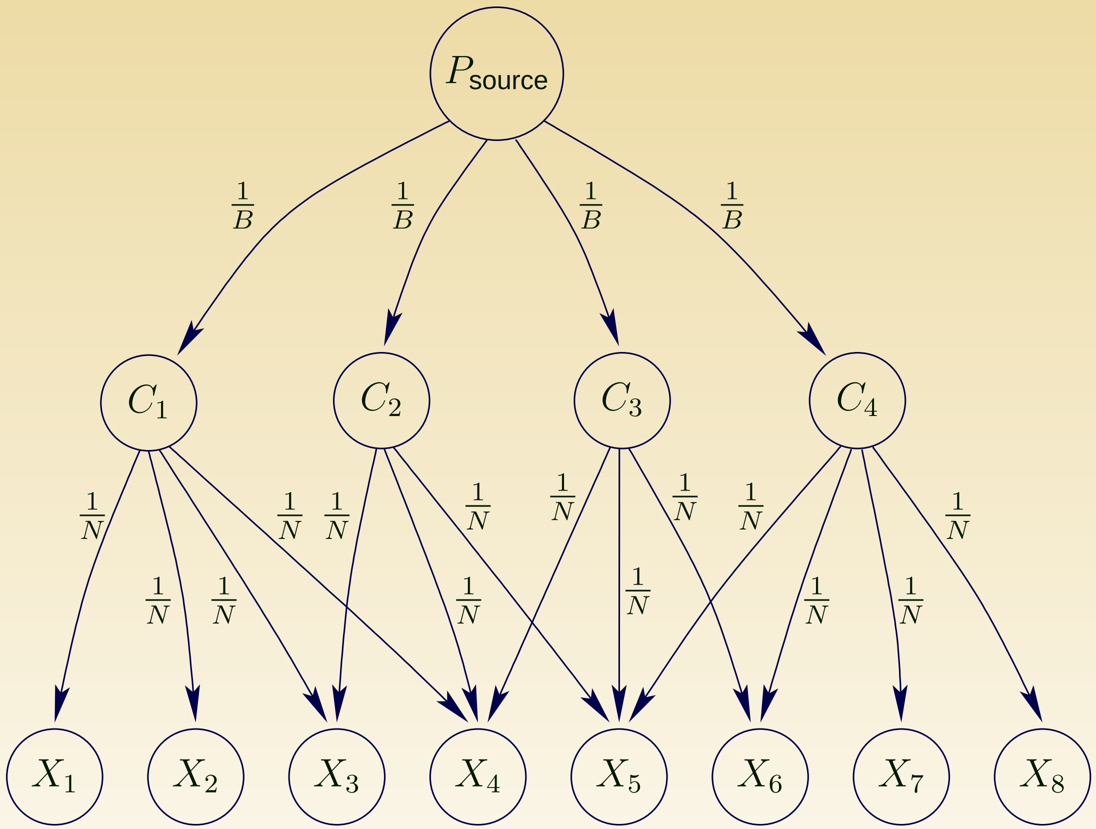

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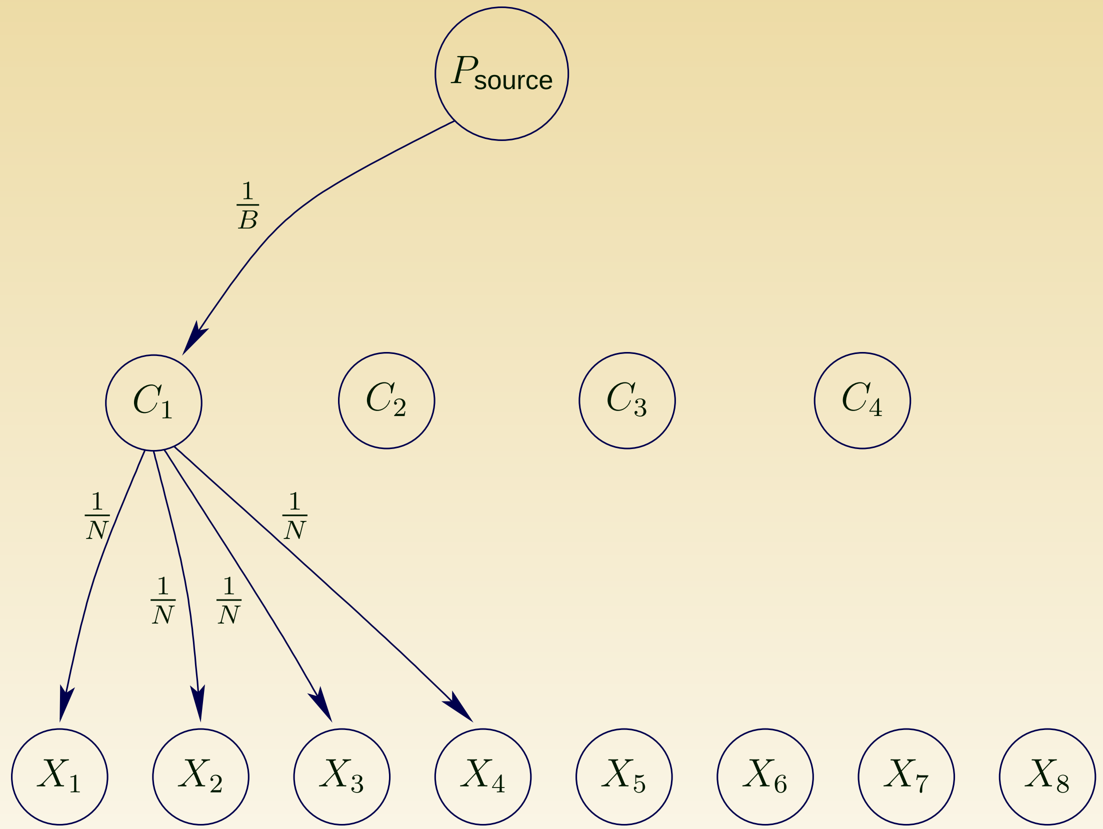

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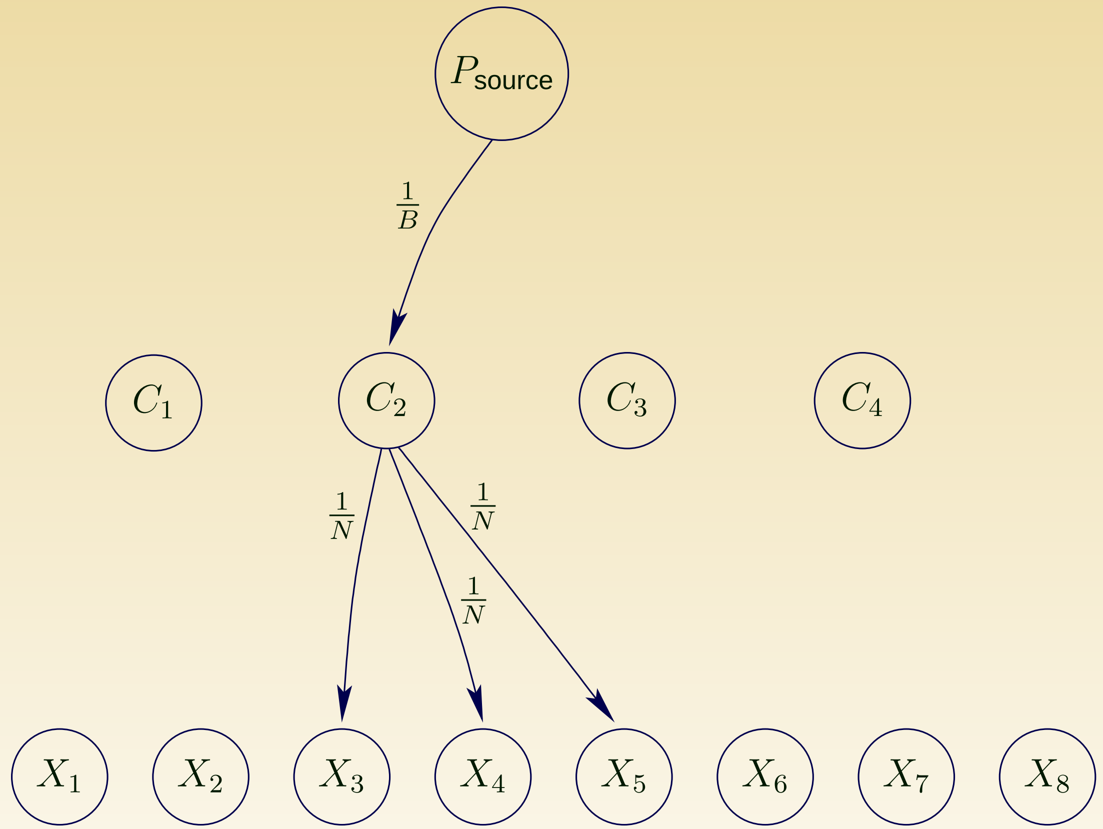

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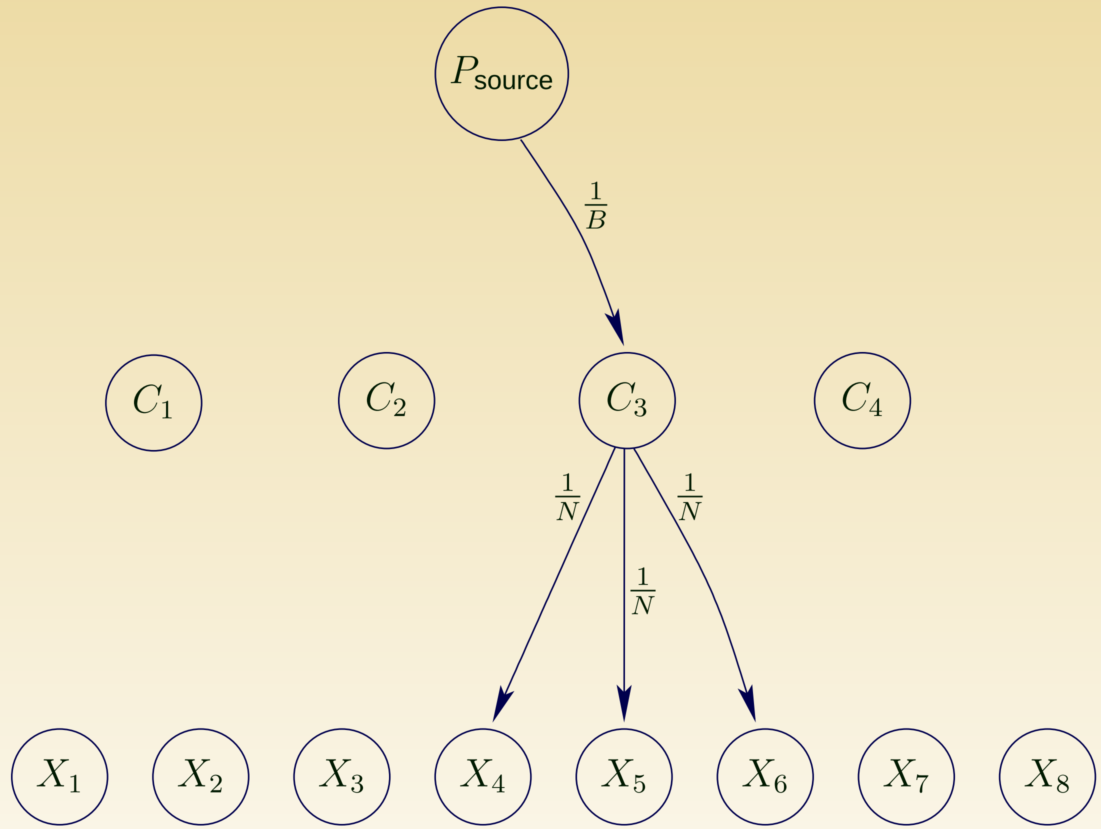

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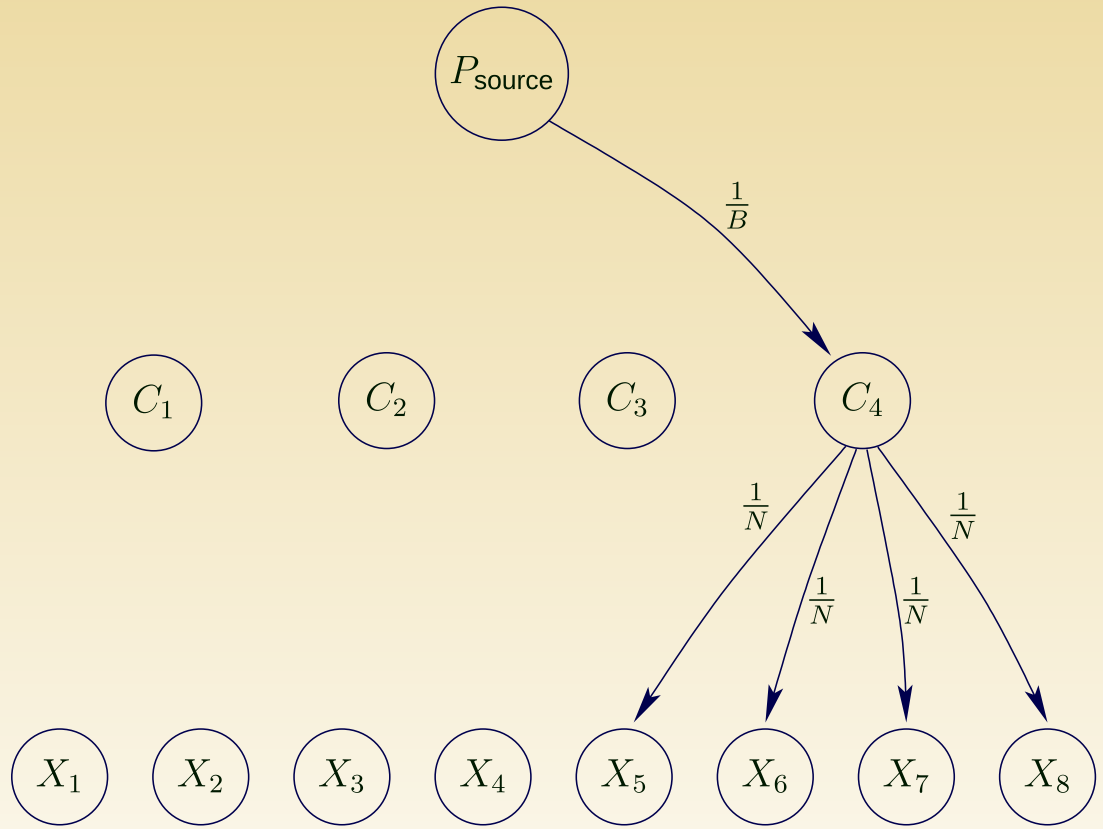

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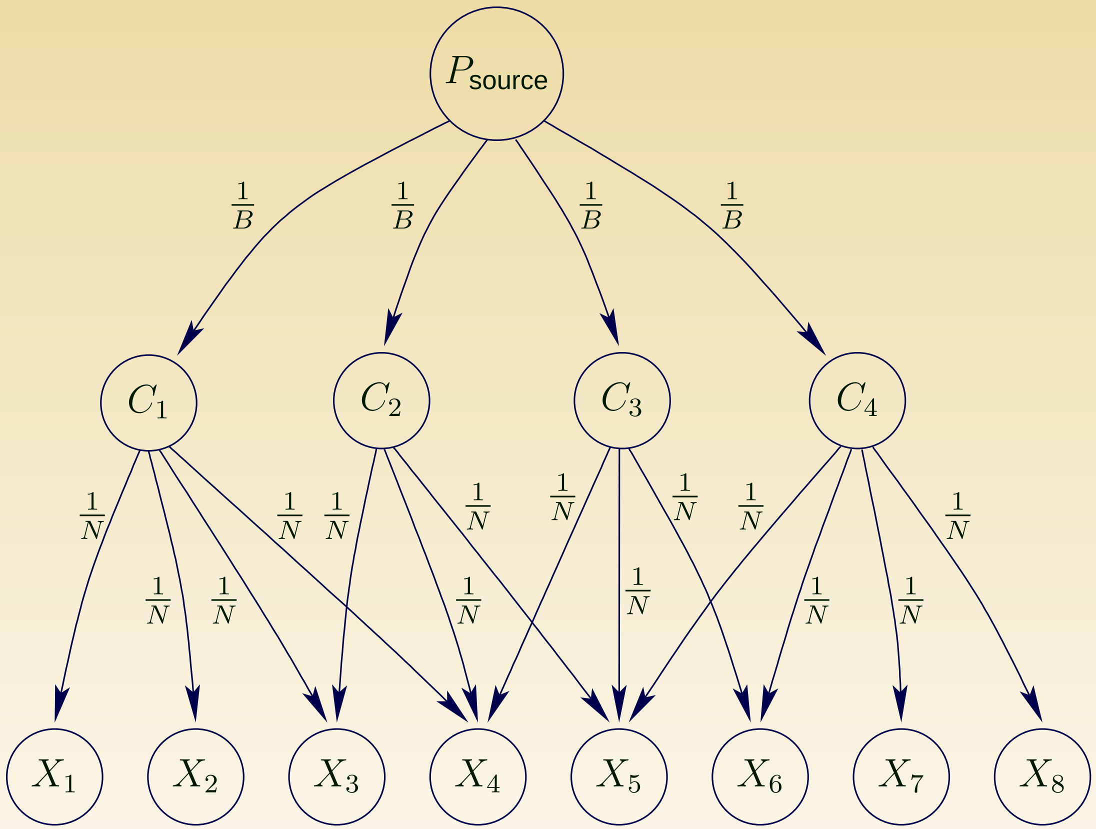

Heuristic based on linear programming

Straightforward heuristics

- lower bound on the throughput $\rightarrow$ scatter heuristic
  $|\mathcal{P}_{\text{target}}|$: cardinal of the target set
  scatter has a guarantee factor of $|\mathcal{P}_{\text{target}}|$:

$$\text{throughput}(\text{scatter}) \geq \frac{\text{upper bound}}{|\mathcal{P}_{\text{target}}|}$$

- broadcast on the whole platform
  the optimistic view ($n_{i,j} = \max_k x_{i,j}^k$) leads to a feasible schedule in this case

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Refined heuristics

- Reduced Broadcast:
  1. compute the solution of the broadcast
  2. choose the node P_{min} not in the original $\mathcal{P}_{\text{target}}$ which forwards the minimum of messages:

$$\text{MIN} \sum_{i \in \mathcal{P}_{\text{target}}} \sum_{P_j \in \mathcal{N}^{\text{in}}(P_m)} x_i^{j,m}$$

  3. set $V = V \setminus \{P_{\text{min}}\}$ and start again until the throughput is not improved
- Augmented Multicast:
  1. compute the solution of the scatter
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Tree-based heuristic

Steiner tree

- problem: find a low-cost multicast tree
- cost: sum of the weights of the edges in the tree
- Minimum Steiner Tree: NP-complete
- some heuristics exist, among other the Minimum Cost Path Heuristic:
  - grow a tree until it spans all the target nodes
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Minimum Cost Path Heuristic

- we adapt the previous heuristic to our metric:

$$\max_i(\sum \text{cost of all edges } P_i \rightarrow P_j \text{ in the tree } T)$$

1. $T = (P_{\text{source}}, \emptyset)$
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- we perform experiments on platforms generated by Tiers
- two types of platforms:
  - one “big”: 65 nodes
  - one “small”: 30 nodes
- results: comparison of the throughput of our heuristics over the two bounds:
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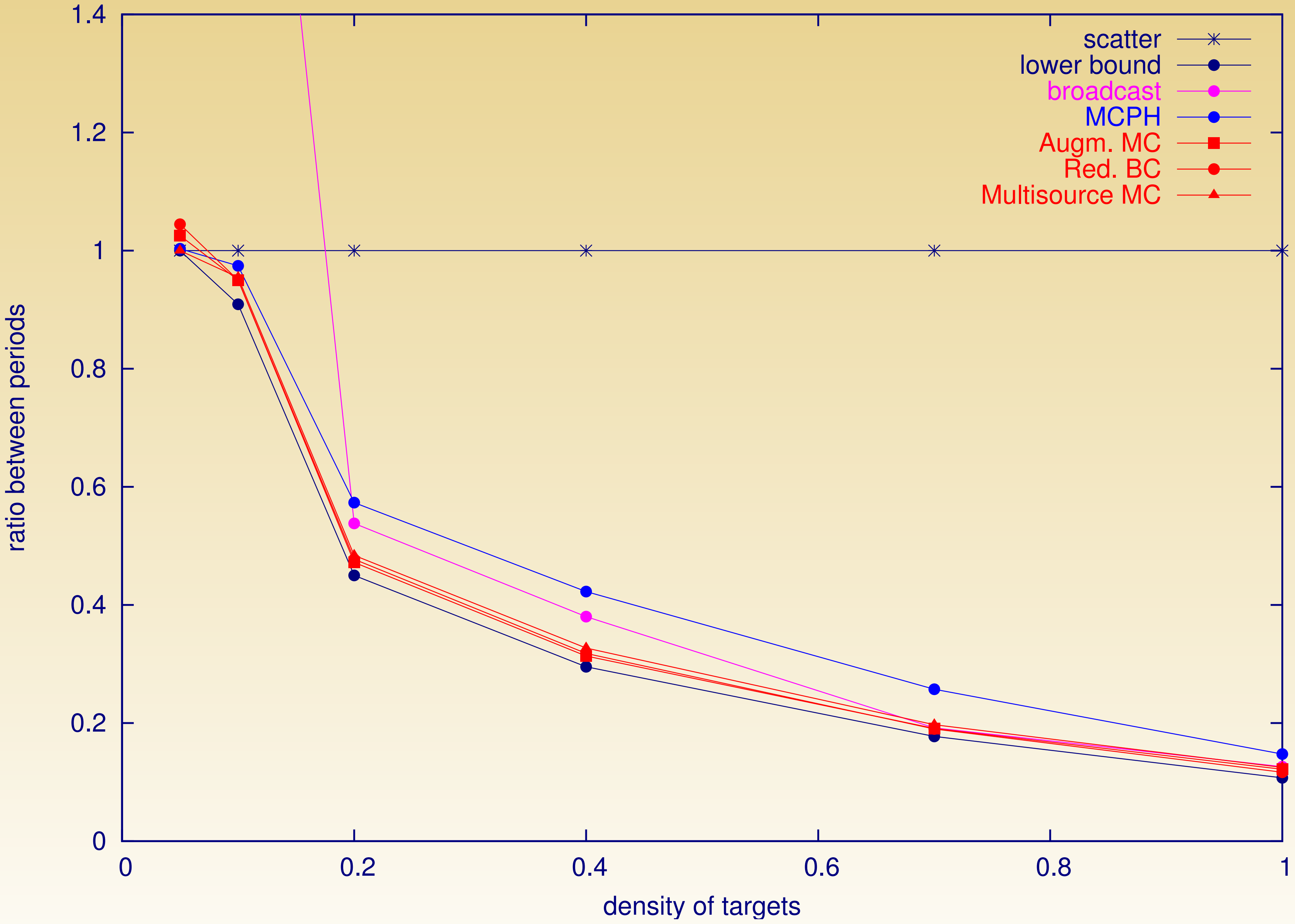
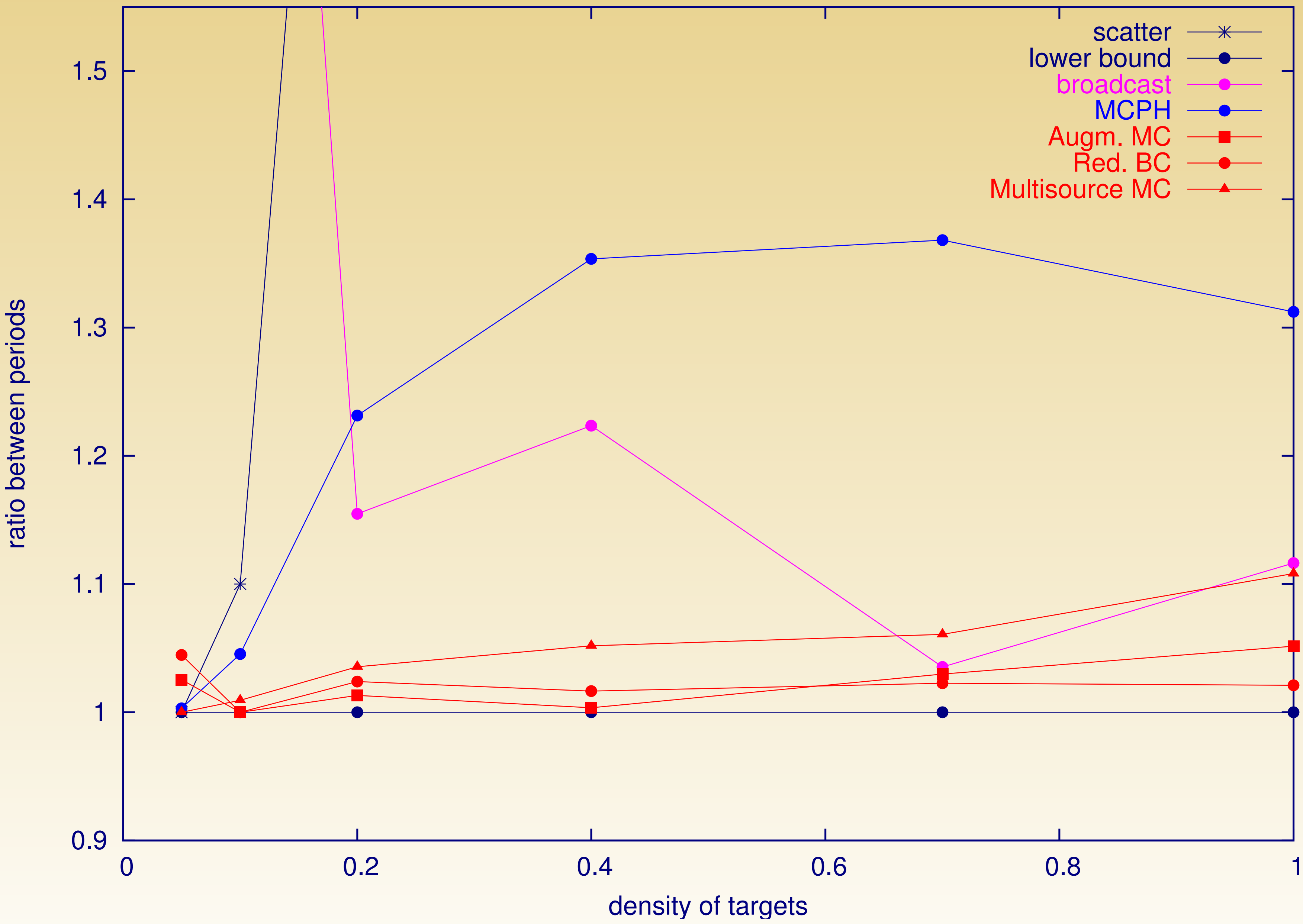
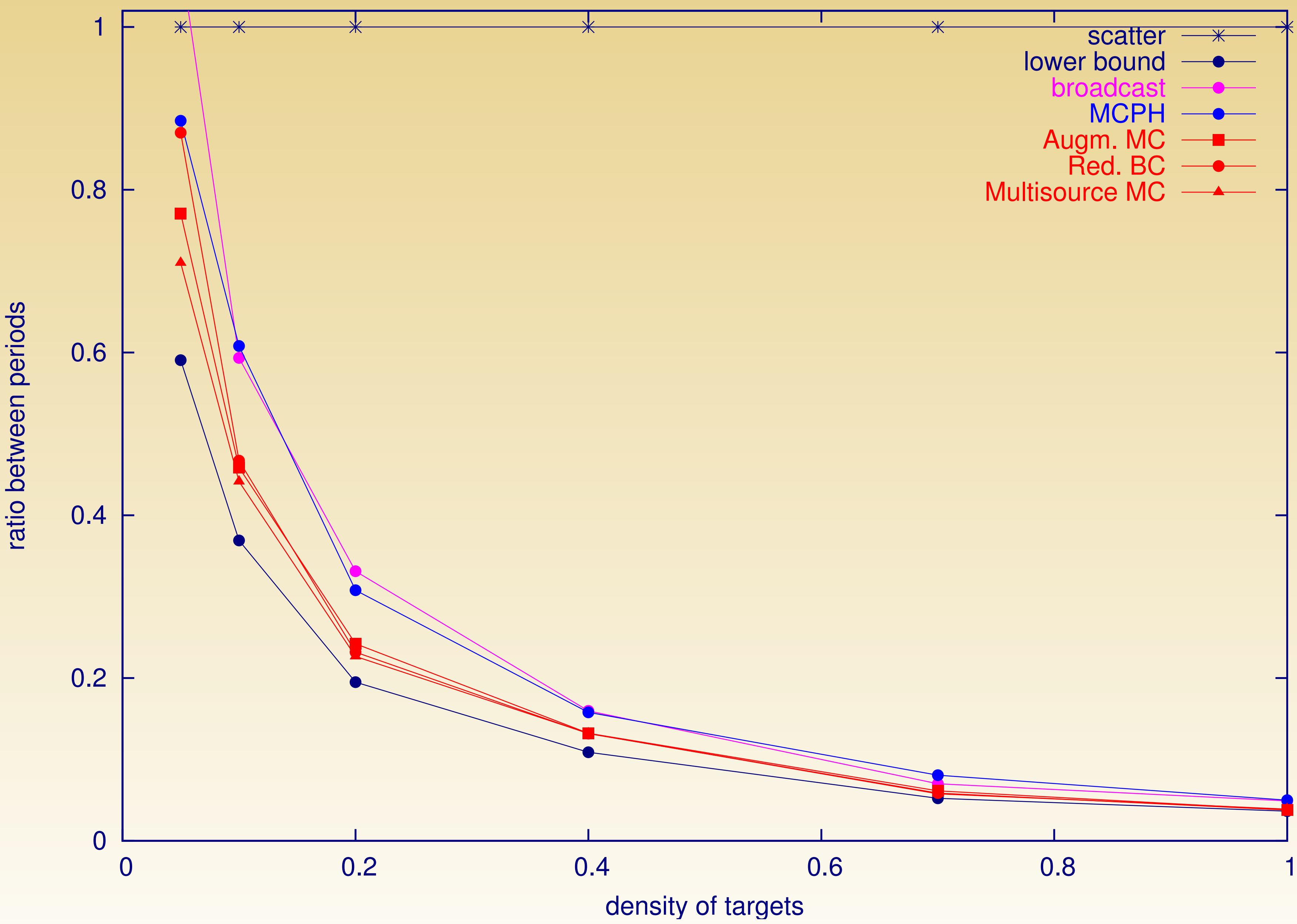
Small platform - comparison over scatter



Small platform - comparison the lower bound



Big platform - comparison scatter



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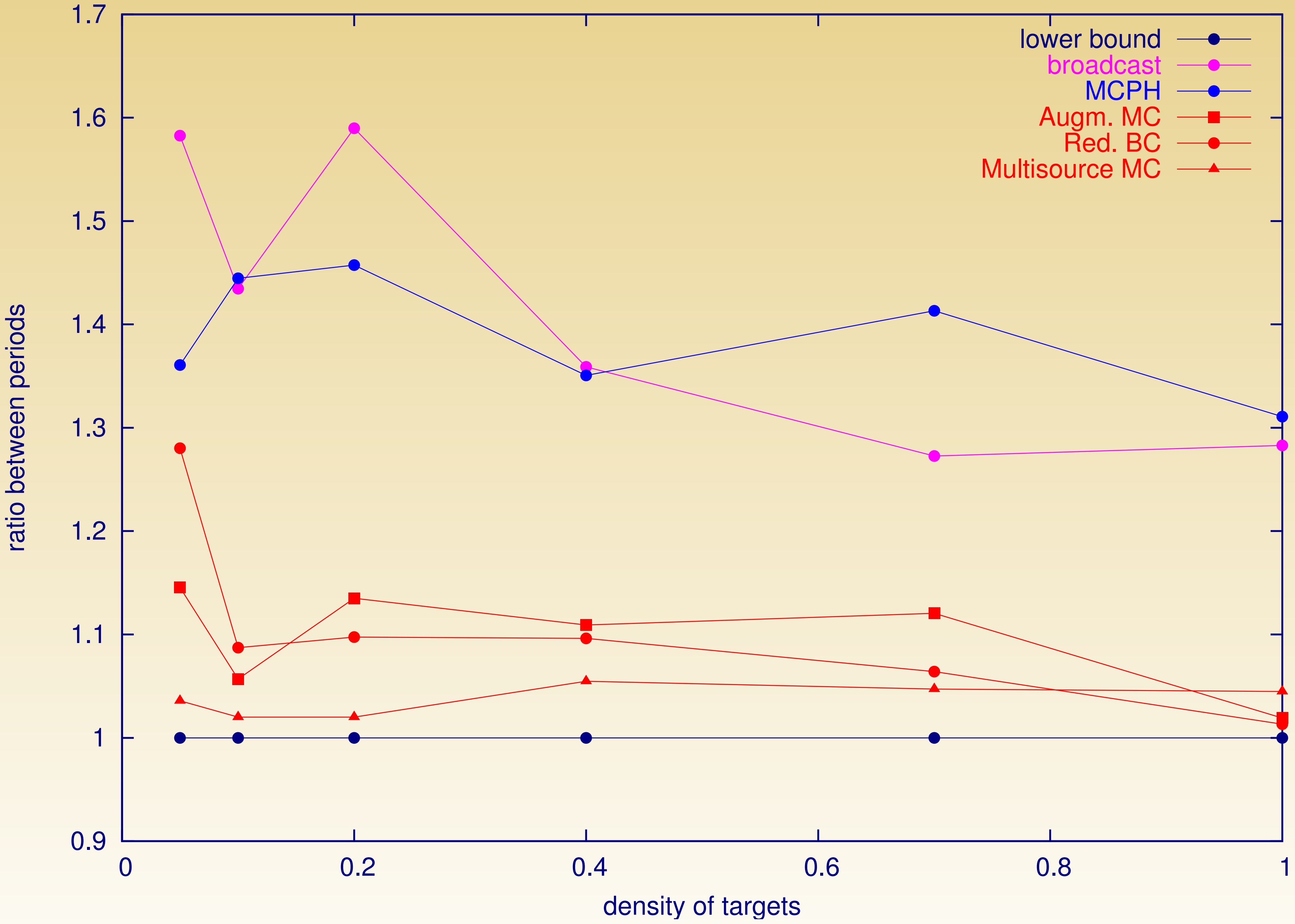

More on complexity ?

Does Multicast belong to NP?

- we have shown that multicast is NP-hard, but multicast $\in$ NP?
- problem: check that a set of multicast trees is a valid solution
  $\rightarrow$ time linear in the size of the set, potentially in $2^{|V|}$
  $\rightarrow$ check if all communications can be orchestrated
- we prove that at most $2 \times |V|$ of those trees are useful (with throughput $\neq 0$)
- suppose the solution is made of weighted trees (T_i, y_i)
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Does Multicast belong to NP?

- we have shown that multicast is NP-hard, but multicast $\in$ NP?
- problem: check that a set of multicast trees is a valid solution
  - $\rightarrow$ time linear in the size of the set, potentially in $2^{|V|}$
  - $\rightarrow$ check if all communications can be orchestrated
- we prove that at most $2 \times |V|$ of those trees are useful (with throughput $\neq 0$)
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$$\textbf{Maximize} \sum_k y_k,$$

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- V is the solution of a $|\mathcal{T}| \times |\mathcal{T}|$ linear system such that at V, at least $\mathcal{T}$ inequalities (among $\mathcal{T} + 2|V|$) are tight
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  - one-port constraints are satisfied
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- is this enough to build a valid schedule?
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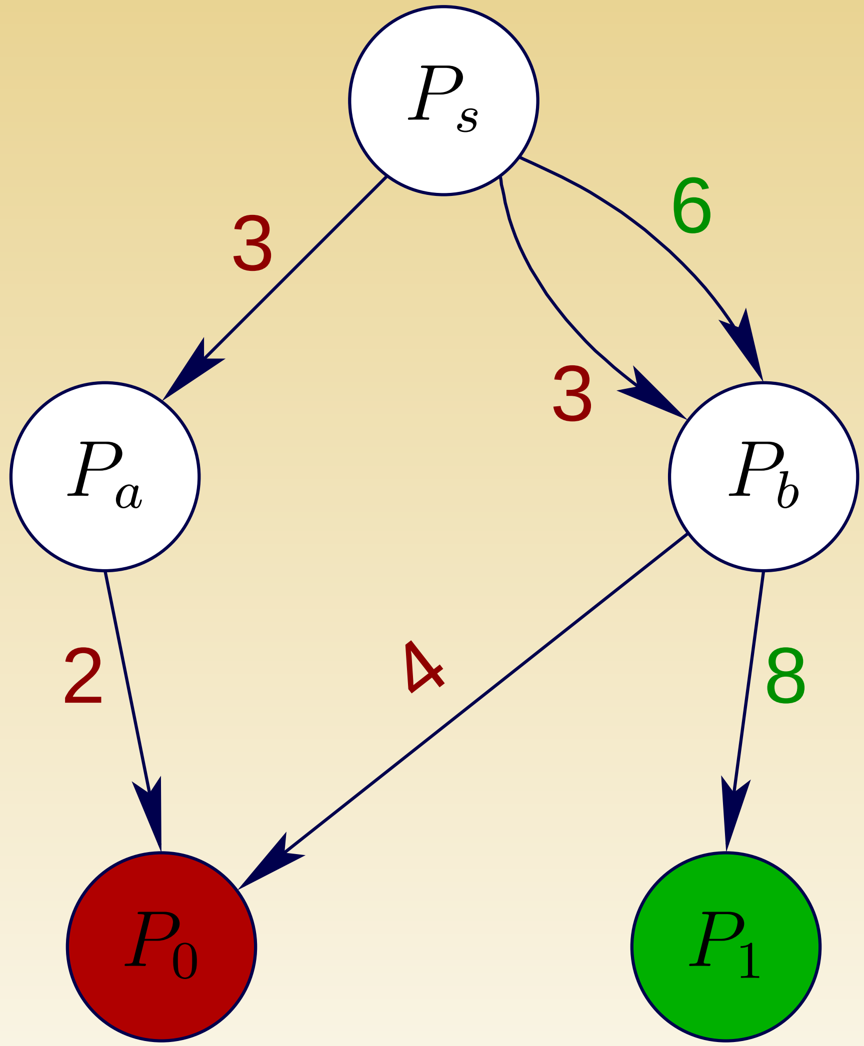
Use of a bipartite graph - example



Communications

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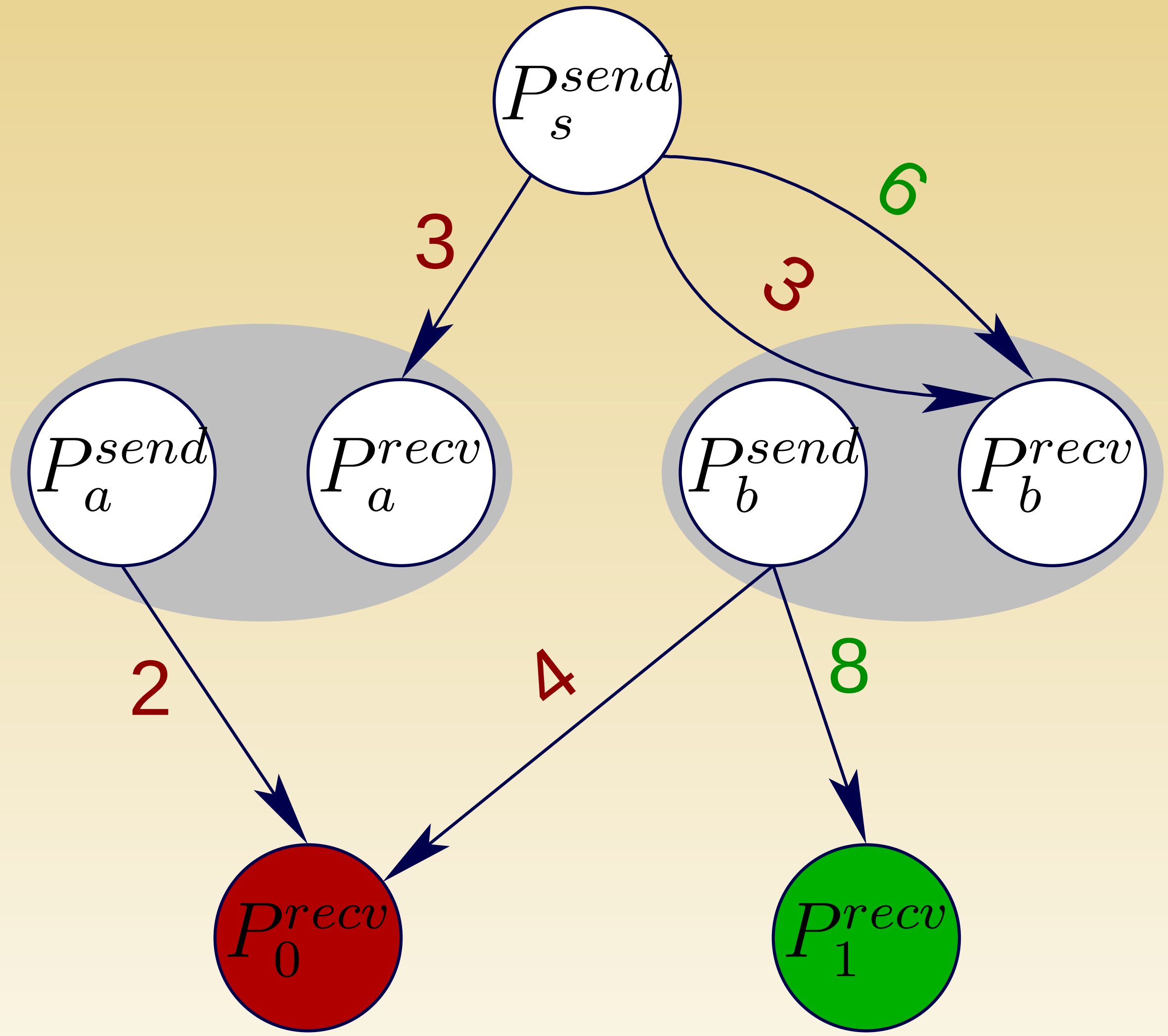

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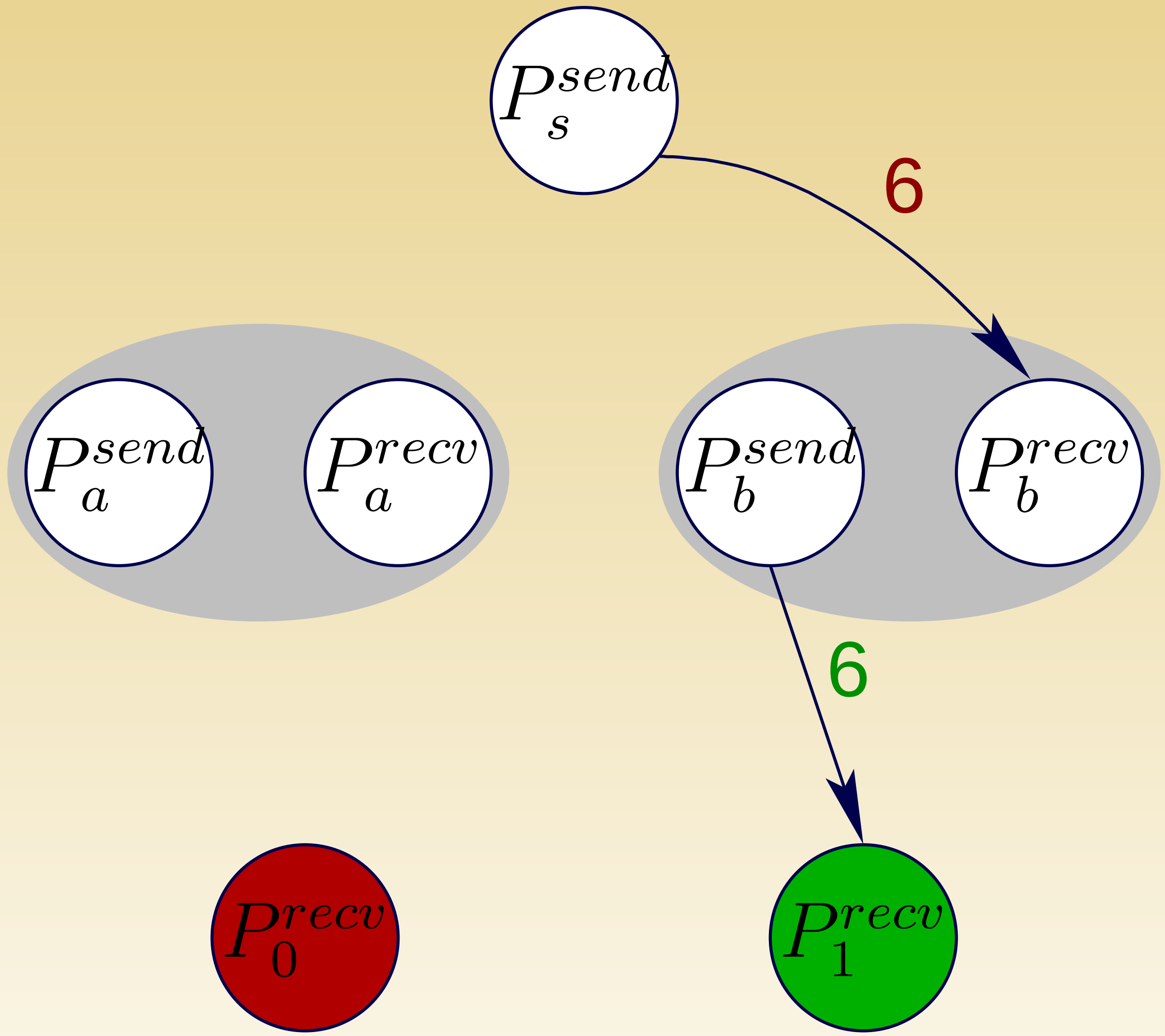

matching 1 (weight 6)

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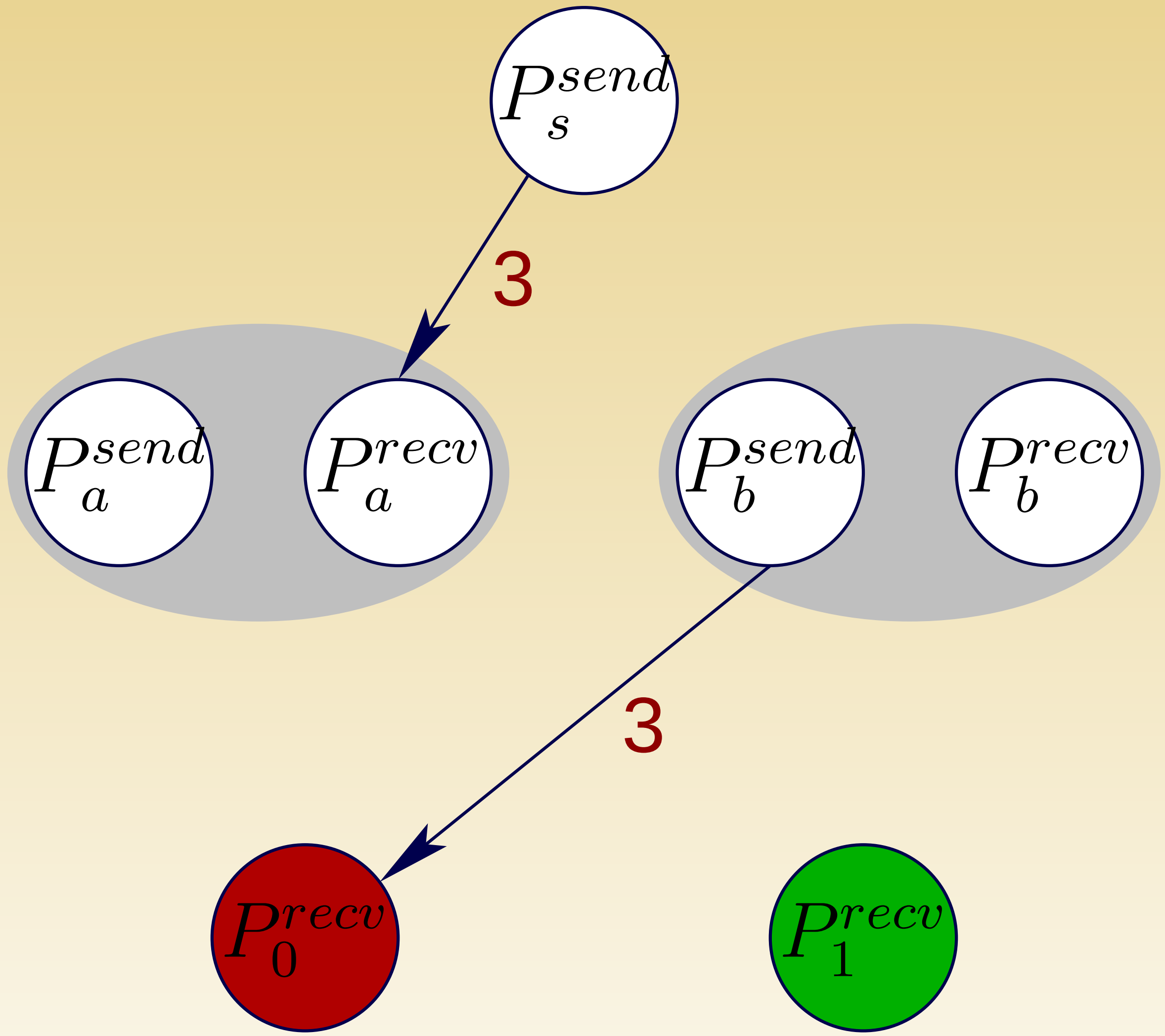
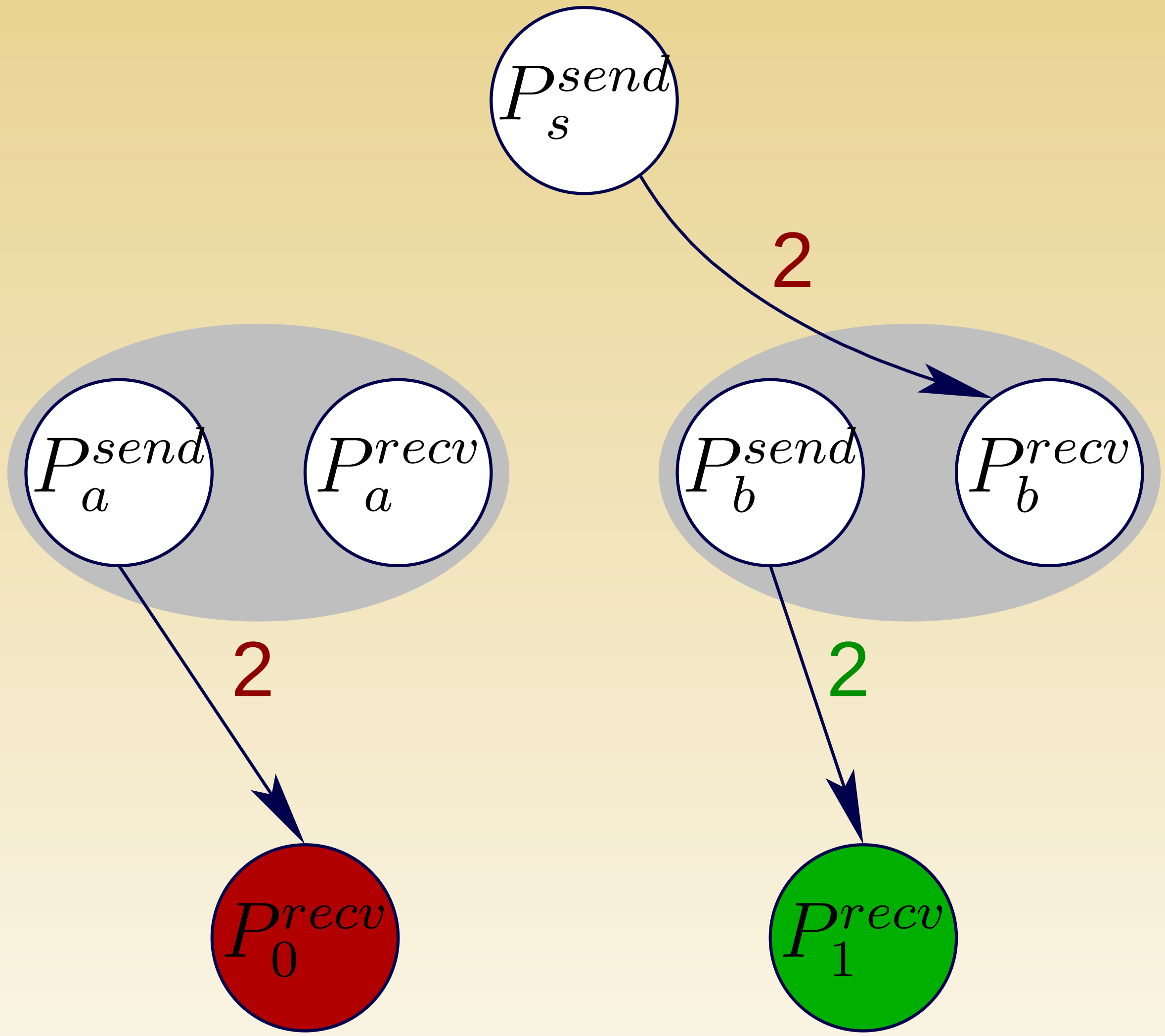


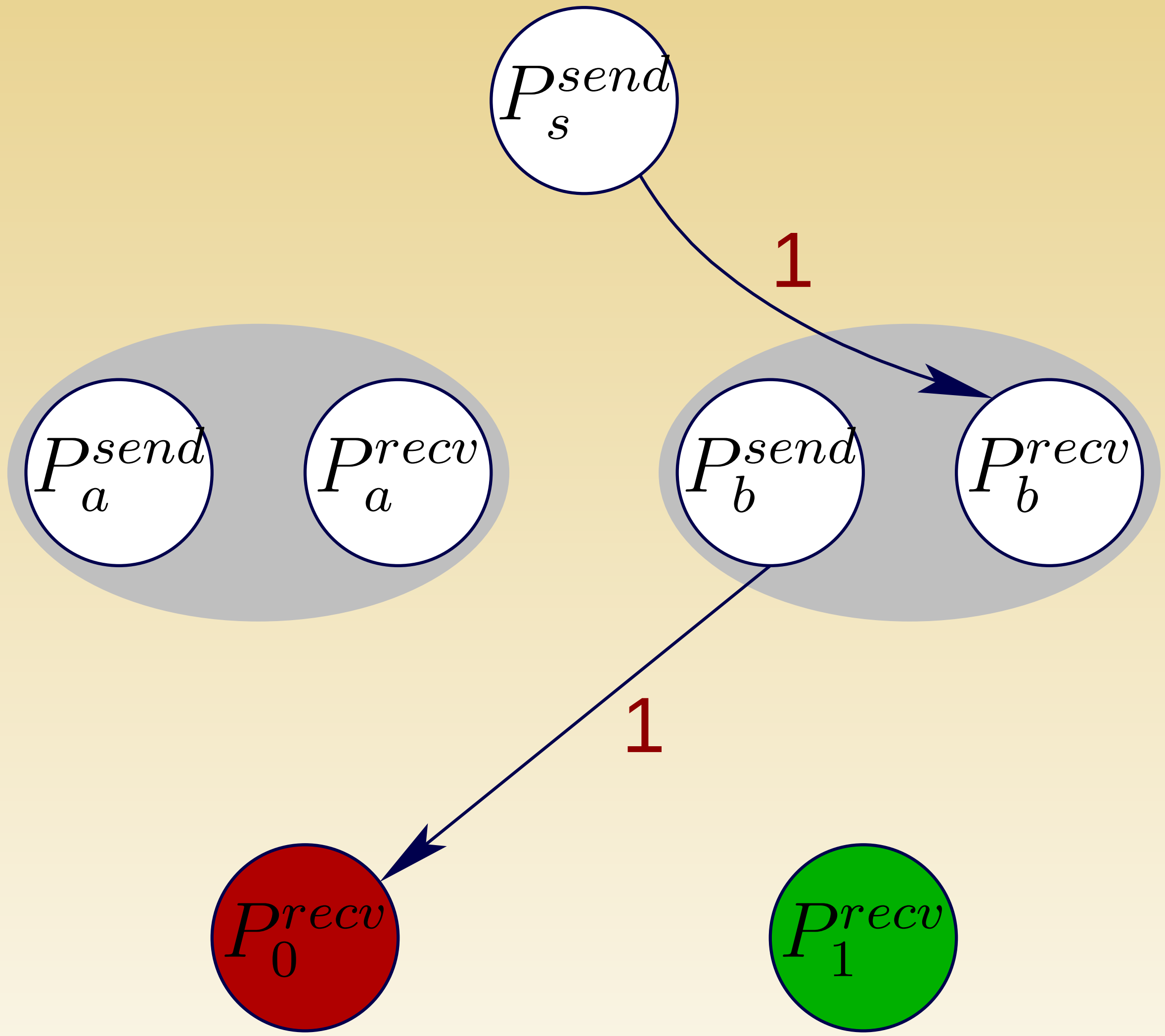
matching 2 (weight 3)

Use of a bipartite graph - example



matching 3 (weight 2)

Use of a bipartite graph - example



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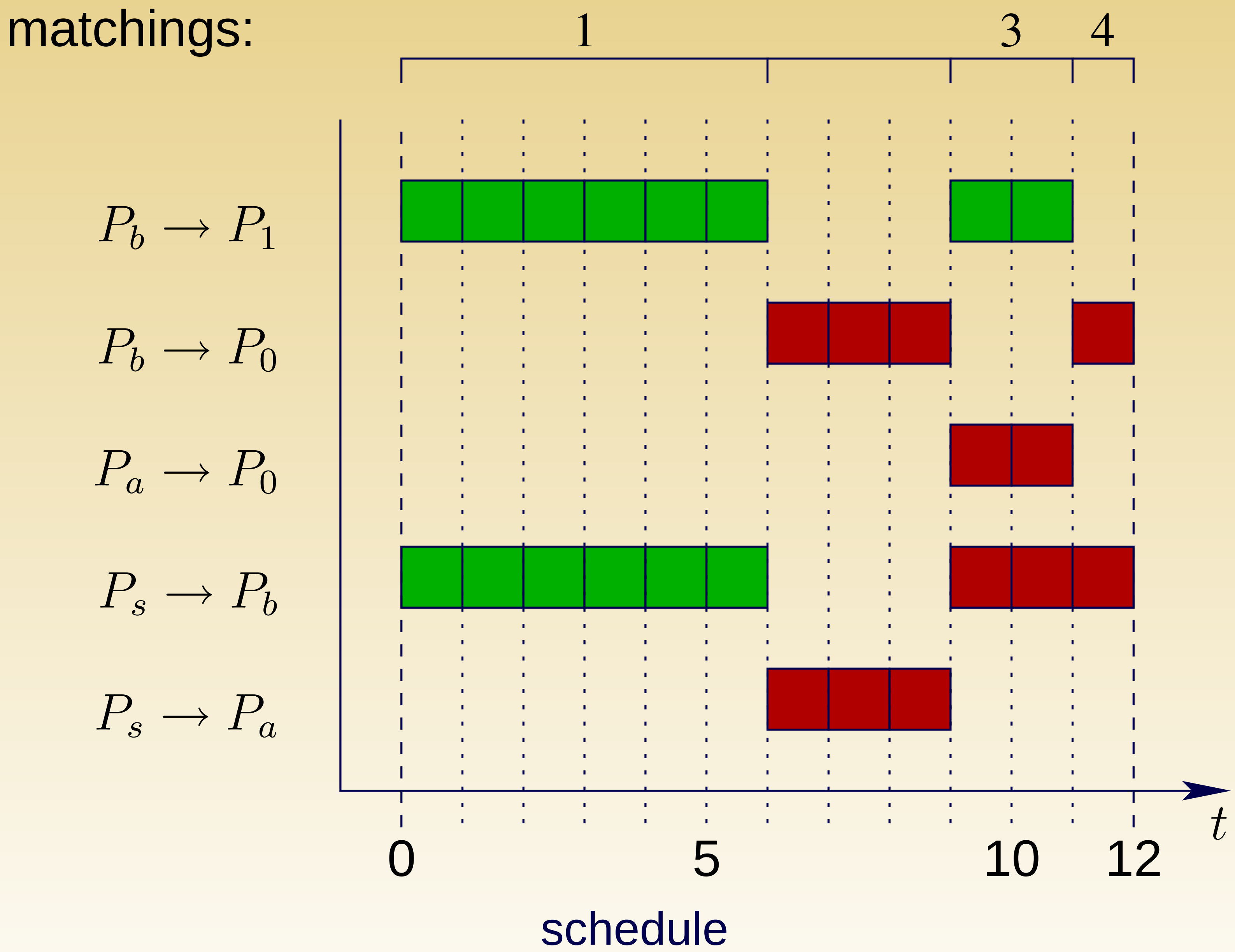
Use of a bipartite graph - example



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- *There exists an oracle-polynomial time algorithm that solves the weak violation problem for every circumscribed convex body* (K, n, R) *given by a weak separation oracle*
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