VoroNet: A scalable object network based on Voronoi tessellations

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Outline

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Peer-to-peer overlay

What is peer-to-peer ?

- paradigm to organize distributed ressources (peers)
- overlay network: logical organization
- core functionnality: search objects in the system
- distributed hashtables (DHT) (Chord, Pastry,...)
- hash function gives identifiers for **peers** and **objects**
- choice of hash function to get uniform distribution

main goals:

- scalability ☺ but restricted to exact search ☹
- fault tolerance ☺ highly depends on the hash function ☹
- efficient search ☺

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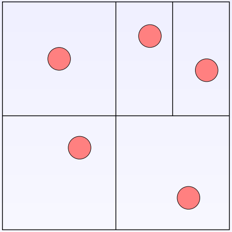
Peer-to-peer overlay

Content-based topologies:

- CAN (Content Adressable Network)
  - d-dimensional torus
  - degree $O(d)$
  - diameter $O(N^{1/d})$
  - not really "content addressed": location (of objects and peers) computed with hash function (to ensure homogeneous distribution)



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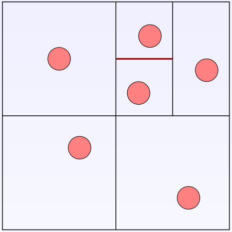

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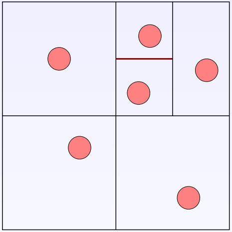

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- Object-based peer-to-peer overlay
  - objects are linked rather than peers
  - an object is held by the node which published it
- Content-based topology:
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- d-dimensional attribute space
- VoroNet topology is inspired from:
  - Voronoi diagram in the attribute space
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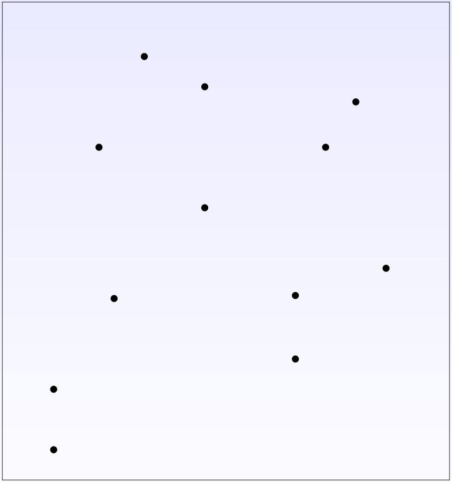

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- consider object at point M
- region of points closer from M than
- do the same for all objects
- Voronoi neighbors: when cells share a border
- graph of Voronoi neighbors: Delaunay triangularization

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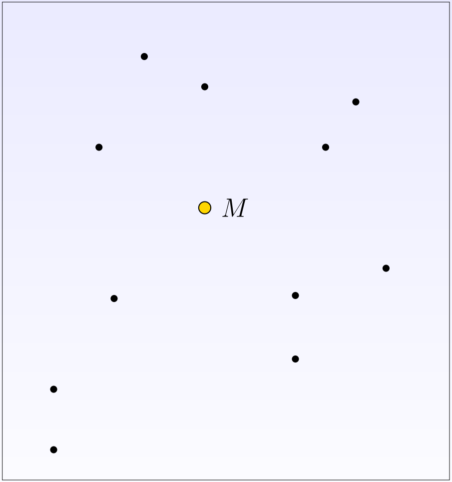

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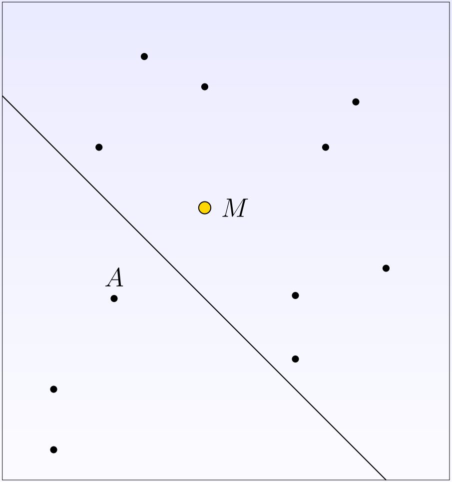

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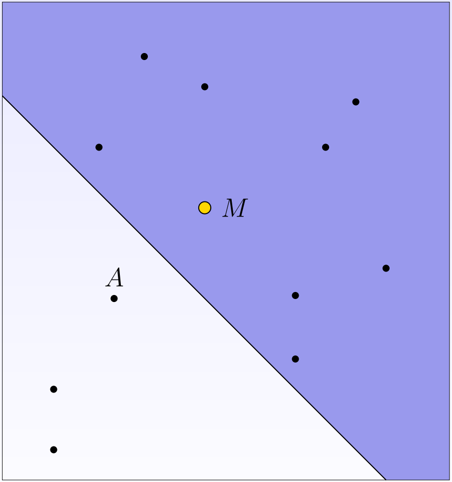

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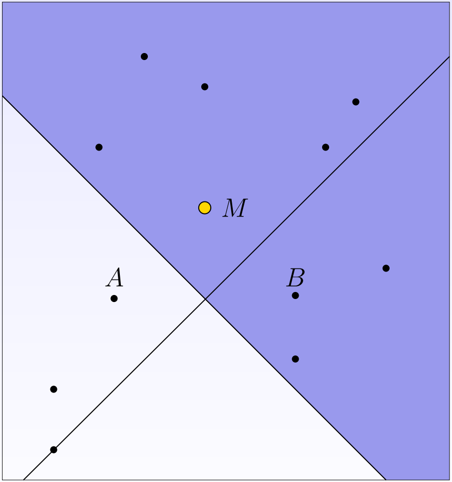

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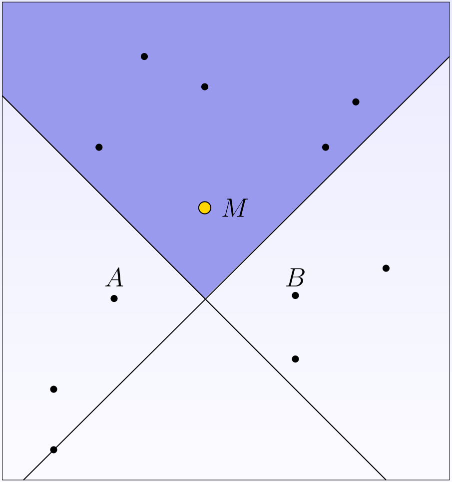

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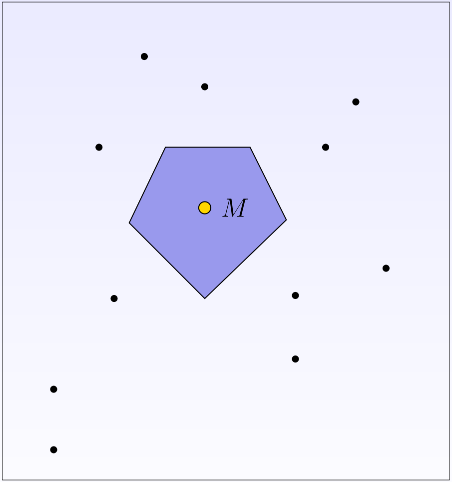

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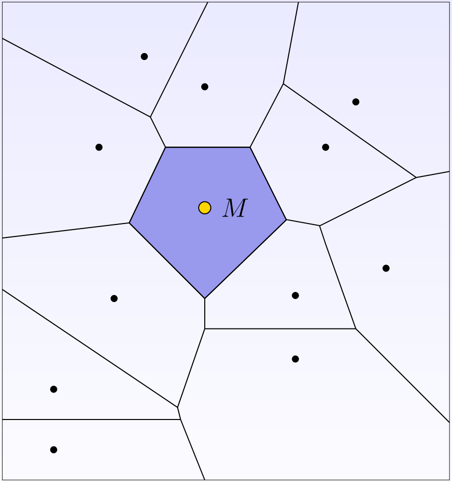

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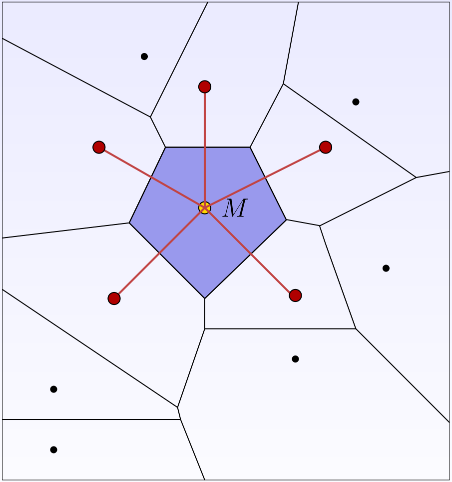

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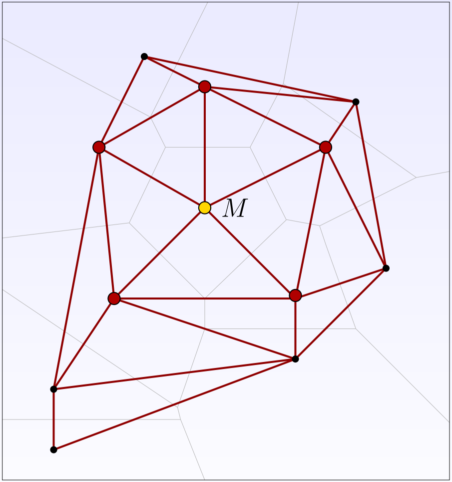

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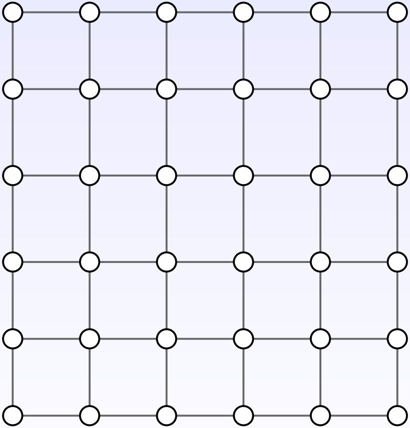

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  - routing in $O(\sqrt{N})$
- add random long range links:
  - probability for a long link to be at distance l: $\propto \frac{1}{l^2}$
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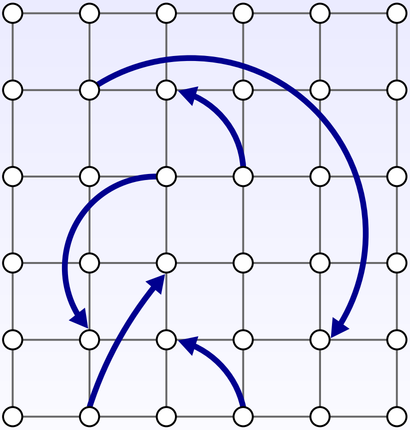

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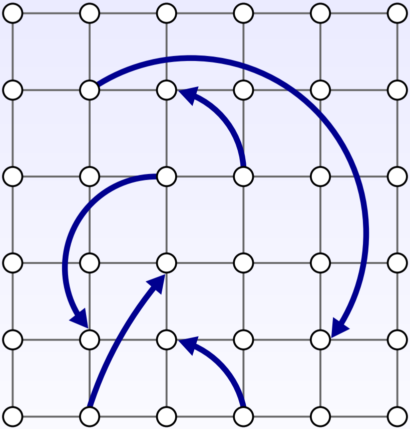

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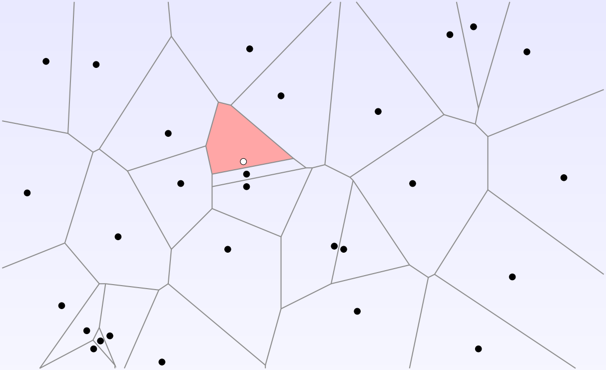

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  - randomly chose a target point t
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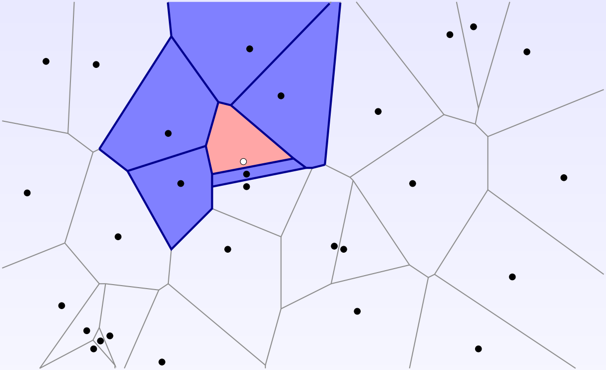

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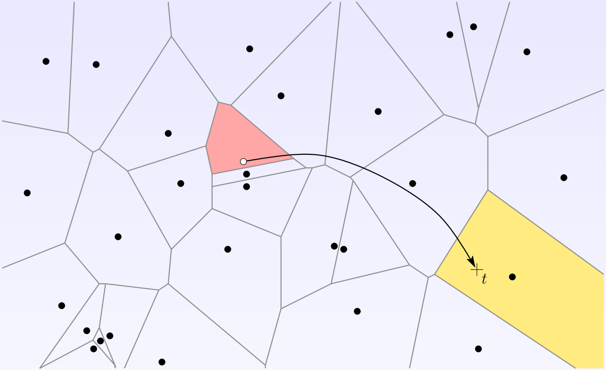

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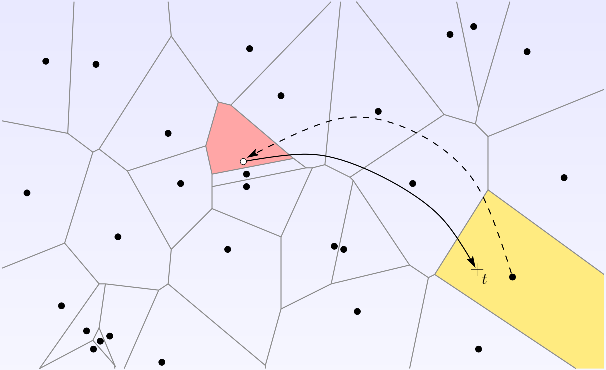

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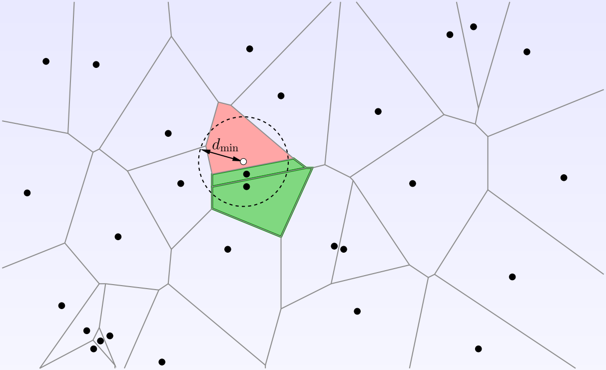

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- Space: 2-dimensional torus: $[0,1] \times [0,1]$
- Long link target of object x: distribution in $1/d^2$

$$\mathsf{Pr}\Big[\mathsf{target}(x) \in \mathcal{B}(y, dr)\Big] = \alpha \frac{\pi r^2}{d(x,y)^2}$$

- Link always points on the closest object from target.
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Size of the neighborhood

- Voronoi neighbors:
  - graph of the Voronoi neighbors is planar $\Rightarrow$ average degree $\leqslant 6$
  - $O(1)$ size
  - mean value $\leqslant 6$ (max $= N$)
- Close neighbors: number of objects in $\mathcal{B}(o, d_{\min})$
  - $O(1)$ size for a "reasonable" distribution
  - mean value $= 1$ (max $= N$)
- Long range neighbors:
  - one long range neighbor per object
  - $O(1)$ backward links for a "reasonable" distribution
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size of data stored at each node: $O(1)$ mean value $\leqslant 9$
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Overlay maintenance

How to insert an object x ?

- Update the Voronoi diagram:
  - Find the closest existing object (route to x)
  - Add a new Voronoi cell
  - Find and update the Voronoi neighbors
- Find and the close neighbors
  sufficient to consider close neighbors of Voronoi neighbors
- create a long range target point t, find the corresponding object:
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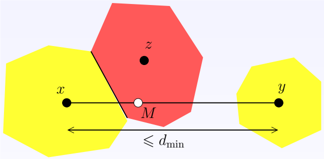

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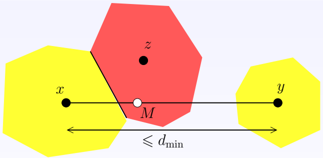

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General routing scheme

$\textsc{Route}$ ($Target$):

find the object responsible for the Voronoi cell where $Target$ is.

$\textsc{Route}(Target)$

$z = \textsc{DistanceToRegion}(Target)$

if $d(z, Target) > \frac{1}{3}d(Target, CurrentObject)$ **and** $d(Target, CurrentObject) > d_{\min}$ **then**

Spawn($\textsc{Route}, Target, \textsc{GreedyNeighbor}(Target)$)

else

$\textsc{AddVoronoiRegion}(z)$

$\textsc{AddVoronoiRegion}(Target)$

Perform some local computations depending on the operation at z

$\textsc{RemoveVoronoiRegion}(z)$

(depending on the operation, $\textsc{RemoveVoronoiRegion}(Target)$)

return

Polylogarithmic routing - sketch of proof - 1/2

Lemma 1

The probability for the long link of x to be chosen in a disk of center y and radius fr, where $r = d(x, y)$ is lower bounded by $\frac{\pi f^2}{K(1+f)^2}$.

- For $f = 1/6$: probability lower bounded by: $\dfrac{1}{98\ln(\sqrt{2}\pi N_{\max})}$
- X: number of hops necessary to reach the disk of center $Target$ and radius $\frac{d}{6}$.

$$E(X) = \sum_{i=1}^{+\infty} \Pr[X \geqslant i] \leqslant \sum_{i=1}^{+\infty} \left(1 - \frac{1}{98\ln(\sqrt{2}\pi N_{\max})}\right)^{i-1}$$

$$E(X) \leqslant 98\ln(\sqrt{2}\pi N_{\max}).$$

but **link target** $\neq$ **link destination**

This accounts for a **super-step**.

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The probability for the long link of x to be chosen in a disk of center y and radius fr, where $r = d(x,y)$ is lower bounded by $\frac{\pi f^2}{K(1+f)^2}$.

- For $f = 1/6$: probability lower bounded by: $\dfrac{1}{98\ln(\sqrt{2}\pi N_{\max})}$
- X: number of hops necessary to reach the disk of center $Target$ and radius $\frac{d}{6}$.

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\begin{aligned}
E(X) &= \sum_{i=1}^{+\infty} \Pr[X \geqslant i] \leqslant \sum_{i=1}^{+\infty} \left(1 - \frac{1}{98\ln(\sqrt{2}\pi N_{\max})}\right)^{i-1} \\
E(X) &\leqslant 98\ln(\sqrt{2}\pi N_{\max}).
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but **link target** $\neq$ **link destination**

This accounts for a **super-step**.

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- During a super-step, the distance to the target is divided by $5/6$
- Number of super-steps bounded by

$$\frac{\ln(\frac{\sqrt{2}}{d_{\min}})}{\ln(\frac{6}{5})} = \frac{\ln(\sqrt{2}\pi N_{max})}{\ln(\frac{6}{5})}$$

- Expectation of number of steps:

$$E(N) \leqslant \alpha \ln^2(N_{max})$$

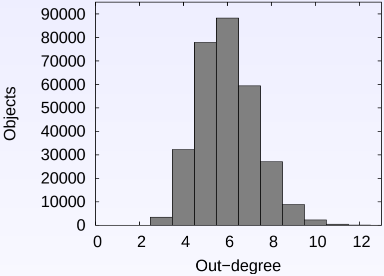
Outline

Experimental framework

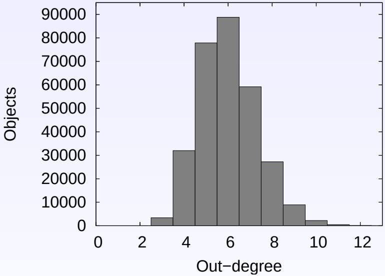
- Simulations
  - 300.000 objects
  - objects are not leaving the overlay (for now)
- Distribution of object
  - uniform
  - skewed (powerlaw with parameter $\alpha = 1, 2, 5$)
- We observe:
  - number of neighbors
  - polylogarithmic routing
  - what happens if we add several long range links

Outgoing degree

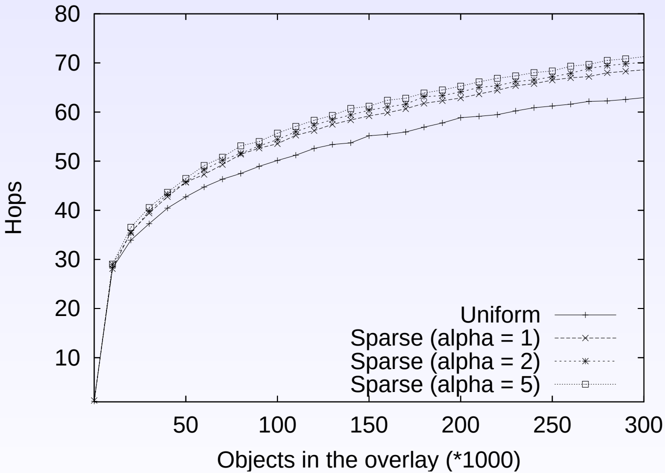
Uniform distribution



Skewed distribution ($\alpha = 5$)



Polylogarithmic routing



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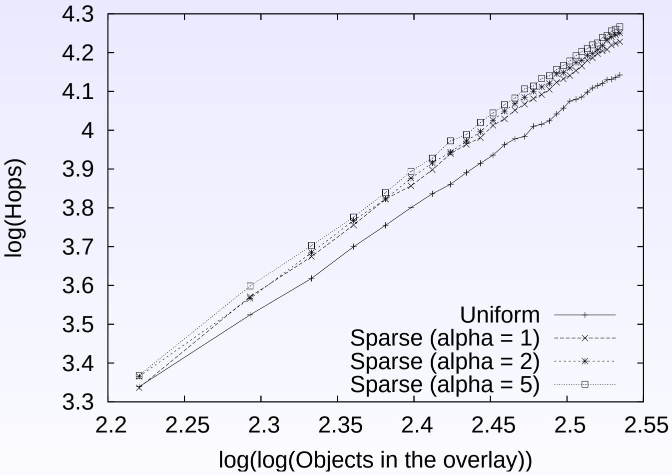

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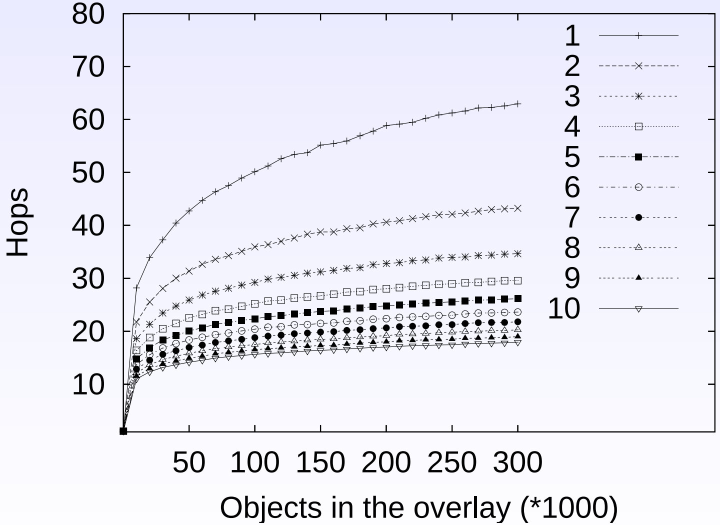
Using several long links to improve routing



Outline

Extension to higher dimensional cases

- No bound on the number of neighbors, $O(1)$ data size ?
- Extend small world property seems possible
- Compute Voronoi diagram: geometric algorithms costly and sensitive to computation errors
- No need to have complete description of Voronoi cells, only compute neighborhood
- Use other techniques:
lifting in dimension $d+1$ and LP

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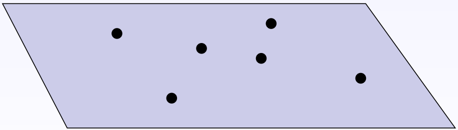

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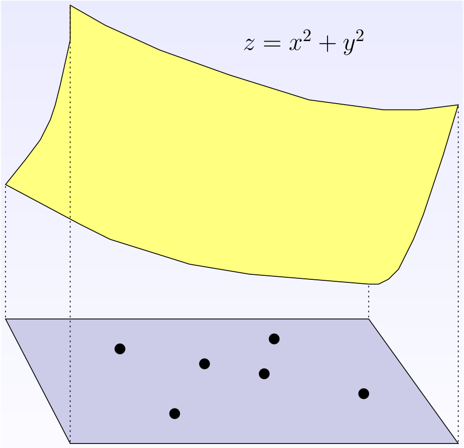

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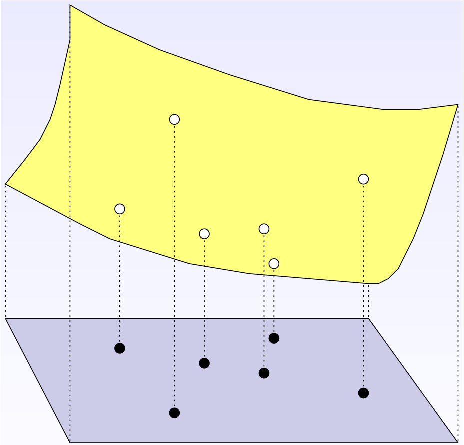

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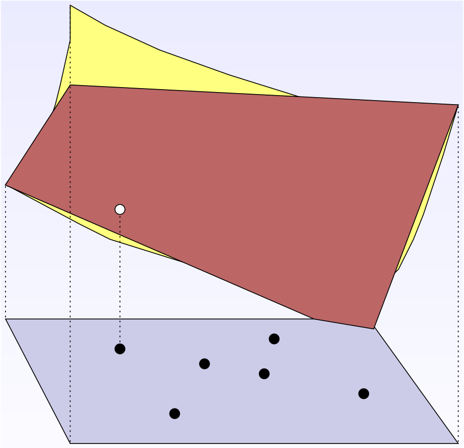

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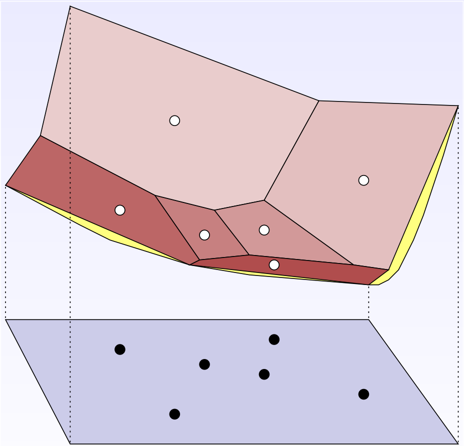

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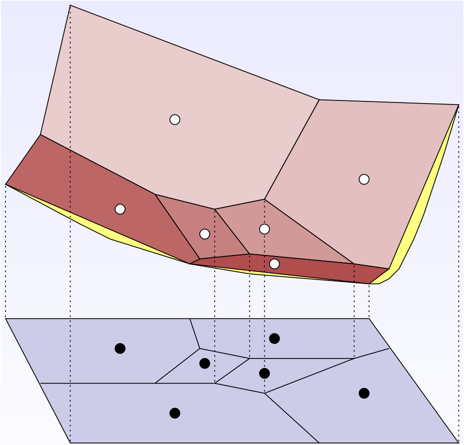

Conclusion

Perspectives:

- Range queries
- Queries by proximity: all objects within d from a given object
- Fault tolerance ?

VoroNet in a nutshell:

- Object-to-object overlay
- Efficient routing
- Distributed construction and management
- Reasonable size of neighborhood
- Insensitive to object distribution
- Base for complex requests

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