

How to deal with uncertainties and dynamicity ?

<http://graal.ens-lyon.fr/~lmarchal/scheduling/>

19 octobre 2011

Outline

- 1 The problem
- 2 Analyzing sensitivity and robustness
- 3 Dynamic load-balancing
- 4 How useful is old information ?
- 5 Conclusion

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The problem : the world is not perfect !

- ▶ Uncertainties
 - ▶ On the platforms' characteristics
(Processor power, link bandwidth, etc.)
 - ▶ On the applications' characteristics
(Volume computation to be performed, volume of messages to be sent, etc.)
- ▶ Dynamicity
 - ▶ Of network (interferences with other applications, etc.)
 - ▶ Of processors (interferences with other users, other processors of the same node, other core of the same processor, hardware failure, etc.)
 - ▶ Of applications (on which detail should the simulation focus?)

Solutions : to prevent or to cure ?

To prevent

- ▶ Algorithms tolerant to uncertainties and dynamicity.

To cure

- ▶ Algorithms auto-adapting to actual conditions.

Leitmotiv : the more the information, the more precise we can statically define the solutions, the better our chances to “succeed”

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Analyzing the sensitivity

Question : we have defined a solution, how is it going to behave “in practice” ?

Possible approach

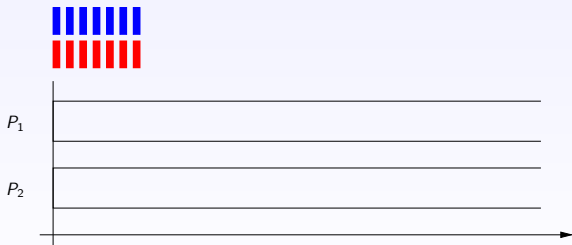
- 1 Definition of an algorithm \mathcal{A} .
- 2 Modeling the uncertainties and the dynamicity.
- 3 Analyzing the sensitivity of \mathcal{A} as follows :
 - ▶ For each theoretical instance of the problem
 - ▶ Evaluate the solution found by \mathcal{A}
 - ▶ For each “actual” instance corresponding to the given theoretical instance, find the optimal solution and the relative performance of the solution found by \mathcal{A} .

Sensitivity of \mathcal{A} : worst relative performance, or (weighted) average relative performance, etc.

Analyzing the sensitivity : an example

Problem

- ▶ Master-slave platform with two identical processors
- ▶ Flow of two types of identical tasks
- ▶ Objective function : maximum minimum throughput between the two applications (*max-min fairness*)

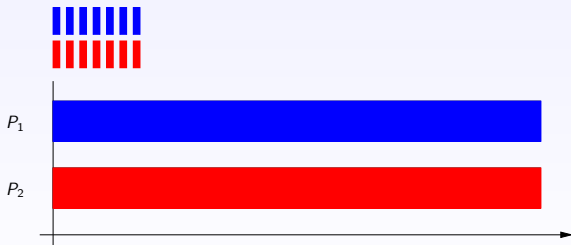


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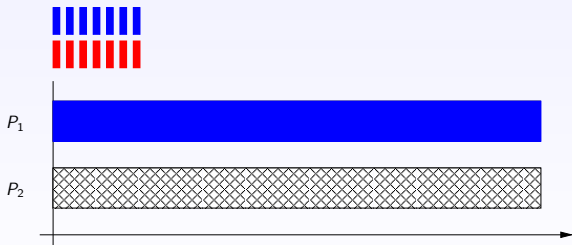


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Analyzing the sensitivity : the case of Backfilling (1)

Context :

- ▶ cluster shared between many users
- ▶ need for an allocation policy, and a reservation policy
- ▶ job request : number of processors + maximal utilization time
- ▶ (A job exceeding its estimate is automatically killed)

Simplistic policy : First Come First Served :

- ▶ lead to waste some resources

Analyzing the sensitivity : the case of Backfilling (2)

The EASY backfilling scheme

- ▶ The jobs are considered in First-Come First-Served order
- ▶ Each time a job arrives or a job completes, a reservation is made for the first job that cannot be immediately started, later jobs that can be started immediately are started.
- ▶ In practice jobs are submitted with runtime estimates. A job exceeding its estimate is automatically killed.

Analyzing the sensitivity : the case of Backfilling (3)

The set-up

- ▶ 128-node IBM SP2 (San Diego Supercomputer Center)
- ▶ Log from May 1998 to April 2000 log : 67,667 jobs
Parallel Workload Archive (www.cs.huji.ac.il/labs/parallel/workload/)
- ▶ Job runtime limit : 18 hours.
(Some dozens of seconds may be needed to kill a job.)
- ▶ Performance measure : average slowdown (=average stretch).

Bounded slowdown : $\max\left(1, \frac{T_w + T_r}{\max(10, T_r)}\right)$

Execution is simulated based on the trace : enable to change task duration (or scheduling policy).

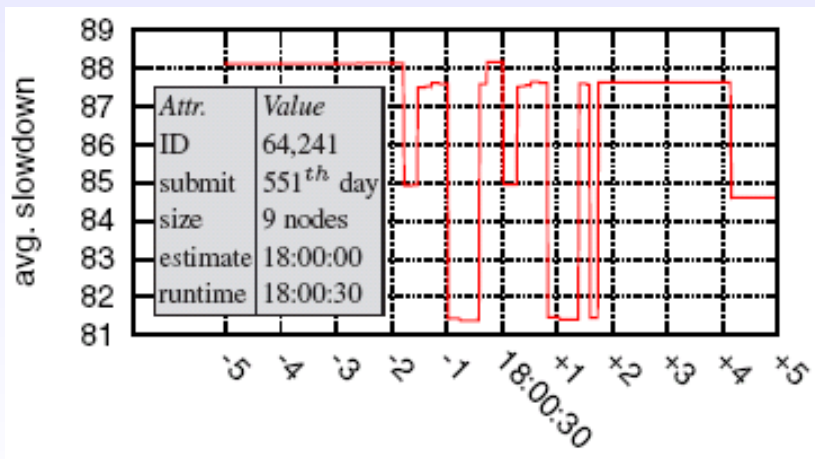
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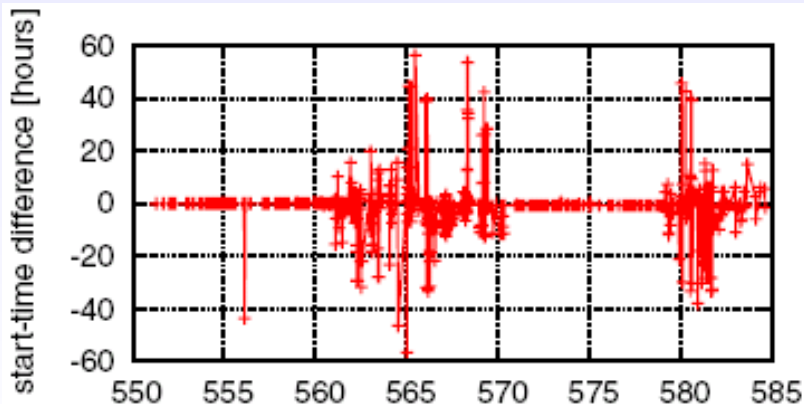
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Analyzing the sensitivity : the case of Backfilling (4)

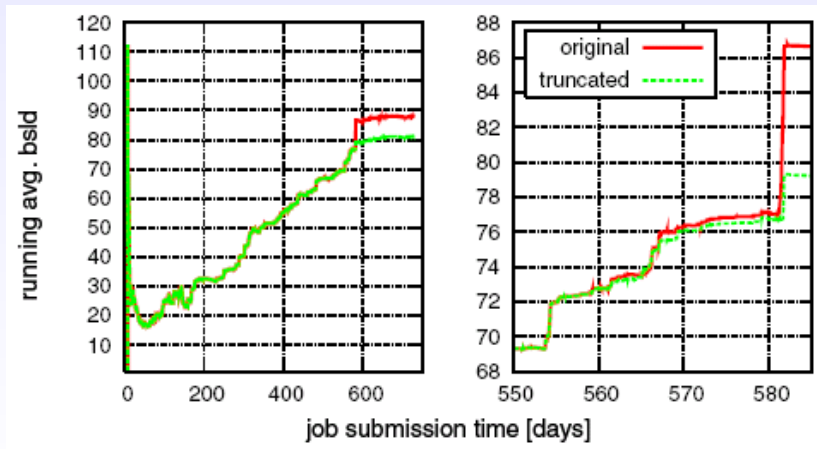


The length of a job running for 18 hours and 30 seconds is shortened by 30 seconds.

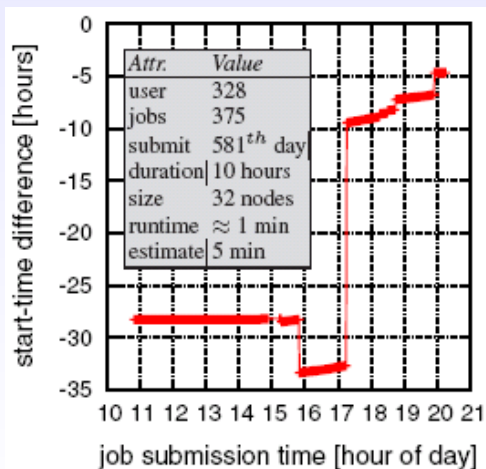
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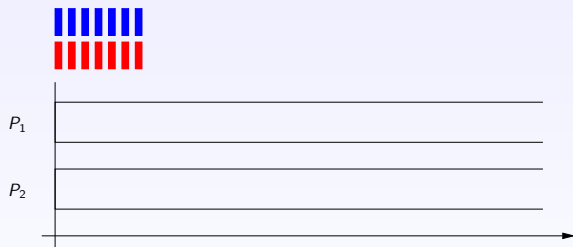
Robust solutions

An algorithm is said to be robust if its solutions stay close to the optimal when the actual parameters are slightly different from the theoretical parameters.

This solution stays optimal whatever the variations in the processors' performance : it is not sensitive to this parameter !

Robust solutions

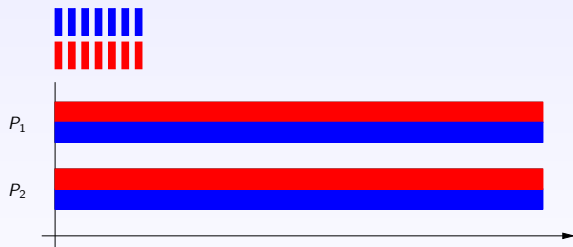
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An example of resolution : the problem

The problem :

- ▶ A master which has an output bandwidth B .
- ▶ p slaves, slave P_i being link with a bandwidth b_i and having a computational speed of c_i .
- ▶ n flows of tasks, each flow being a set of identical tasks.
A task from the flow k needs β_k units of communications and γ_k units of computation.
- ▶ $\rho_i^{(k)}$: throughput of application k on processor i .
 $\rho^{(k)} = \sum_i \rho_i^{(k)}$ is the throughput of application k on the whole platform.
- ▶ Objective : maximize $\min_k \rho^{(k)}$.

Dynamicity : a processor may fail (definitively).

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$$\forall i, \quad \sum_k \rho_i^{(k)} \gamma_k \leq c_i$$

- 2 Resource constraint : processors' bandwidths

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- 3 Resource constraint : the master output bandwidth

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$\textcircled{4}$ Objective if processor P_p fails :

$$\rho_{\bar{p}} = \min_k \sum_{i \neq p} \rho_i^{(k)}$$

$\textcircled{5}$ Objective : Maximize $\min \left\{ \min_p \frac{\rho_{\bar{p}}}{\rho_{\bar{p}}^{(\text{opt})}}, \min_k \sum_i \frac{\rho_i^{(k)}}{\rho_i^{(\text{opt})}} \right\}$

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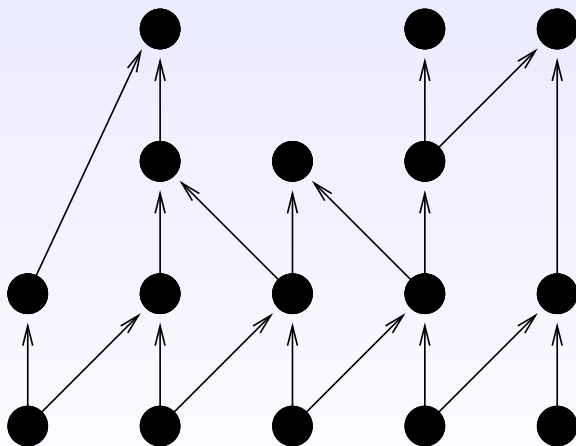
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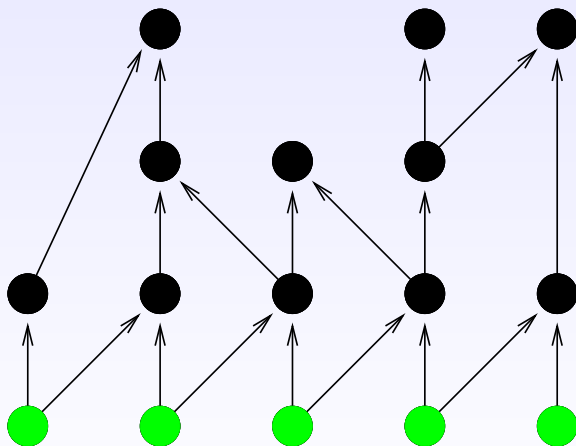
Giving priority to robustness : an extreme solution

Internet-Based Computing : problem motivation



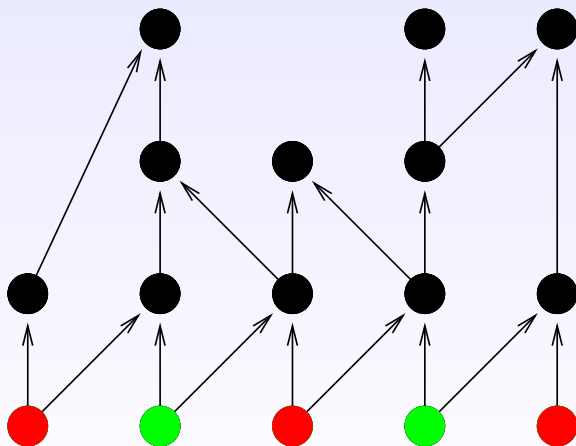
A possible schedule
(enabled, in process, completed)

Internet-Based Computing : problem motivation



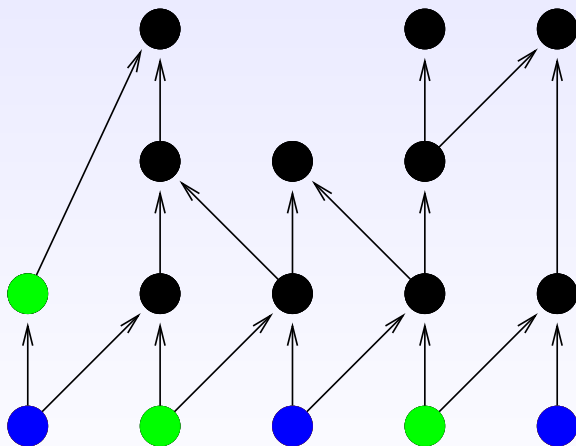
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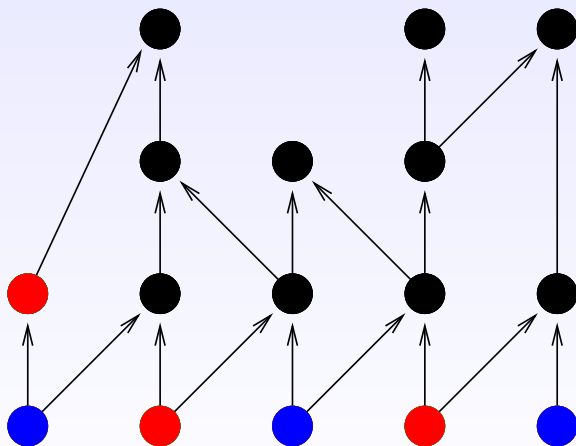
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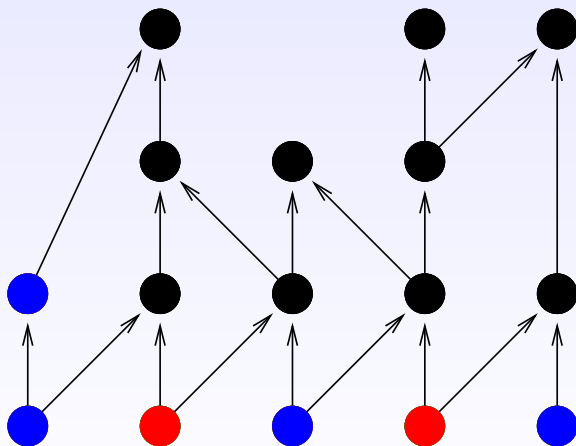
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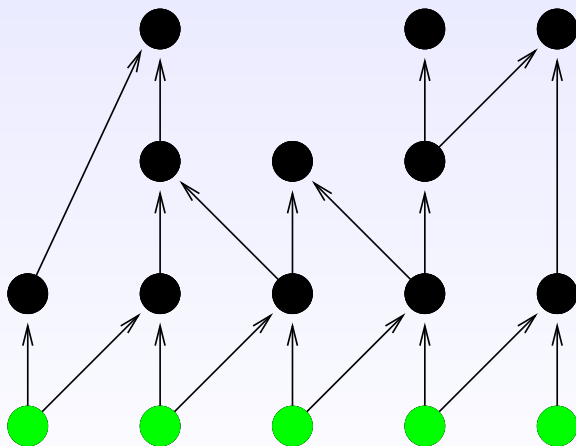
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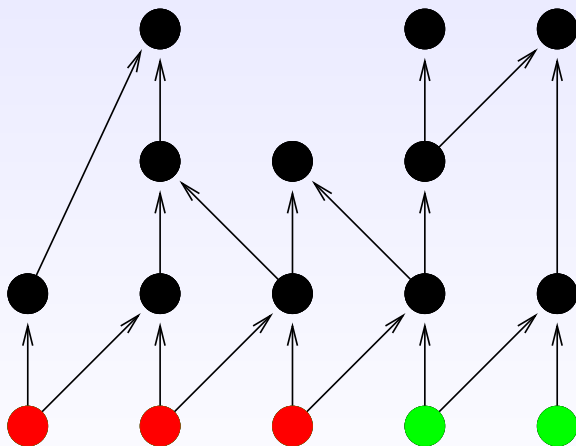
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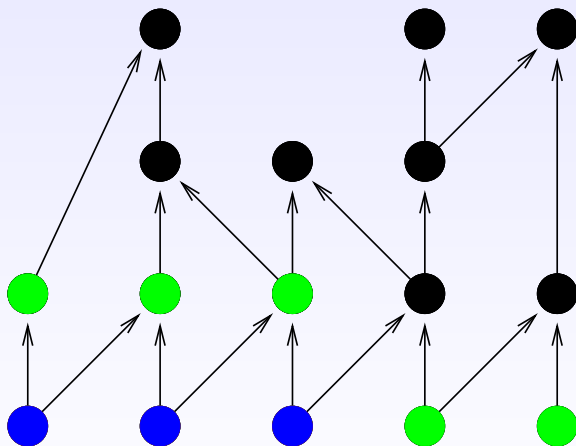
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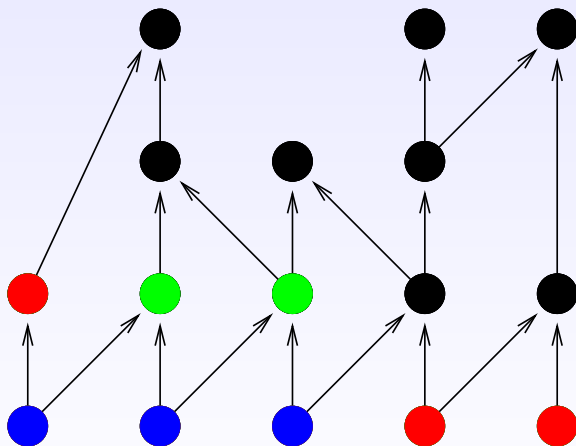
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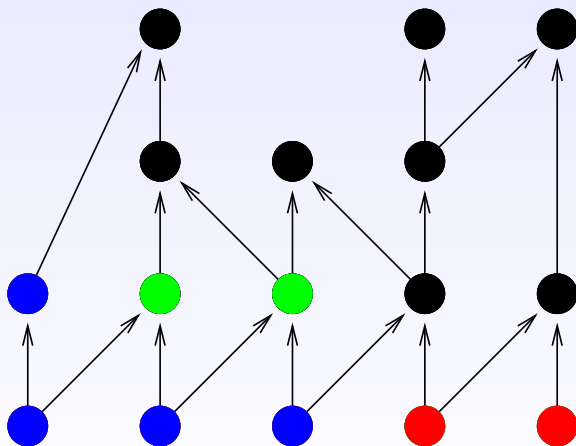
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Internet-Based Computing : the principle

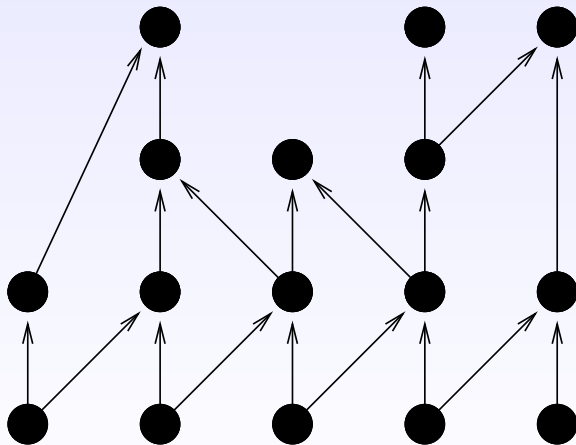
Framework

- ▶ Internet-Based computing : processor capacities are unknown, running times are thus unpredictable.
- ▶ Execution of a task graph (aka workflow) containing n tasks.

The principle

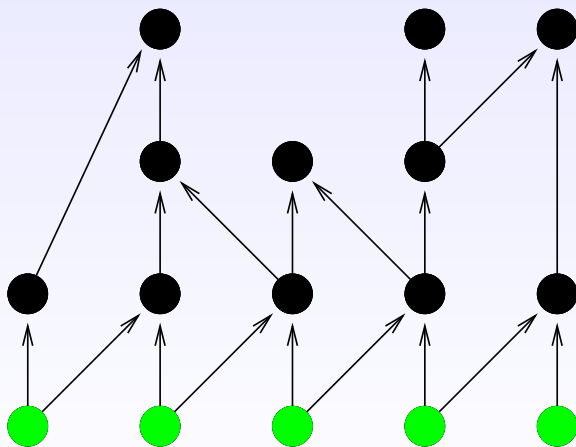
- ▶ Motivation : lessening the likelihood of the “gridlock” that can arise when a computation stalls pending computation of already allocated tasks.
- ▶ IC-optimal schedule : after t tasks have been executed, the number of eligible (=executable) tasks is maximal, for any $t \in [1, n]$

Internet-Based Computing : back to the example



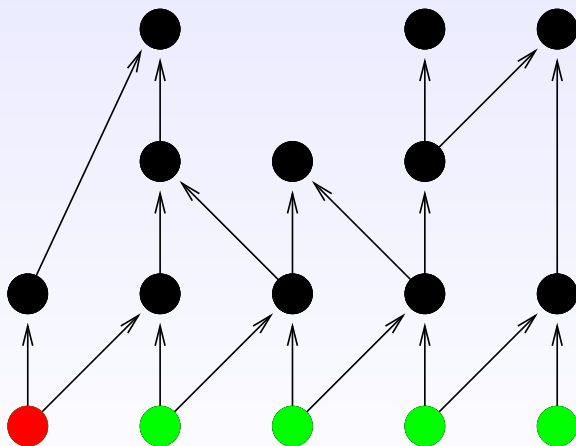
The IC-optimal schedule

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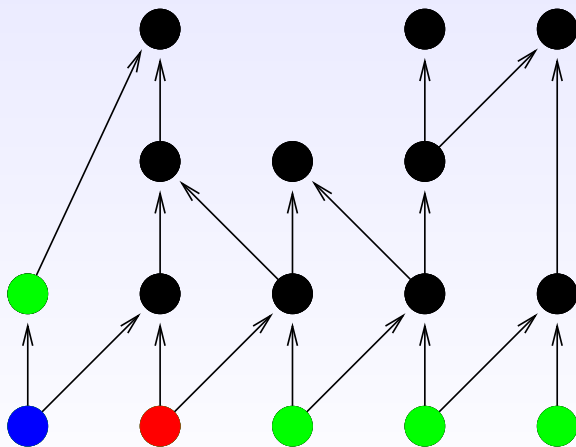
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Internet-Based Computing : back to the example



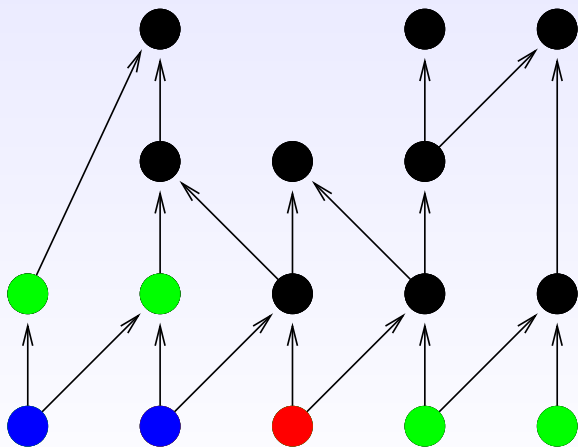
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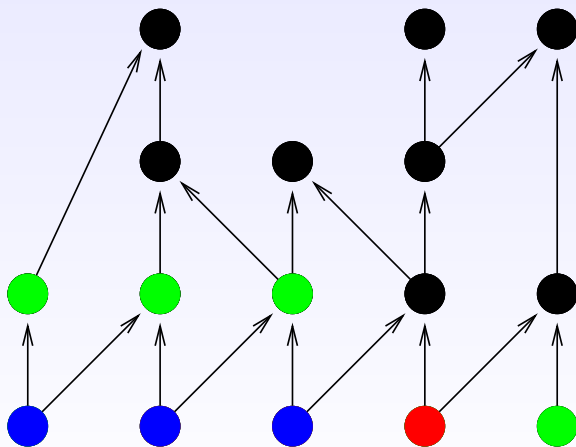
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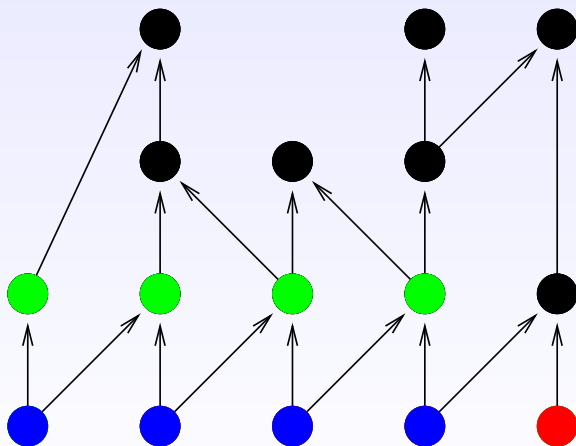
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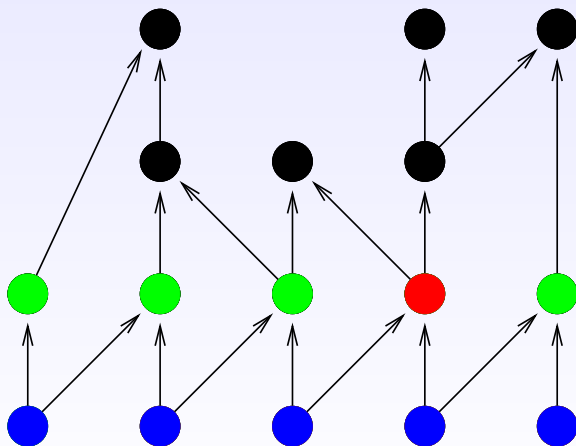
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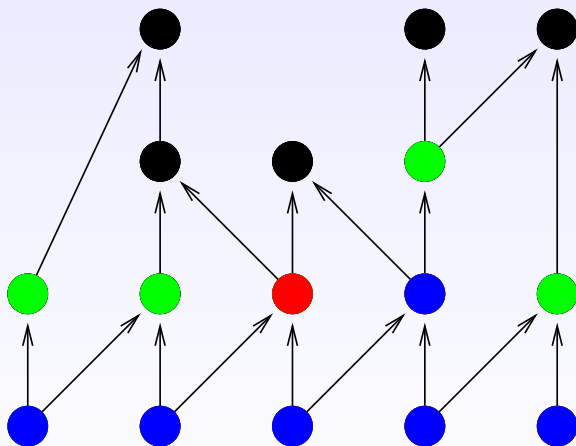
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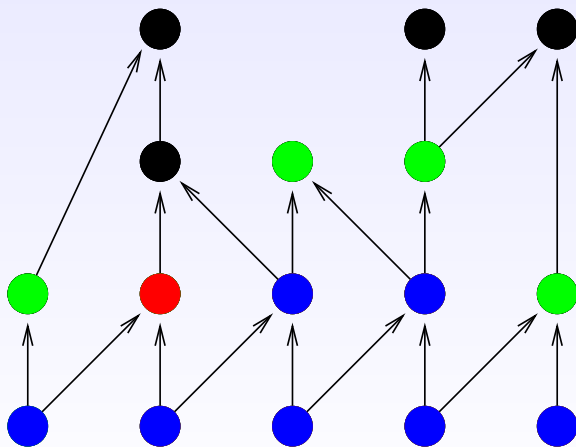
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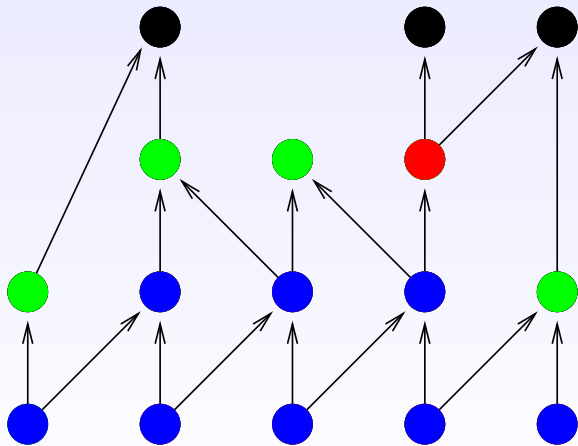
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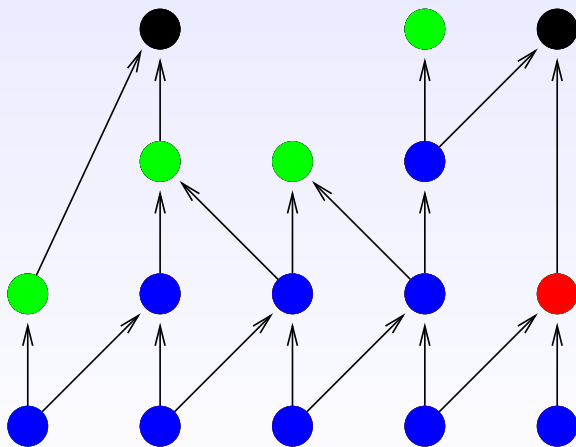
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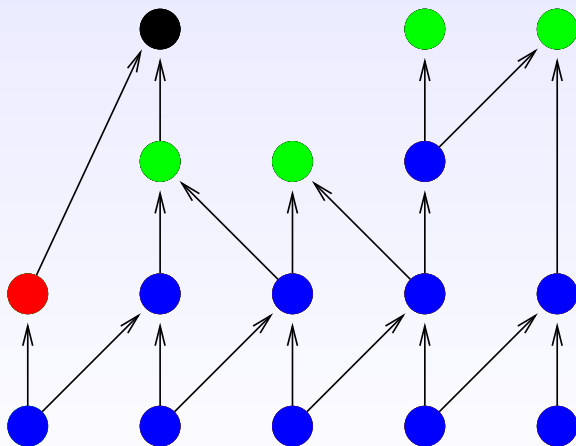
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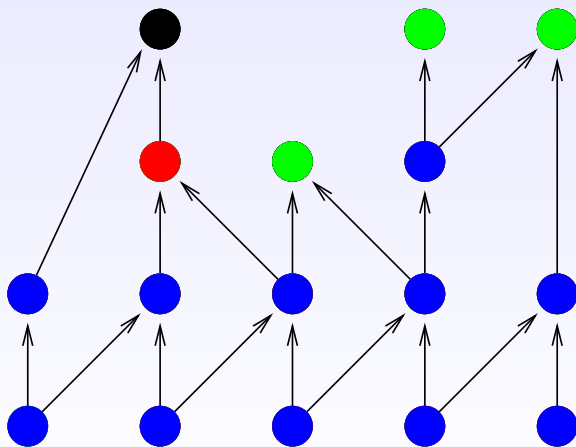
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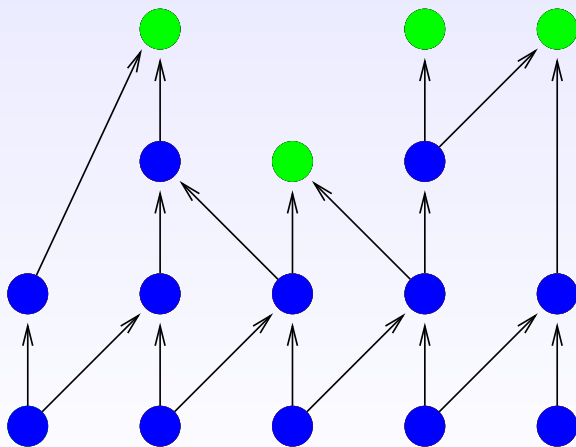
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The IC-optimal schedule

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Dynamic load-balancing

To cure (and no longer preventing) : the algorithm balance the load to take into account uncertainties and dynamicity.

General scheme

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If the objective is to minimize the running time, the comparison is obvious. How do we compare a time and some QoS ?

General scheme

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Each invocation has a cost : the invocations should only take place at “useful” instants
 - ▶ Compute a good solution using the observed parameters.
How do we predict the future from the past ?
 - ▶ Evaluate the cost of balancing the load
 - ▶ If the gain is larger than the cost : load-balance
If the objective is to minimize the running time, the comparison is obvious. How do we compare a time and some QoS ?

Network Weather Service

Distributed system which periodically monitors/records network and processor performance.

Also, allows to predict the future performance of the network and of the processors.

Does the past enable to predict the future ?

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Outline

- 1 The problem
- 2 Analyzing sensitivity and robustness
- 3 Dynamic load-balancing
- 4 How useful is old information ?**
- 5 Conclusion

How useful is old information ?

The problem

- ▶ The values used when taking decisions have already “aged” .
- ▶ Is it a problem ? Should we take this ageing into account ?

Framework : the platform

- ▶ A set of n servers.
- ▶ Tasks arrive according to a Poisson law of throughput λn , $\lambda < 1$.
- ▶ Task execution time : exponential law of mean 1.
- ▶ Each server executes in FIFO order the tasks it receives.
- ▶ We look at the time each task spent in the system (=flow).

Framework : information

There is a *bulletin board* on which are displayed the loads of the different processors.

This information may be wrong or approximate.

We only deal with the case in which this information is *old*.

This is the only information available to the tasks : they cannot communicate between each other and have some coordinated behavior.

The obvious strategies

- ▶ Random and uniform choice of the server.
 - ▶ Low overhead, finite length of queues.
- ▶ Random and uniform choice of d servers, the task being sent on the least loaded of the d servers.
 - ▶ Better than random, practical in distributed settings (poll a small number of processors)
- ▶ Task sent on the least loaded server.
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First model : periodic updates

- ▶ Each T units of time the bulletin board is updated with correct information.
- ▶ $P_{i,j}(t)$: fraction of queues with true load j but load i on the board, at time t
- ▶ $q_i(t)$ rate of arrivals at a queue with size i on the board at time t

System dynamics :

$$\frac{dP_{i,j}(t)}{dt} = P_{i,j-1}(t) \times q_i(t) + P_{i,j+1}(t) - P_{i,j}(t) \times q_i(t) - P_{i,j}(t)$$

First model : specific strategies

fractions of servers with (apparent) load i : $b_i(t) = \sum_j P_{i,j}(t)$

- ▶ choose the least loaded among d random servers

$$q_i(t) = \lambda \frac{\left(\sum_{j \geq i} b_j(t)\right)^d - \left(\sum_{j > i} b_j(t)\right)^d}{b_i(t)}$$

- ▶ choose the shortest queue (assume there is always a server with load 0)

$$\begin{aligned} q_0(t) &= \frac{\lambda}{b_0(t)} \\ q_i(t) &= 0 \quad i \neq 0 \end{aligned}$$

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Two possible resolutions

1. Theoretical :

- ▶ fixed point *when* $\frac{dP_{i,j}(t)}{dt} = 0$?
- ▶ fixed cycle on $[kT, (k+1)T]$
- ▶ can be solved using waiting queue theory (close form, but complex)

2. Practice with the above differential system :

- ▶ simulations, on truncated version of the system (bounding i and j)

3. Practice without the differential system :

- ▶ simulate 100 queues
- ▶ can use every distribution you want

After using 2 and 3 : comparable results on the same set of parameters.

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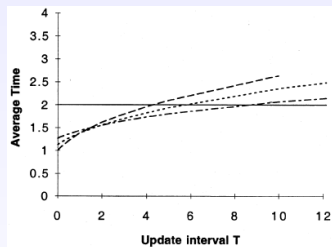
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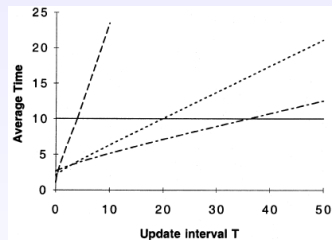
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First model : results

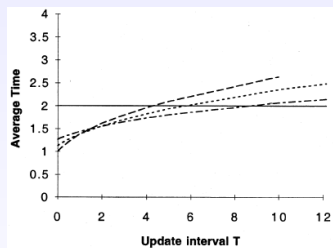


$n = 100$ and $\lambda = 0.5$

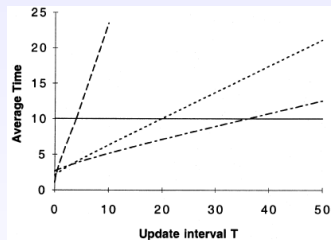


$n = 100$ and $\lambda = 0.9$

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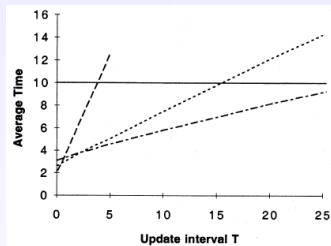


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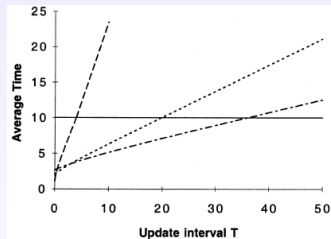


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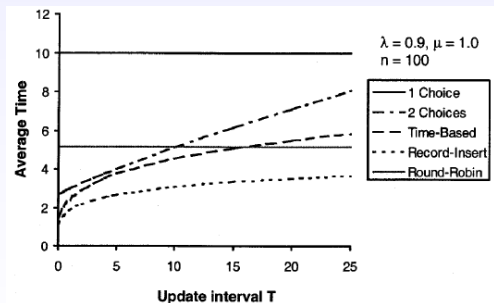
$n = 8$ and $\lambda = 0.9$



$n = 100$ and $\lambda = 0.9$

First model : more elaborated strategies

- ▶ *Time-based* : random choice among the servers which are supposed to be the least loaded.
- ▶ *Record-Insert* : centralized service in which each task updates the bulletin board by indicating on which server it is sent.

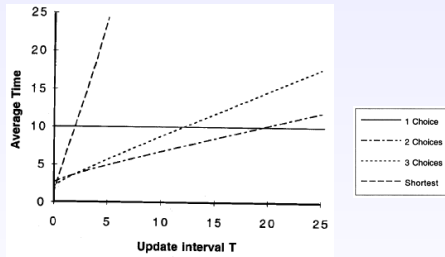


$n = 100$ and $\lambda = 0.9$

Second model : continuous updates

Model : continuous updates, but the information used is T units of time old.

$$n = 100 \text{ and } \lambda = 0.9$$

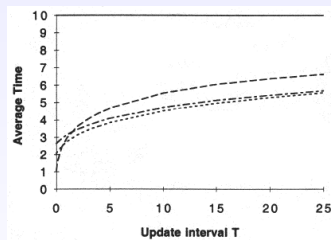
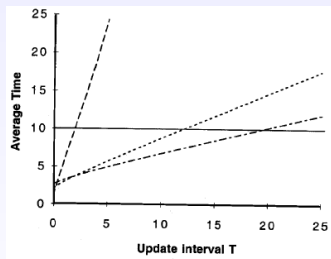


Age of information :
exactly T

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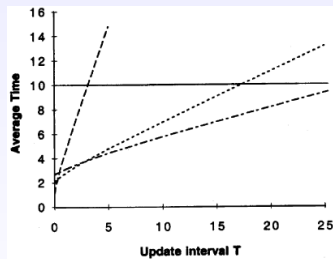
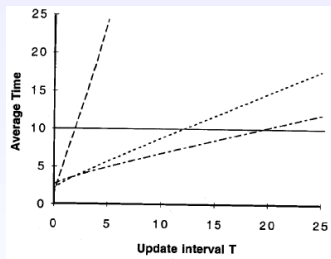
exactly T

exponential distribution, average T

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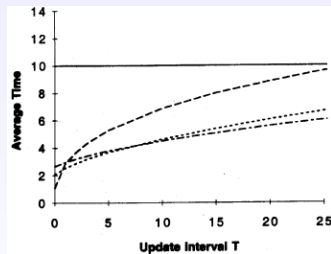
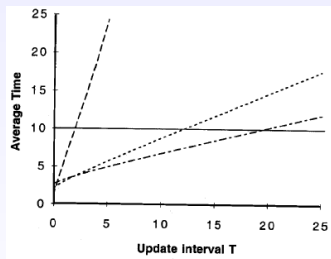
exactly T

uniform distribution on $[\frac{T}{2}; \frac{3T}{2}]$

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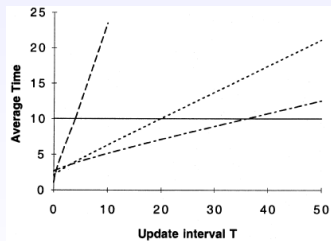
exactly T

uniform distribution on $[0; 2T]$

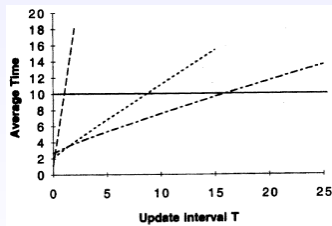
Third model : de-synchronized updates

The different servers updates their information in a de-synchronized manner, each following an exponential law of average T .

$$n = 100 \text{ and } \lambda = 0.9$$



Regular updates



De-synchronized updates.

And if some were cheating?

With a probability p a task does not choose between two randomly determined servers, but takes the least loaded of all servers.

T	p	Avg. Time All Tasks	Avg. Time 2 Choices	Avg. Time Shortest	Variance All Tasks	Variance 2 Choices	Variance Shortest
1	0.00	3.23286					
1	0.01	3.21072	3.22093	2.19877	5.73117	5.74186	3.63718
1	0.05	3.17061	3.21389	2.34814	5.62948	5.67621	4.02956
1	0.10	3.14132	3.20978	2.52474	5.58554	5.65450	4.54205
1	0.25	3.20098	3.25693	3.03311	6.05849	5.94553	6.35980
5	0.00	4.94051					
5	0.01	4.95386	4.95677	4.66575	13.8029	13.8821	11.8131
5	0.05	5.05692	5.05668	5.06154	14.4591	14.5105	13.4837
5	0.10	5.21456	5.17956	5.52974	15.6083	15.6597	15.7552
5	0.25	6.06968	5.70758	7.15609	23.6380	22.0182	26.9240
10	0.00	6.74313					
10	0.01	6.80669	6.80588	6.88703	26.4946	26.5391	22.0827
10	0.05	7.00344	6.97692	7.50776	28.4836	28.6189	25.6448
10	0.10	7.36957	7.26152	8.34185	32.7326	32.7395	31.6201
10	0.25	8.91193	8.23577	10.9422	54.8097	52.0265	57.6721

Some memory always help

- ▶ Studied scenario : a task is allocated to the “best” of two randomly determined servers.
- ▶ New scenario : a task is allocated to the “best” of two servers, one being randomly chosen and the other one being the least-loaded one —after the previous task was allocated— of the two processors considered by the previous task.
- ▶ The problem : the memorization requires some communications and centralization.

Complete vs. incomplete information

Complete information

- ▶ Requires some centralization (or total replication);
- ▶ Communications of the most remote elements to the “center” ;
- ▶ Obsolescence of the information.

Decentralized schedulers

- ▶ The local data are more up-to-date ;
- ▶ A local optimization does not always lead to a global optimization...

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Conclusion

- ▶ An obvious need to be able to cope with the dynamicity and the uncertainties.
- ▶ Crucial need to be able to model the dynamicity and the uncertainty.
- ▶ The static world is already complex enough !
- ▶ Where is the trade-off between the precision of the models and their usability ?
- ▶ Trade-off between static and dynamic approaches ?