Reclaiming the Energy of a Schedule, Models and Algorithms

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based on a work done with Anne Benoit, Fanny Dufossé and Yves Robert

4th December 2012

Cours d'ordonnancement (CR-07)

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Contents

Motivation

- Scheduling = Makespan minimization
  Difficulty of scheduling is to chose the right processor to assign the task to.

- **General mapping**
  If we are not tight on deadline, why not take our time?
  - Economical + environmental reasons: Energy consumption.
  - Affinities or security reasons: what if the tasks are pre-assigned to a processor?

Goal: "efficiently" use speed scaling

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Task graph model

Consider a task graph (directed acyclic graph) to be executed on a set of processors. Assume that the mapping is given.

Useful definition in a task graph

For every task T_i we define

- w_i its size/work
- s_i the speed of the processor which has task T_i assigned to.
- t_i the time when the computation of T_i ends.
- d_i the time it took to compute task T_i.
- $d_i s_i^3$ the energy consumed on task T_i by the system.

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Speed models

- CONTINUOUS: any speeds in $[0, s_{max}]$. A processor can change speed at any time.

- DISCRETE: set of speed: $\{s_1, ..., s_m\}$. Constant speed during the computation of a task, but it can change from task to task.

- VDD-HOPPING: close to the previous model, difference: we can switch speeds during a computation.

- INCREMENTAL: DISCRETE model where $s_1 = s_{min}$, $s_m = s_{max}$, and for all i, $s_i = s_{min} + i \cdot \delta$ for some δ.

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Gauss Fact

When Gauss wife asked him "How much do you love me?", he quantified it with an irrational number.
Unfortunately a computer will never be as good as Gauss.

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Example

Consider this DAG, with $s_{max} = 6$. Suppose deadline is $D = 1.5$.

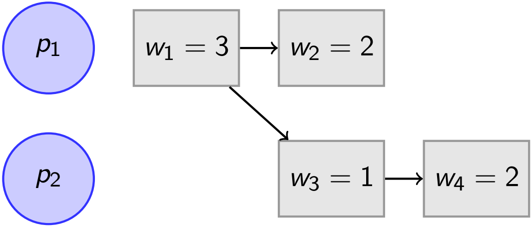

Figure : Execution graph for the example.

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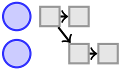


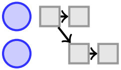
- CONTINUOUS: $(s_{max} = 6)$ $E_{opt}^{(c)} \simeq 109.6$.
  With the CONTINUOUS model, the optimal speeds are non rational values, and we obtain

$$s_1 = \frac{2}{3}(3 + 35^{1/3}) \simeq 4.18; \quad s_2 = s_1 \times \frac{2}{35^{1/3}} \simeq 2.56;$$

$$s_3 = s_4 = s_1 \times \frac{3}{35^{1/3}} \simeq 3.83.$$

- DISCRETE: $(s_1 = 2,\ s_2 = 5,\ s_3 = 6)$ $E_{opt}^{(d)} = 170$.
- INCREMENTAL: $(\delta = 2,\ s_{min} = 2,\ s_{max} = 6)$ $E_{opt}^{(i)} = 128$.
- VDD-HOPPING: $(s_1 = 2,\ s_2 = 5,\ s_3 = 6)$ $E_{opt}^{(v)} = 144$.

Example



- CONTINUOUS: $(s_{max} = 6)$ $E_{opt}^{(c)} \simeq 109.6$.
- DISCRETE: $(s_1 = 2,\ s_2 = 5,\ s_3 = 6)$ $E_{opt}^{(d)} = 170$.
  For the DISCRETE model, if we execute all tasks at speed $s_2^{(d)} = 5$, we obtain an energy $E = 8 \times 5^2 = 200$. A better solution is obtained with $s_1 = s_3^{(d)} = 6$, $s_2 = s_3 = s_1^{(d)} = 2$ and $s_4 = s_2^{(d)} = 5$, which turns out to be optimal.
- INCREMENTAL: $(\delta = 2,\ s_{min} = 2,\ s_{max} = 6)$ $E_{opt}^{(i)} = 128$.
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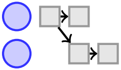

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- INCREMENTAL: $(\delta = 2,\ s_{min} = 2,\ s_{max} = 6)$ $E_{opt}^{(i)} = 128$. For the INCREMENTAL model, the reasoning is similar to the DISCRETE case, and the optimal solution is obtained by an exhaustive search: all tasks should be executed at speed $s_2^{(i)} = 4$.
- VDD-HOPPING: $(s_1 = 2,\ s_2 = 5,\ s_3 = 6)$ $E_{opt}^{(v)} = 144$.

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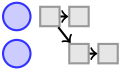

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- VDD-HOPPING: $(s_1 = 2,\ s_2 = 5,\ s_3 = 6)$ $E_{opt}^{(v)} = 144$.
  With the VDD-HOPPING model, we set $s_1 = s_2^{(d)} = 5$; for the other tasks, we run part of the time at speed $s_2^{(d)} = 5$, and part of the time at speed $s_1^{(d)} = 2$ in order to use the idle time and lower the energy consumption.

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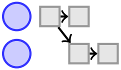

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Optimization goal

Energy-Performance-oriented objective

- Constraint on Deadline
- Minimize Energy Consumption:

Today's talk: comparison of all speed models in this regard.

We assume the mapping is already fixed.

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- Minimize Energy Consumption: $\sum_{i=1}^{n} w_i \times s_i^2$

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Hardness

The problem of minimizing energy when the scheduled is already fixed on p processors is:

- CONTINUOUS: Polynomial for some special graphs, geometric optimization in the general case.
- DISCRETE: NP-complete (reduction from 2-partition). We give an approximation.
- INCREMENTAL: NP-complete (reduction from 2-partition). We give an approximation.
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General problem: geometric programming

Reminder

For each task T_i we define

- w_i its size/work
- s_i the speed of the processor which has task T_i assigned to.
- t_i the time when the computation of T_i ends.

Objective function

$$\begin{aligned} &\text{Minimize} \quad \sum_{i=1}^{n} s_i^2 \times w_i \\ &\text{subject to (i) } t_i + \frac{w_j}{s_j} \leq t_j \text{ for each } (T_i, T_j) \in E \\ &\qquad\qquad \text{(ii) } t_i \leq D \text{ for each } T_i \in V \end{aligned}$$

Results for continuous speeds

- MinEnergy(G,D) can be solved in polynomial time when G is a tree
- MinEnergy(G,D) can be solved in polynomial time when G is a series-parallel graph (assuming $s_{max} = +\infty$)

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Linear program for VDD-HOPPING

Definition

G, n tasks, D deadline;

$s_1, ..., s_m$ be the set of possible processor speeds;

t_i is the finishing time of the execution of task T_i;

$\alpha_{(i,j)}$ is the *time* spent at speed s_j for executing task T_i

This makes us a total of $n(m+1)$ variables for the system.
Note that the total execution time of task T_i is $\sum_{j=1}^{m} \alpha_{(i,j)}$.

The objective function is:

$$\min \left(\sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{(i,j)} s_j^3 \right)$$

Linear program for Vdd-Hopping

The constraints are:

- $\forall 1 \leq i \leq n,\ t_i \leq D$: the deadline is not exceeded by any task;
- $\forall 1 \leq i, i' \leq n$ s.t. $T_i \rightarrow T_{i'},\ \ t_i + \sum_{j=1}^{m} \alpha_{(i',j)} \leq t_{i'}$: a task cannot start before its predecessor has completed its execution;
- $\forall 1 \leq i \leq n,\ \sum_{j=1}^{m} \alpha_{(i,j)} \times s_j \geq w_i$: task T_i is completely executed.
- $\forall 1 \leq i \leq n,\ t_i \geq \sum_{j=1}^{m} \alpha_{(i,j)}$: each task cannot finish until all work is done;

NP-completeness

Theorem

With the INCREMENTAL *model (and hence the* DISCRETE *model), finding the speed distribution that minimizes the energy consumption while enforcing a deadline D is NP-complete.*

NP-completeness

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PROOF: Reduction from 2-PARTITION,

- 1 processor, n independent tasks of weight (a_i).
- 2 speeds : $s_1 = 1/2$, $s_2 = 3/2$
- $D = 2W = \sum_{i=1}^{n} a_i$
- $E = W((3/2)^2 + (1/2)^2)$

Approximation results for DISCRETE and INCREMENTAL.

Proposition (Polynomial-time Approximation algorithms.)

- *With the* DISCRETE *model, for any integer $K > 0$, the* MINENERGY*(G,D) problem can be approximated within a factor*

$$(1+\frac{\alpha}{s_1})^2 \times (1+\frac{1}{K})^2$$

where $\alpha = \max_{1 \leq i < m}\{s_{i+1} - s_i\}$, in a time polynomial in the size of the instance and in K.

- *With the* INCREMENTAL *model, the same result holds where $\alpha = \delta$ ($s_1 = s_{min}$).*

Approximation results for DISCRETE and INCREMENTAL.

Proposition (Comparaison to the optimal solution:)

For any integer $\delta > 0$, any instance of MINENERGY*(G,D) with the* CONTINUOUS *model can be approximated within a factor* $(1 + \frac{\delta}{s_{min}})^2$ *in the* INCREMENTAL *model with speed increment δ.*

TOC

The problem of minimizing energy when the scheduled is already fixed on p processors is:

CONTINUOUS: Polynomial for some special graphs, geometric optimization in the general case.

DISCRETE and INCREMENTAL: NP-complete. However we were able to give an approximation.

VDD-HOPPING: Polynomial (linear programming).

- Bi-criteria Energy/Deadline optimization problem
- Mapping already given.
- Theoretical foundations for a comparative study of energy models.

Thanks for listening. Any questions?