Multi-criteria Scheduling of Pipeline Workflows

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Introduction and motivation

- Mapping applications onto parallel platforms
  Difficult challenge
- Heterogeneous clusters, fully heterogeneous platforms
  Even more difficult!
- Structured programming approach
  - Easier to program (deadlocks, process starvation)
  - Range of well-known paradigms (pipeline, farm)
  - Algorithmic skeleton: help for mapping

Mapping pipeline skeletons onto
communication homogeneous platforms

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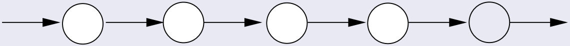
Multi-criteria scheduling of workflows

Workflow



Several consecutive data-sets enter the application graph.

Multi-criteria?

Period: time interval between the beginning of execution of two consecutive data sets

Latency: maximal time elapsed between beginning and end of execution of a data set

Bi-criteria!

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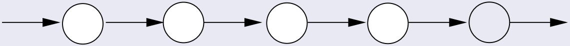

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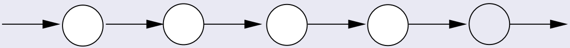

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Why restrict to pipelines?

Pipeline: linear application graph

Chains-on-chains partitioning problem

- no communications
- identical processors

Load-balance **contiguous** tasks

5 7 3 4 8 1 3 8 2 9 7 3 5 2 3 6

With $p = 4$ identical processors?

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$T_{\text{period}} = 20$

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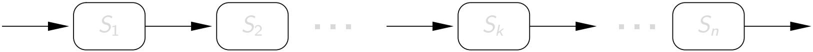
Rule of the game

- Map each pipeline stage on a single processor
- Goal: minimize period AND minimize latency
- Several mapping strategies



The pipeline application

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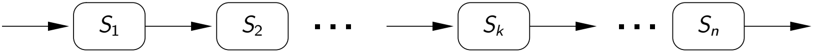

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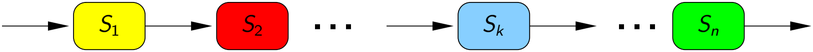

ONE-TO-ONE MAPPING

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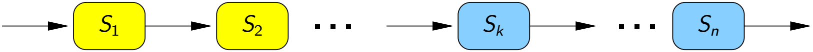

INTERVAL MAPPING

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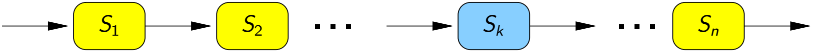

General Mapping

Major contributions

Theory
- Definition of bi-criteria mapping
- Problem complexity
- Linear programming formulation

Practice
- Heuristics for INTERVAL MAPPING on clusters
- Experiments to compare heuristics and evaluate their performance
- Simulation of a real world application

Major contributions

Theory	Definition of bi-criteria mapping Problem complexity Linear programming formulation
Practice	Heuristics for INTERVAL MAPPING on clusters Experiments to compare heuristics and evaluate their performance Simulation of a real world application

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2. Complexity results
3. Linear programming formulation
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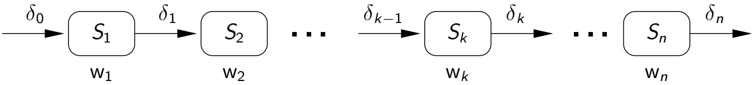
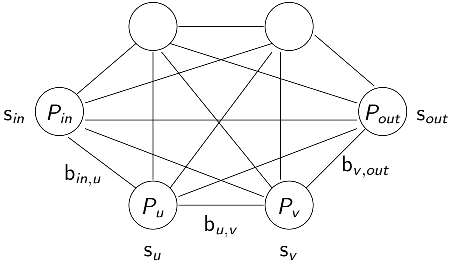
The application



- n stages $\mathcal{S}_k$, $1 \leq k \leq$ n
- $\mathcal{S}_k$:
  - receives input of size δ_{k-1} from $\mathcal{S}_{k-1}$
  - performs w_k computations
  - outputs data of size δ_k to $\mathcal{S}_{k+1}$
- $\mathcal{S}_0$ and $\mathcal{S}_{n+1}$: virtual stages representing the outside world

The platform



- p processors P_u, $1 \leq u \leq$ p, fully interconnected
- s_u: speed of processor P_u
- bidirectional link $\text{link}_{u,v} : P_u \rightarrow P_v$, bandwidth $b_{u,v}$
- one-port model: each processor can either send, receive or compute at any time-step

Different platforms

Fully Homogeneous – Identical processors ($s_u = s$) and links ($b_{u,v} = b$): typical parallel machines

Communication Homogeneous – Different-speed processors ($s_u \neq s_v$), identical links ($b_{u,v} = b$): networks of workstations, clusters

Fully Heterogeneous – Fully heterogeneous architectures, $s_u \neq s_v$ and $b_{u,v} \neq b_{u',v'}$: hierarchical platforms, grids

In this talk we restrict to *Communication Homogeneous* platforms!

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Mapping problem: Interval Mapping

- Partition of [1..n] into m intervals $I_j = [d_j, e_j]$ (with $d_j \leq e_j$ for $1 \leq j \leq m$, $d_1 = 1$, $d_{j+1} = e_j + 1$ for $1 \leq j \leq m - 1$ and $e_m = \text{n}$)
- Interval I_j mapped onto processor $P_{\text{alloc}(j)}$

$$T_{\text{period}} = \max_{1 \leq j \leq m} \left\{ \frac{\delta_{d_j - 1}}{\text{b}} + \frac{\sum_{i=d_j}^{e_j} \text{w}_i}{\text{s}_{\text{alloc}(j)}} + \frac{\delta_{e_j}}{\text{b}} \right\}$$

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Objective function?

Mono-criterion

- Minimize T_{period}
- Minimize T_{latency}

Bi-criteria

- How to define it?
  Minimize $\alpha.T_{\text{period}} + \beta.T_{\text{latency}}$?
- Values which are not comparable

- Minimize T_{period} for a fixed latency
- Minimize T_{latency} for a fixed period

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Complexity results

Lemma

The optimal mapping which minimizes latency can be determined in polynomial time.

Assign whole pipeline to fastest processor!
No communications to pay in this case.

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Complexity results

Minimize the period?

Chains-on-chains problem with different speed processors!

Definition (HETERO-1D-PARTITION-DEC)

Given n elements $a_1, a_2, \ldots, a_n$, p values $s_1, s_2, \ldots, s_p$ and a bound K, can we find a partition of $[1..n]$ into p intervals $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_p$, with $\mathcal{I}_k = [d_k, e_k]$ and $d_k \leq e_k$ for $1 \leq k \leq p$, $d_1 = 1$, $d_{k+1} = e_k + 1$ for $1 \leq k \leq p-1$ and $e_p = n$, and a permutation σ of $\{1, 2, \ldots, p\}$, such that

$$\max_{1 \leq k \leq p} \frac{\sum_{i \in \mathcal{I}_k} a_i}{s_{\sigma(k)}} \leq K \quad ?$$

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Complexity results

Theorem 1

The HETERO-1D-PARTITION-DEC problem is NP-complete.

Involved reduction

Theorem 2

The period minimization problem for pipeline graphs is NP-complete.

Direct consequence from Theorem 1

All bi-criteria optimization problems are NP-complete on *Communication Homogeneous* platforms.

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Integer linear programming

- Integer LP to solve INTERVAL MAPPING on *Communication Homogeneous* platforms
- Many integer variables: no efficient algorithm to solve
- Approach limited to small problem instances
- Absolute performance of the heuristics for such instances

Linear program: variables

- T_{opt}: period or latency of the pipeline, depending on the objective function

Boolean variables:

- $x_{k,u}$: 1 if S_k on P_u
- $y_{k,u}$: 1 if $\mathcal{S}_k$ and S_{k+1} both on P_u
- $z_{k,u,v}$: 1 if $\mathcal{S}_k$ on P_u and S_{k+1} on P_v

Integer variables:

- first_u and last_u: integer denoting first and last stage assigned to P_u (to enforce interval constraints)

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Linear program: constraints

Constraints on procs and links:

- $\forall k \in [0..\mathrm{n}+1], \quad \sum_u x_{k,u} = 1$
- $\forall k \in [0..\mathrm{n}], \quad \sum_{u \neq v} z_{k,u,v} + \sum_u y_{k,u} = 1$
- $\forall k \in [0..\mathrm{n}], \forall u, v \in [1..\mathrm{p}] \cup \{in, out\}, u \neq v, x_{k,u} + x_{k+1,v} \leq 1 + z_{k,u,v}$
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Constraints on intervals:

- $\forall k \in [1..\mathrm{n}], \forall u \in [1..\mathrm{p}], \quad \mathrm{first}_u \leq k.x_{k,u} + \mathrm{n}.(1 - x_{k,u})$
- $\forall k \in [1..\mathrm{n}], \forall u \in [1..\mathrm{p}], \quad \mathrm{last}_u \geq k.x_{k,u}$
- $\forall k \in [1..\mathrm{n}-1], \forall u, v \in [1..\mathrm{p}], u \neq v,$
  $\mathrm{last}_u \leq k.z_{k,u,v} + \mathrm{n}.(1 - z_{k,u,v})$
- $\forall k \in [1..\mathrm{n}-1], \forall u, v \in [1..\mathrm{p}], u \neq v, \quad \mathrm{first}_v \geq (k+1).z_{k,u,v}$

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Linear program: constraints

$$\forall u \in [1..p], \sum_{k=1}^{n} \left\{ \left(\sum_{t \neq u} \frac{\delta_{k-1}}{b} z_{k-1,t,u} \right) + \frac{w_k}{s_u} x_{k,u} + \left(\sum_{v \neq u} \frac{\delta_k}{b} z_{k,u,v} \right) \right\} \leq T_{\text{period}}$$

$$\sum_{u=1}^{p} \sum_{k=1}^{n} \left[\left(\sum_{t \neq u, t \in [1..p] \cup \{in,out\}} \frac{\delta_{k-1}}{b} z_{k-1,t,u} \right) + \frac{w_k}{s_u} x_{k,u} \right] + \left(\sum_{u \in [1..p] \cup \{in\}} \frac{\delta_n}{b} z_{n,u,out} \right) \leq T_{\text{latency}}$$

Min period with fixed latency

$T_{\text{opt}} = T_{\text{period}}$

T_{latency} is fixed

Min latency with fixed period

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Heuristics

- Target clusters: *Communication Homogeneous* platforms and INTERVAL MAPPING
- n stages
- p processors

Two sets of heuristics

- Minimizing latency for a fixed period
- Minimizing period for a fixed latency

Minimizing Latency for a Fixed Period (1/2)

Sp mono P: Splitting mono-criterion

- Map the whole pipeline on the fastest processor.
- At each step, select used processor j with largest period.
- Try to split its stage interval, giving some stages to the next fastest processor j' in the list (not yet used).
- Split interval at any place, and either assign the first part of the interval on j and the remainder on j', or the other way round. Solution which minimizes $max(period(j), period(j'))$ is chosen if better than original solution.
- Break-conditions:
  Fixed period is reached or period cannot be improved anymore.

Minimizing Latency for a Fixed Period (2/2)

3-Explo mono: 3-Exploration mono-criterion – Select used processor j with largest period and split its interval into three parts.

3-Explo bi: 3-Exploration bi-criteria – More elaborated choice where to split: split the interval with largest period so that $max_{i \in \{j,j',j''\}}(\frac{\Delta latency}{\Delta period(i)})$ is minimized.

Sp bi P: Splitting bi criteria – Binary search over latency: at each step choose split that minimizes $max_{i \in \{j,j'\}}(\frac{\Delta latency}{\Delta period(j)})$ within the authorized latency increase.

$\Delta latency$: T_{latency} after split - T_{latency} before split

$\Delta period$: $T_{\text{period}}(j)$ before split - $T_{\text{period}}(j)$ after split

Minimizing Period for a Fixed Latency

Sp mono L: Splitting mono-criterion – Similar to **Sp mono P** with different break condition: splitting is performed as long as fixed latency is not exceeded.

Sp bi L: Splitting bi criteria – Similar to **Sp mono L**, but at each step choose solution that minimizes $max_{i \in \{j,j'\}}(\frac{\Delta latency}{\Delta period(i)})$ while fixed latency is not exceeded.

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2. Complexity results
3. Linear programming formulation
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Plan of experiments

- Assess performance of polynomial heuristics
- Random applications, $n \in \{5, 10, 20, 40\}$ stages
- Random *Communication Homogeneous* platforms, $p = 10$ and $p = 100$ processors
- $b = 10$, proc. speed between 1 and 20
- Relevant parameters: ratios $\frac{\delta}{b}$ and $\frac{w}{s}$
- Average over 50 similar random appli/platform pairs

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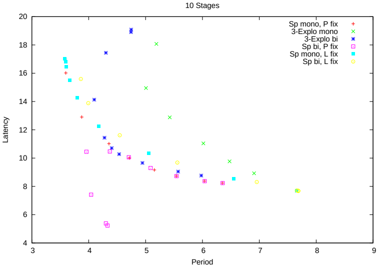
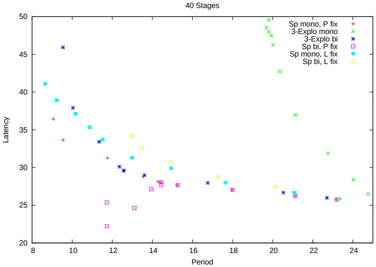
Experiment 1 - balanced comm/comp, hom comm

- communication time $\delta_i = 10$
- computation time between 1 and 20
- 10 processors



10 stages.
☺ Sp bi P
☹ 3-Explo mono



40 stages.
☺ Sp mono P
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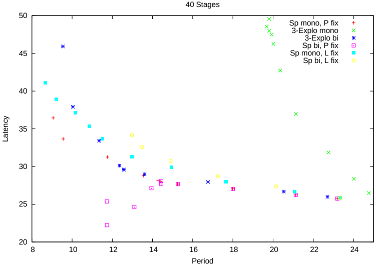
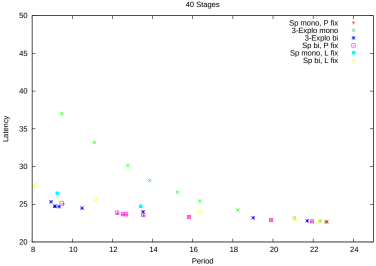
Experiment 1 - balanced comm/comp, hom comm

- communication time $\delta_i = 10$
- computation time between 1 and 20
- 10 vs. 100 processors



40 stages, 10 procs.
☺ Sp mono P
☹ 3-Explo mono



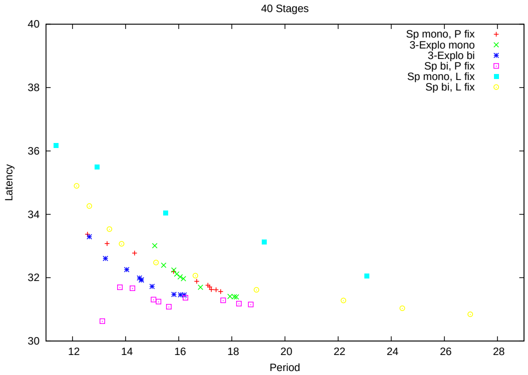
40 stages, 100 procs.
☺ 3 Explo bi
☹ 3-Explo mono

Experiment 2 - balanced comm/comp, het comm

- communication time between 1 and 100
- computation time between 1 and 20

100 processors.
40 stages.

☺ Sp bi P
☹ 3-Explo mono



Experiment 3 - large computations

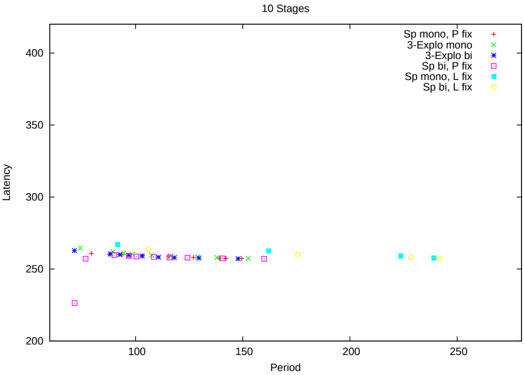
- communication time between 1 and 20
- computation time between 10 and 1000

100 processors.

5 stages.

☺ Sp bi P

☹ Sp mono L



Experiment 4 - small computations

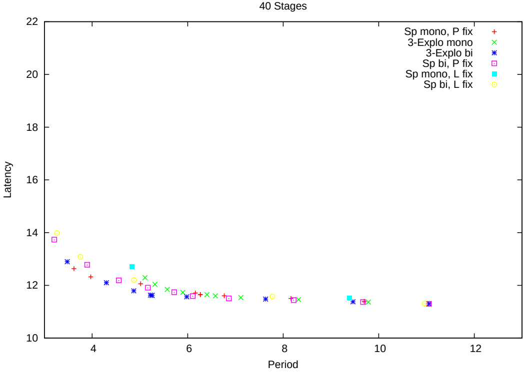
- communication time between 1 and 20
- computation time between 0.01 and 10

100 processors.

5 stages.

☺ 3-Explo bi

☹ Sp mono L



Failure Thresholds for 10 procs

Failure threshold: largest fixed value (latency or period) for which a heuristic does not find a solution.

Exp.	Heuristic	Number of stages			
		5	10	20	40
E1	Sp mono P	3.0	3.3	5.0	5.0
	3-Explo mono	3.0	4.7	9.0	18.0
	3-Explo bi	3.0	4.0	5.0	5.0
	Sp bi P	3.3	3.3	6.0	10.0
	Sp mono L	4.5	6.0	13.0	25.0
	Sp bi L	4.5	6.0	13.0	25.0
E3	Sp mono P	50.0	70.0	100.0	250.0
	3-Explo mono	50.0	140.0	450.0	950.0
	3-Explo bi	50.0	90.0	250.0	400.0
	Sp bi P	100.0	140.0	300.0	650.0
	Sp mono L	140.0	270.0	500.0	1000.0
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Small values are good !

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Summary of experiments

- Performance of bi-criterion heuristics highly depends on the number of available processors.
- Small number of processors:
  - Sp mono P and Sp mono L
  - Small latencies: Sp bi P
- Increasing number of processors:
  - Sp bi P and Sp bi L

Summary of experiments

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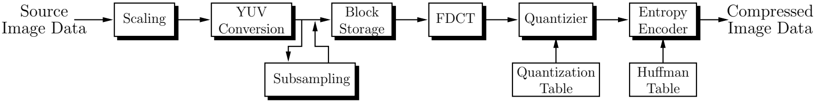
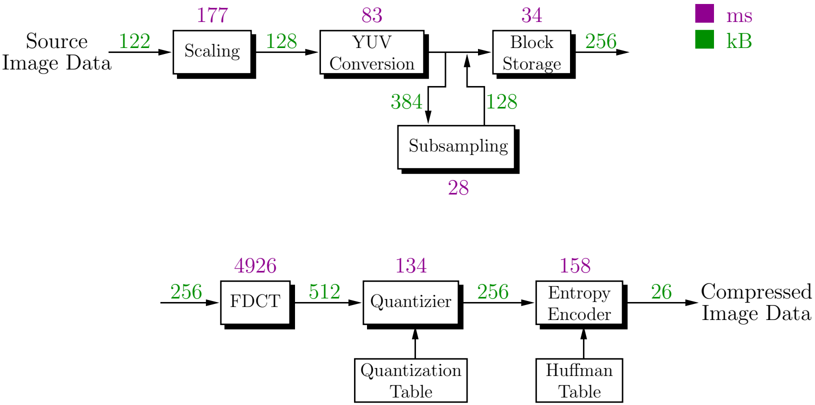
Real World Application

The JPEG encoder

- Image processing application
- JPEG: standardized interchange format
- Data compression
- 7 stages



JPEG Encoder



Simulation environment

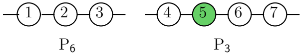
- MPI application
- Message passing + sleep()
- Homogeneous processors (Salle Europe)
- Simulation of heterogenity
- Mapping 7 stages on 10 processors

Influence of the fixed parameter on the solution

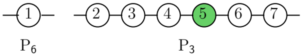
LP solutions:

minimize latency

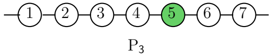
$P_{fix} = 310$ $L_{opt} = 337,575$



$P_{fix} = 320$ $L_{opt} = 336,729$

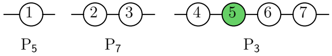


$P_{fix} = 330$ $L_{opt} = 322,700$

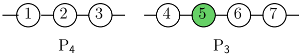


minimize period

$L_{fix} = 370$ $P_{opt} = 307,319$



$L_{fix} = 340$ $P_{opt} = 307,319$



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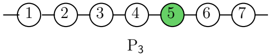

Overview of the different solutions

Minimize latency with $T_{period} = 310$

Algorithm	Intervals	Processors	Latency	Simu
LP	[1-3][4-7]	6,3	337,575	
Sp mono P	[1-3][4-7]	4,3	337,575	308,2
3-Explo mono	[1][2-3][4-7]	4,6,3	350,57	310,02
3-Explo mono	[1][2-3][4-7]	6,4,3	350,57	310,06
Sp bi P	does not succeed		(322,7)	307,02

Overview of the different solutions

Minimize period with $T_{\text{latency}} = 370$

Algorithm	Intervals	Processors	Period	Simu
LP	[1][2-3][4-7]	5,7,3	307,319	
Sp mono L	[1-3][4-7]	4,3	307,319	308,15
Sp bi L	[1-7]	3	322,7	307,00

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Related work

Subhlok and Vondran– Extension of their work (pipeline on hom platforms)

Mapping pipelined computations onto clusters and grids– DAG [Taura et al.], DataCutter [Saltz et al.]

Energy-aware mapping of pipelined computations [Melhem et al.], three-criteria optimization

Mapping pipelined computations onto special-purpose architectures– FPGA arrays [Fabiani et al.]. Fault-tolerance for embedded systems [Zhu et al.]

Mapping skeletons onto clusters and grids– Use of stochastic process algebra [Benoit et al.]

Conclusion

Theoretical side

- Bi-criteria mapping problem on *Communication Homogeneous* platforms
- Pipeline structured applications
- Complexity study
- Linear programming formulation

Practical side

- Design of several polynomial heuristics
- Extensive simulations to compare their performance
- Simulation of a real world application
- Evaluation

Future work

Theory

- Extension to stage replication
- Extension to fork, fork-join and tree workflows
- Multi-criteria: reliability in addition to period and latency

Practice

- Real experiments on heterogeneous clusters with bigger pipeline applications, using MPI
- Comparison of effective performance against theoretical performance