Divisible load scheduling (Scheduling part 2)

Anne Benoit

ENS Lyon

Anne.Benoit@ens-lyon.fr

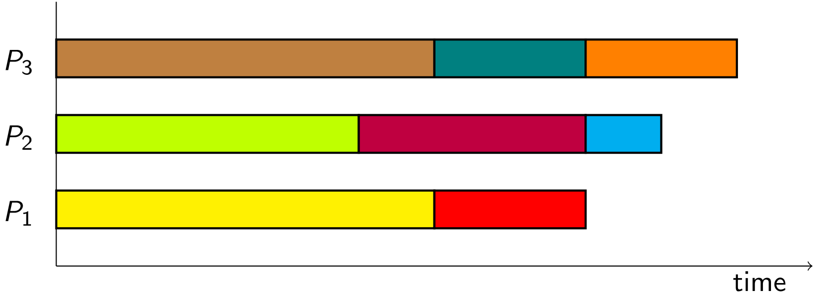
http://graal.ens-lyon.fr/~abenoit

CR02 - 2016/2017

Why are many scheduling problems hard?

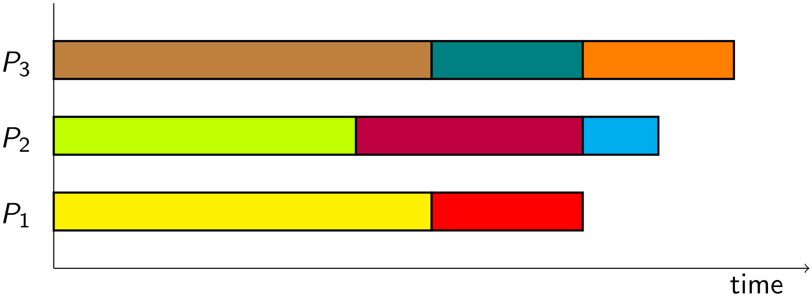
- We have seen in the last class that many scheduling problems are $\mathcal{NP}$-complete
- It turns out that this is often because of integer constraints
  - The same reason why bin packing is difficult: you can't cut boxes into pieces!
- This is somewhat the same idea as the use of preemption
  - $P||C_{max}$ is $\mathcal{NP}$-complete
  - $P|pmtn|C_{max}$ is in $\mathcal{P}$!
- Let's see this on an example

$P3||C_{max}$ example schedule (offline)



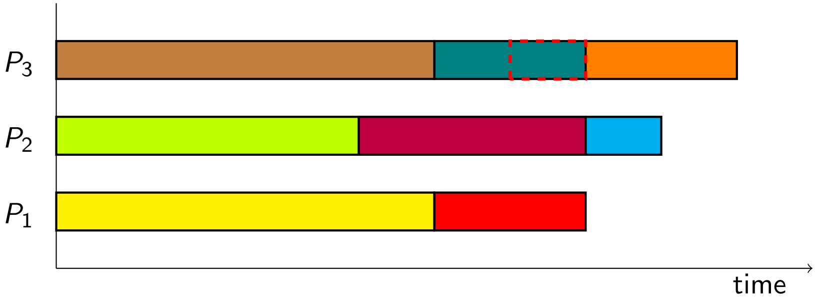
$\sum a_i = 21;\ C_{max} = 8$

$P3|pmtn|C_{max}$ example schedule (offline)

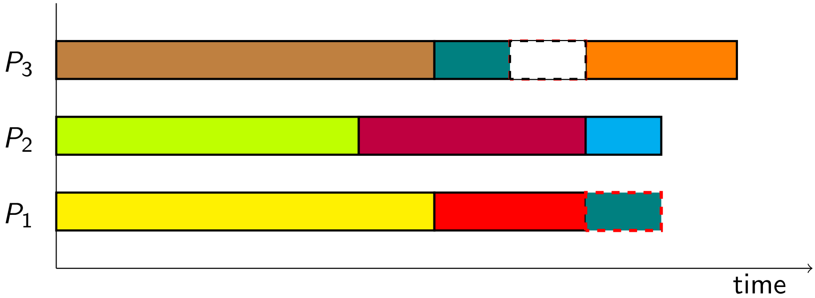


Let's modify the schedule using preemption

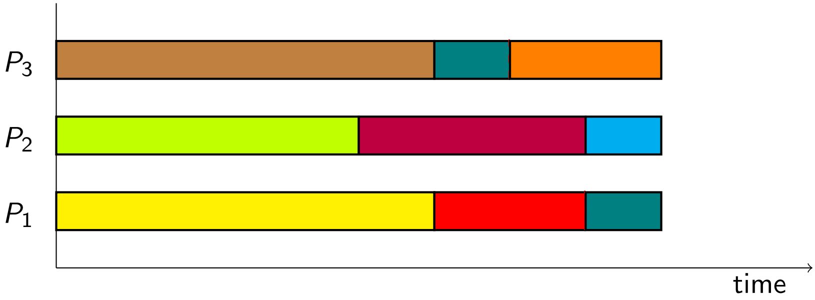
$P3|pmtn|C_{max}$ example schedule (offline)



$P3|pmtn|C_{max}$ example schedule (offline)



$P3|pmtn|C_{max}$ example schedule (offline)



$\sum a_i = 21$; $C_{max} = 7$ (optimal: no idle time)

Cutting tasks

- By "cutting" a task in two, we are able to have all processors finish at the same time
  - Zero idle time means the schedule is optimal
- If we were able to cut all tasks into tiny bits, then we would always be able to achieve zero idle time
  - Again, if you have a knife, bin-packing is easy
- Question: Can this be done for real-world applications?

Divisible load applications

- It turns out that many useful applications consist of very large numbers of small, independent, and identical tasks.
  - task execution time $<<$ application execution time
  - tasks can be completed in any order
  - tasks all do the same thing, but on different data
- Example applications:
  - Ray tracing (1 task = 1 photon)
  - MPEG encoding of a movie (1 task = 1 frame)
  - Seismic event processing (1 task = 1 event)
  - High-energy physics (1 task = 1 particle)
- These applications are termed Divisible Loads (DLs)
  - So fine-grain that a continuous load assumption is valid
- By the previous example, DL scheduling is trivial...

Input data?

- In the previous class, there was no notion of “input data”
  - The implicit assumption was that tasks had access to whatever data they needed
- But in many real-world applications, including DLs, there is some input data for each task
- This input data is stored at some location (the hard drive of a computer)
- If the DL is large, one wants to enroll multiple computers
- Problem: The data must be transferred over the network, which takes time

Here comes the network

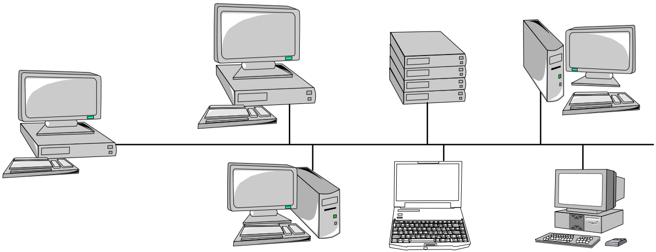
- When scheduling applications on processors within a single machines (multi-core), one often ignores data transfers (questionable)
- When scheduling applications on distributed platforms, one has to schedule both computation and communication
- Many theoretical scheduling results ignore the network component
    - In some cases, communication can be seen as computation, e.g., a computation task depends on a communication task and each type of task can only run on a subset of the "resources"
- Let us define first a very simple execution and platform model. . .

Master-worker execution

- The computer that holds all input data is called the master (P_0)
- All m other computers are called the workers ($P_1, \ldots, P_m$)
- All P_i's can compute (master and workers)
- P_0 initially holds W_{total} units of load
- P_0 allocates n_i units of load to worker P_i
- $\sum_i n_i = W_{total}$
- For now, we completely ignore output data (assume it has size zero)

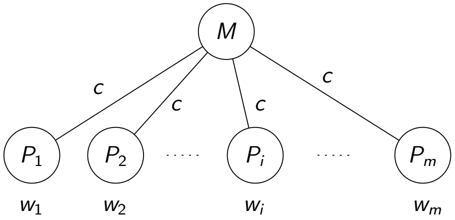
Outline

Bus-shaped platform - practice



A bit 1980's ☺

Bus-shaped platform - theory

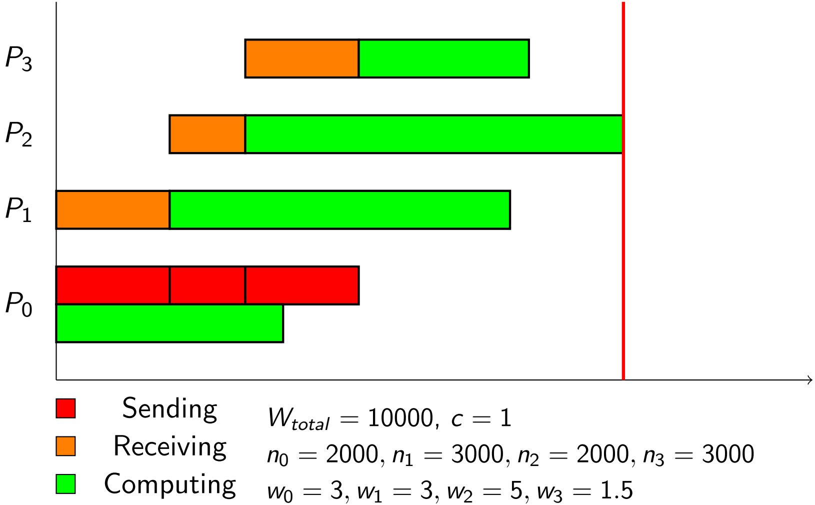


- P_i computes one unit of load (one infinitesimal task) in w_i seconds
- P_0 sends one unit of load to a worker in c seconds

Computation-communication model

- P_0 can compute and communicate at the same time
- $P_i, i > 0$ must have received all data before beginning computation
  - Questionable assumption, but will make sense with network latencies
- P_0 can only communicate with one worker at a time
  - Other versions allow communication to a bounded number of workers
  - We will talk about such models in other contexts
- Let's now draw an example schedule...

Example schedule

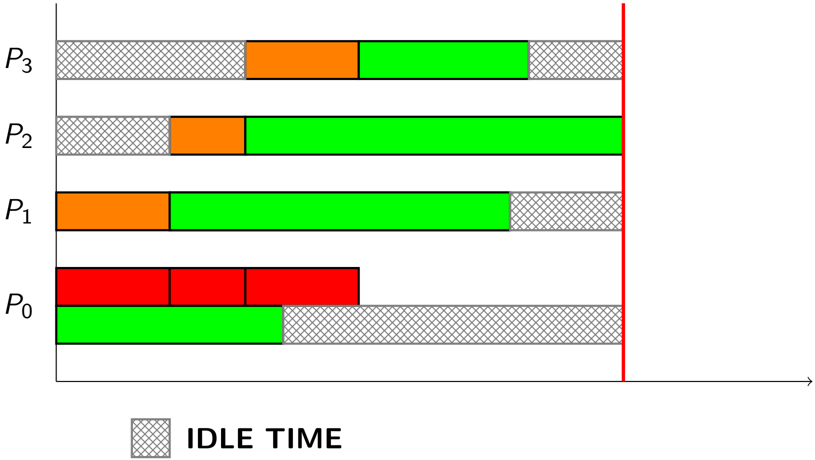


$W_{total} = 10000,\ c = 1$

$n_0 = 2000, n_1 = 3000, n_2 = 2000, n_3 = 3000$

$w_0 = 3, w_1 = 3, w_2 = 5, w_3 = 1.5$

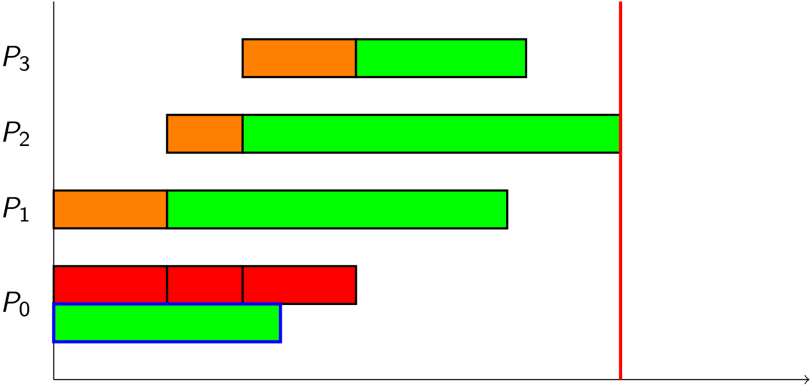
Example schedule



Recursion

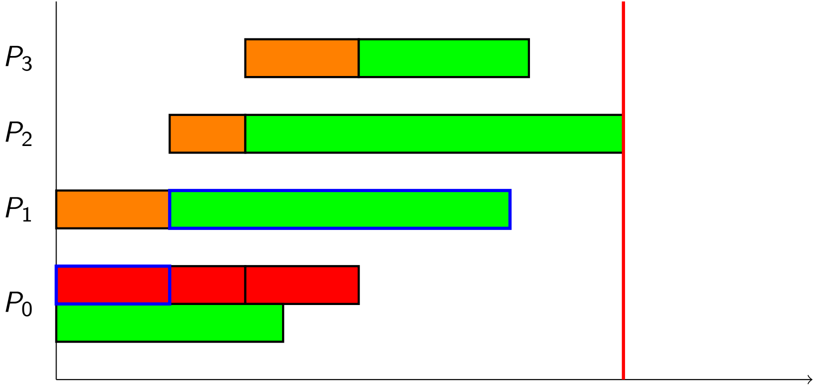
- Let's call T_i the finish time of processor P_i
- We can write a recursion with the T_i's, n_i's, w_i's and c
- Let's see it on a picture

Example schedule



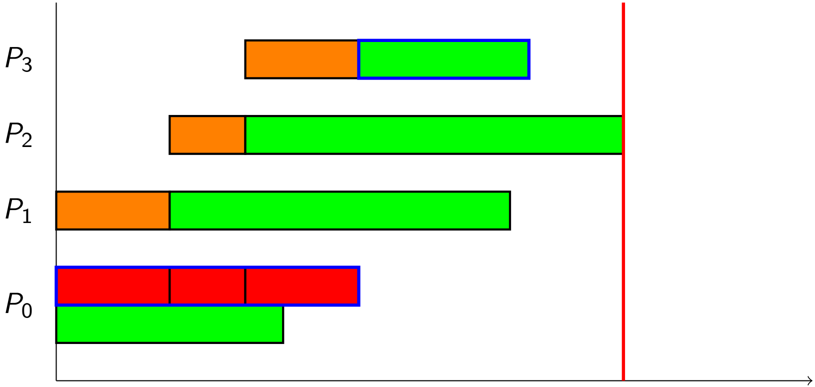
- P_0: $T_0 = n_0 w_0$

Example schedule



- P_0: $T_1 = n_1 c + n_1 w_1$

Example schedule



- P_i: $T_i = \sum_{j=1}^{i} n_j c + n_i w_i$

Recursion and Dynamic Programming

- Given the recursion, we have the makespan, T, as:

  $T = \max_{0 \leq i \leq m} \left(\sum_{j=1}^{i} n_j c + n_i w_i \right)$
- which can be rewritten as:

  $T = \max \left(n_0 w_0, \max_{1 \leq i \leq m} \left(\sum_{j=1}^{i} n_j c + n_i w_i \right) \right)$
- which suggests a dynamic programming solution
  - An optimal schedule for $p+1$ processors is constructed from an optimal schedule for p processors

We are stuck

- We now face many difficulties:
  - We don't have a closed form solution
  - The order of the processors is fixed!
    - We would have to try all $m!$ orders to find the best one
  - The complexity of the dynamic programming solution is $O(W_{total}^2 m)$
    - The time to compute the schedule could be longer than the time to execute the application!
  - If we know the optimal schedule for $W_{total} = 1000$, we have to recompute a whole schedule for $W_{total} = 1001$
- Okay, we get it, scheduling is hard ☺

The DL scheduling approach

- The fact that the n_i's are integers is the root cause of the difficulties
- But in the case of DLs, since $\sum n_i >> n_i$, a reasonable approximation is to reason on fractions, i.e., rational numbers
- Let $\alpha_i \geq 0$ be the rational fraction of W_{total} allocated to processor P_i
  - $n_i = \alpha_i W_{total}$
- $\sum_i \alpha_i = 1$

The DL scheduling approach

- We can now rewrite the recursion in terms of the α_i's
$T = \max_{0 \leq i \leq m} \left(\sum_{j=1}^{i} \alpha_j c + \alpha_i w_i \right) W_{total}$
- It turns out that with rational α_i's, we can prove two important lemmas

Lemma (1)

In an optimal solution, all processors participate and finish at the same time

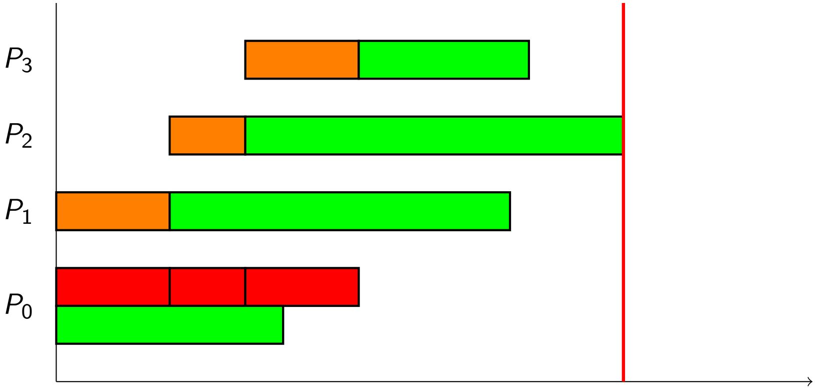
Lemma (2)

If one can choose the master processor, it should be the fastest processor. The order of the worker processors does not matter

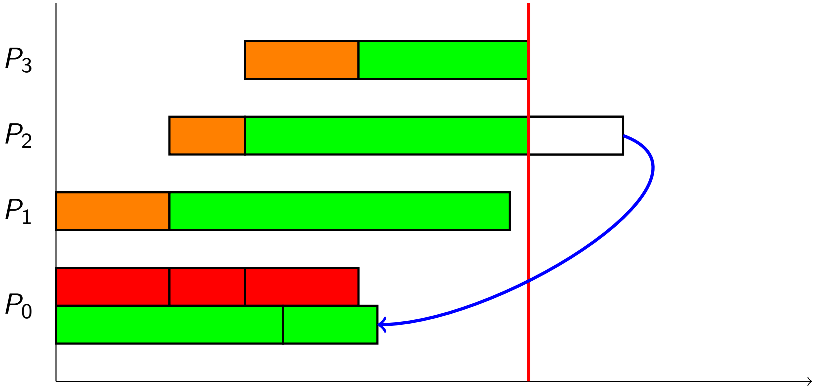
Proof sketch of Lemma 1

- Take some load from the processor that finishes last, give it to another processor (that perhaps does not yet participate)
- Obtain a better schedule, and repeat until all processors finish at the same time
- Let's see this (informally) on our example schedule...
  - The formal proof is not difficult but not particularly interesting

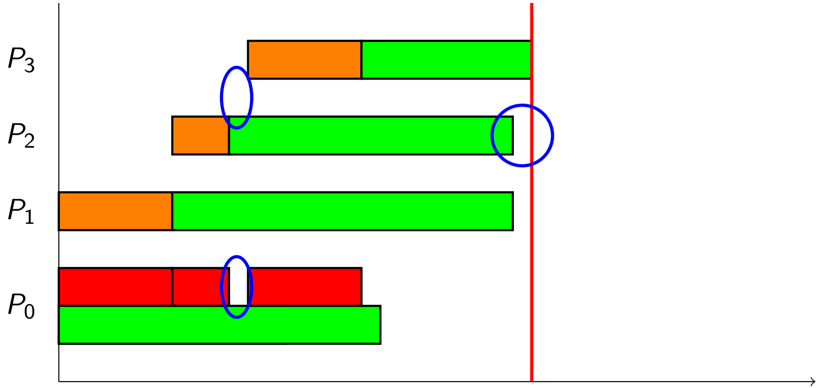
Example schedule



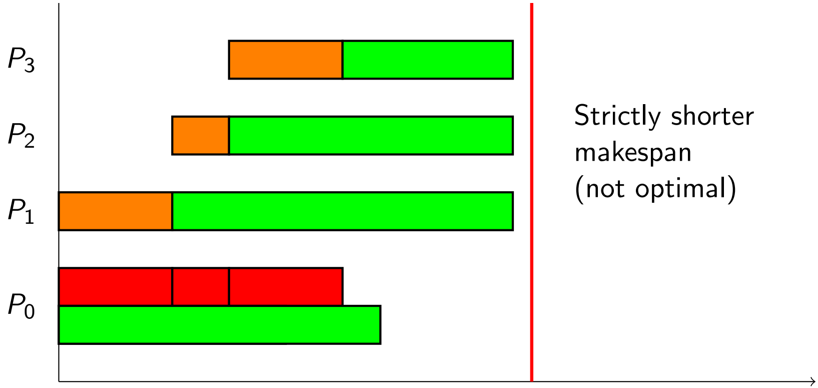
Example schedule



Example schedule



Example schedule



Strictly shorter makespan (not optimal)

Proof of Lemma 2

- The master should be the fastest processor, and the order of the workers doesn't matter
- In an optimal schedule, we know that $T = T_0 = T_1 = \ldots = T_m$ (Lemma 1)
- Therefore:

$$
\begin{array}{l}
T = \alpha_0 w_0 W_{total} \\
T = \alpha_1 (c + w_1) W_{total} \Rightarrow \alpha_1 = \frac{w_0}{c+w_1}\alpha_0 \\
T = (\alpha_1 c + \alpha_2 (c + w_2)) W_{total} \Rightarrow \alpha_2 = \frac{w_1}{c+w_2}\alpha_1 \\
\ldots \\
\Rightarrow \quad \forall i \geq 0 \quad \alpha_i = \prod_{j=1}^{i} \frac{w_{j-1}}{c+w_j}\alpha_0
\end{array}
$$

$$\sum_{i=0}^{m} \alpha_i = 1 \quad \Rightarrow \quad \alpha_i = \frac{\prod_{j=1}^{i} \frac{w_{j-1}}{c+w_j}}{\sum_{k=0}^{m}\left(\prod_{j=1}^{k} \frac{w_{j-1}}{c+w_j}\right)}$$

Proof of Lemma 2

- Let us compute the "work" done in time T by processors P_i and P_{i+1} for $0 \leq i \leq m-1$
- To ease notations let's define $c_0 = 0$ and $c_i = c$ for $i > 0$
- We have:

  $$T = T_i = \left(\left(\sum_{j=0}^{i-1} \alpha_j c_j \right) + \alpha_i w_i + \alpha_i c_i \right) W_{total}$$

  and

  $$T = T_{i+1} = \left(\left(\sum_{j=0}^{i-1} \alpha_j c_j \right) + \alpha_i c_i + \alpha_{i+1} w_{i+1} + \alpha_{i+1} c_{i+1} \right) W_{total}$$

Proof of Lemma 2

- Let's define $K = \frac{T - W_{total}\left(\sum_{j=0}^{i-1} \alpha_j c_j\right)}{W_{total}}$
- We now have $\alpha_i = \frac{K}{w_i + c_i}$ and $\alpha_{i+1} = \frac{K - \alpha_i c_i}{w_{i+1} + c_{i+1}}$
- The total fraction of work processed by P_i and P_{i+1} is equal to:

  $\alpha_i + \alpha_{i+1} = \frac{K}{w_i + c_i} + \frac{K}{w_{i+1} + c_{i+1}} - \frac{c_i K}{(w_i + c_i)(w_{i+1} + c_{i+1})}$
- If $i > 0$, then $c_i = c_{i+1} = c$, and the expression above is symmetric in w_i and w_{i+1}
- Therefore <u>the order of the workers does not matter</u>

Proof of Lemma 2

- Since $\alpha_i = \frac{K}{w_i + c_i}$ and $\alpha_{i+1} = \frac{K - \alpha_i c_i}{w_{i+1} + c_{i+1}}$
  the total fraction of work processed by P_0 and P_1 is
  $\alpha_0 + \alpha_1 = \frac{K}{w_0} + \frac{K}{w_1 + c}$
- The above is maximized when w_0 is smaller than w_1
- By induction, we find that it is better to pick the fastest processor as the master
  - Perhaps counter-intuitive?

Overall theorem

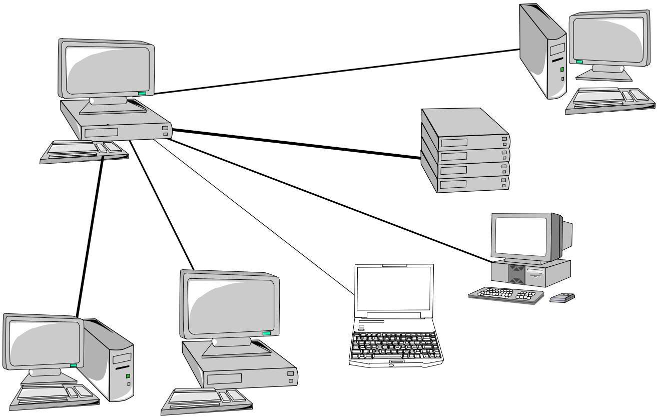
Theorem

For Divisible Load applications on bus-shaped networks, in an optimal schedule, the fastest computing processor is the master processor, the order of the communications to the workers has no impact on the quality of a solution, and all processors participate and finish simultaneously. The fraction α_i of load allocated to each processor is:

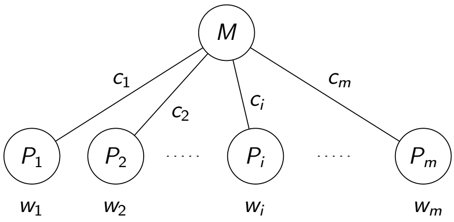
$$\forall i \in \{0, \ldots, m\} \quad \alpha_i = \frac{\prod_{j=1}^{i} \frac{w_{j-1}}{c+w_j}}{\sum_{k=0}^{m} \left(\prod_{j=1}^{k} \frac{w_{j-1}}{c+w_j} \right)}$$

Outline

Star-shaped platforms - practice



Star-shaped platforms - theory



- P_i computes one unit of load (one infinitesimal task) in w_i seconds
- P_0 sends one unit of load to worker P_i in c_i seconds
- P_0 does not compute (easier to write equations, and no loss of generality as we can add a worker with $c_i = 0$)

Two lemmas revisited

Lemma (1)

In an optimal schedule, all workers participate

- Simple proof based on the notion of giving some load from the last processor to an unused processor so as to reduce the makespan

Lemma (2)

There is a unique optimal schedule, and in that schedule, workers finish at the same time

- Rather technical proof based on a linear programming formulation and reasoning on the extremal solutions

Two lemmas revisited

Lemma (1)

In an optimal schedule, all workers participate

- Simple proof based on the notion of giving some load from the last processor to an unused processor so as to reduce the makespan

Lemma (2)

There is a unique optimal schedule, and in that schedule, workers finish at the same time

- Rather technical proof based on a linear programming formulation and reasoning on the extremal solutions

A third lemma

Lemma (3)

In the optimal schedule, the workers are served in non-decreasing order of the c_i's (the w_i's don't matter!)

- Proof: using the same computation as in the proof of Lemma 2 for bus-shaped platforms, for processors P_i and P_{i+1} we have:

$$\alpha_i = \frac{K}{w_i+c_i} \quad \text{and} \quad \alpha_{i+1} = \frac{K-\alpha_i c_i}{w_{i+1}+c_{i+1}}$$

$$\Rightarrow \alpha_i + \alpha_{i+1} = \left(\frac{1}{w_i+c_i} + \frac{1}{w_{i+1}+c_{i+1}}\right) K - \frac{Kc_i}{(w_i+c_i)(w_{i+1}+c_{i+1})}$$

If we exchange P_i and P_{i+1} (order P_{i+1}, P_i), we obtain:

$$\alpha_i + \alpha_{i+1} = \left(\frac{1}{w_i+c_i} + \frac{1}{w_{i+1}+c_{i+1}}\right) K - \frac{Kc_{i+1}}{(w_i+c_i)(w_{i+1}+c_{i+1})}$$

A third lemma

- The difference in processed load between the P_i, P_{i+1} and the P_{i+1}, P_i orders is $\Delta = (c_{i+1} - c_i)\frac{K}{(w_i+c_i)(w_{i+1}+c_{i+1})}$
- The above is not symmetric! Depending on whether c_i is larger/smaller than c_{i+1} the quantity of processed load increases: If $c_{i+1} > c_i$ then Δ is positive, meaning that the P_i, P_{i+1} order is better than the P_{i+1}, P_i order
- It's easy to verify that communication times are the same in both orders $(\alpha_i c_i + \alpha_{i+1} c_{i+1})$
- Conclusion: more load is processed by serving the workers by non-decreasing c_i's

Overall theorem

Theorem

For Divisible Load applications on star-shaped networks, in the optimal schedule, all workers participate, the workers must be served in non-decreasing c_i's, all workers finish at the same time, and the load fractions are given by:

$$\alpha_i = \frac{\frac{1}{c_i+w_i}\prod_{k=1}^{i-1}\left(\frac{w_k}{c_k+w_k}\right)}{\sum_{j=1}^{m}\frac{1}{c_j+w_j}\prod_{k=1}^{j-1}\frac{w_k}{c_k+w_k}}$$

So far... so good

- For bus-shaped platforms, we have solved the problem
- For star-shaped platforms, we have solved the problem
- Other have solved it for other platform shapes (e.g., trees) and variations (e.g., multiple masters)

- A big problem: our model is very naive
- In practice, compute costs and communication costs are rarely linear, but affine

So far... so good

- For bus-shaped platforms, we have solved the problem
- For star-shaped platforms, we have solved the problem
- Other have solved it for other platform shapes (e.g., trees) and variations (e.g., multiple masters)

- A big problem: our model is very naive
- In practice, compute costs and communication costs are rarely linear, but affine

Outline

Latencies

- The time for the master to send α_i units of load to worker P_i is $C_i + c_i \alpha_i W_{total}$
  - e.g., network latency
- The time for worker P_i to compute α_i units of load is $W_i + w_i \alpha_i W_{total}$
  - e.g., overhead to start a process/VM
  - e.g., software overhead to "prepare" the computation

Latencies

- The time for the master to send α_i units of load to worker P_i is $C_i + c_i \alpha_i W_{total}$
  - e.g., network latency
- The time for worker P_i to compute α_i units of load is $W_i + w_i \alpha_i W_{total}$
  - e.g., overhead to start a process/VM
  - e.g., software overhead to "prepare" the computation

Known results

- The addition of latencies makes things much harder
- The problem is $\mathcal{NP}$-complete (even if w_i's are zero)
  - Non-trivial reduction to 2-PARTITION
- All participating workers finish at the same time
  - Easy proof
- If W_{total} is large enough then all workers participate and must be served by non-decreasing c_i's
  - Much more complicated proof
- An optimal solution can be found using a mixed linear program...

Known results

- The addition of latencies makes things much harder
- The problem is $\mathcal{NP}$-complete (even if w_i's are zero)
  - Non-trivial reduction to 2-PARTITION
- All participating workers finish at the same time
  - Easy proof
- If W_{total} is large enough then all workers participate and must be served by non-decreasing c_i's
  - Much more complicated proof
- An optimal solution can be found using a mixed linear program...

Linear Programming

- An Integer Linear Program (ILP):
  - A set of integer variables
  - A set of linear constraints
  - A linear objective function
- A Mixed Integer Linear Program (MILP):
  - A set of integer or rational variables
  - A set of linear constraints
  - A linear objective function
- Both (associated decision problems) are $\mathcal{NP}$-complete
  - Fully rational Linear Programs can be solved in p-time!

Linear programming and scheduling

- MILPs occur frequently when formalizing scheduling problems
- Typical integer variables are binary:
  - $x_{i,j}$: is task i scheduled on processor j?
  - $x_{i,j}$: is the i-th communication for processor j?
  - $\cdots$
- Typical rational variables:
  - $\alpha_{i,j}$: the i-th fraction of load processed on processor j
  - $\alpha_{i,j}$: the fraction of network bandwidth to processor j used for task i
  - $\cdots$

Why are MILP formulations useful?

- After all, solving them is $\mathcal{NP}$-complete
  - And there may be easy optimal algorithms instead
- Reason #1: provide concise problem description
  - Useful when writing an article
- Reason #2: can be relaxed by making all variables rational
  - Solve the rational program in p-time
  - Obtain the (unfeasible) optimal objective function value
  - This value is a bound on optimal, which is useful to gauge the quality of heuristics
  - e.g., for a maximization problem: on this instance my heuristic achieves 92, the upper bound on optimal is 100, so I can say my heuristic is (at most) within 8% of optimal.

Mixed Linear Program for DL with latencies

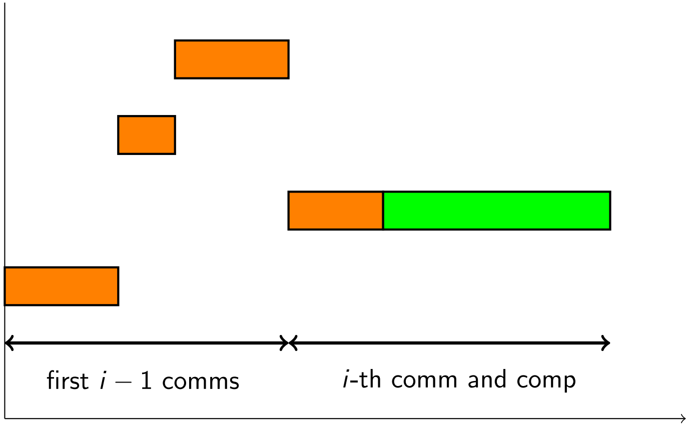
- We define the following variables:
  - $\alpha_j \geq 0$ (rational): load fraction sent to P_j
  - y_j (binary): true if worker P_j participates
  - $x_{i,j}$ (binary): true if worker P_j received the i-th load fraction

- We have the following "setup" constraints:
  - $\sum_i \alpha_i = 1$: the entire load is processed
  - $\forall j \quad \alpha_j \leq y_j$: only participating workers are allocated some load
  - $\forall j \quad \sum_i x_{i,j} = y_j$: a participating worker receives only one fraction of load
  - $\forall i \quad \sum_j x_{i,j} \leq 1$: at most one worker is used for the i-th communication

Mixed Linear Program for DL with latencies

- We define the following variables:
  - $\alpha_j \geq 0$ (rational): load fraction sent to P_j
  - y_j (binary): true if worker P_j participates
  - $x_{i,j}$ (binary): true if worker P_j received the i-th load fraction
- We have the following "setup" constraints:
  - $\sum_i \alpha_i = 1$: the entire load is processed
  - $\forall j \quad \alpha_j \leq y_j$: only participating workers are allocated some load
  - $\forall j \quad \sum_i x_{i,j} = y_j$: a participating worker receives only one fraction of load
  - $\forall i \quad \sum_j x_{i,j} \leq 1$: at most one worker is used for the i-th communication

Main constraint



Main constraint

- The time at which the communication of the $(i-1)$-th load fraction finishes: $\sum_{k=1}^{i-1}\sum_{j=1}^{m} x_{k,j}(C_j + \alpha_j c_j W_{total})$
- The time to communicate and compute the i-th load fraction: $\sum_{j=1}^{m} x_{i,j}(C_j + \alpha_j c_j W_{total} + W_j + \alpha_j w_j W_{total})$
- Let T_f be the finish time (of all processors)
- We have the constraint:

$$\forall i \quad \sum_{k=1}^{i-1}\sum_{j=1}^{m} x_{k,j}(C_j + \alpha_j c_j W_{total}) + \sum_{j=1}^{m} x_{i,j}(C_j + \alpha_j c_j W_{total} + W_j + \alpha_j w_j W_{total}) \leq T_f$$

- And the objective is to minimize T_f

Mixed Linear Program for DL with latencies

Mixed Integer Linear Program

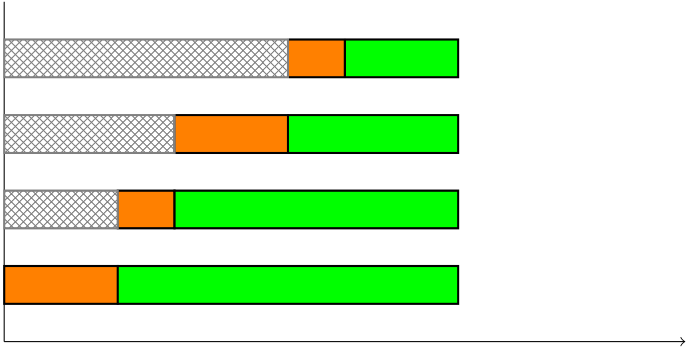
minimize T_f subject to

$$
\left\{
\begin{array}{lrl}
(1) & \forall i, 1 \leq i \leq m, & \alpha_i \geq 0 \\
(2) & & \sum_{i=1}^{m} \alpha_i = 1 \\
(3) & \forall j, 1 \leq j \leq m, & y_j \in \{0,1\} \\
(4) & \forall i,j, 1 \leq i,j \leq m, & x_{i,j} \in \{0,1\} \\
(5) & \forall j, 1 \leq j \leq m, & \sum_{i=1}^{m} x_{i,j} = y_j \\
(6) & \forall i, 1 \leq i \leq m, & \sum_{j=1}^{m} x_{i,j} \leq 1 \\
(7) & \forall j, 1 \leq j \leq m, & \alpha_j \leq y_j \\
(8) & \forall i, 1 \leq i \leq m, & \sum_{k=1}^{i-1} \sum_{j=1}^{m} x_{k,j}(C_j + \alpha_j c_j W_{total}) \\
& & + \sum_{j=1}^{m} x_{i,j}(C_j + \alpha_j c_j W_{total} + W_j + \alpha_j w_j W_{total}) \\
& & \leq T_f
\end{array}
\right.
$$

Outline

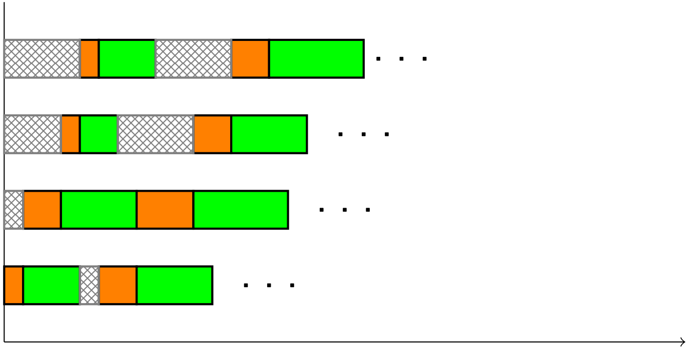
Multiple rounds?

- In everything we've seen so far, there are m communications to m workers
- This leads to a lot of idle time, especially if m is large



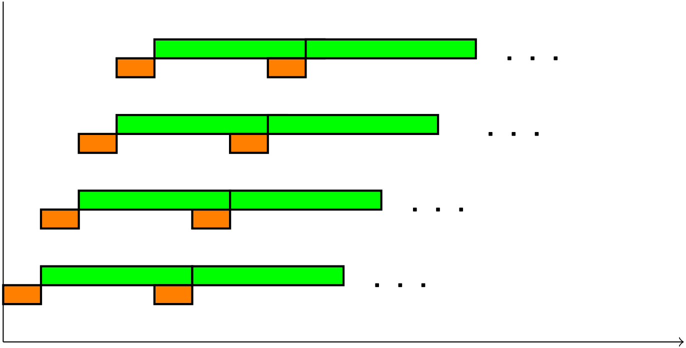
Multiple rounds

- Simple idea: get workers to work early



Multiple rounds

- Even better: hide communication (note the homogeneity)



Multi-round DL scheduling

- Several variations of this problem have been studied
- Many authors have studied the following question: "Given a number of rounds, how much work should be allocated at each round and how?"
  - Worthwhile question with linear or affine models
- More interesting: "How many rounds should be used?"
  - Linear models: an infinite number of rounds!
    - "Obvious" but long and technical proof
    - Surprisingly not acknowledged in early DL literature
  - Affine models: $\mathcal{NP}$-complete
- Let us see the known results for both questions above

Homogeneous bus, given number of rounds

- Assume everything is homogeneous ($c_i = c$, $C_i = C$, $w_i = w$, $W_i = W$) and the number of rounds is M
- At each round, m "chunks" are sent, one per worker
- Each chunk corresponds to a fraction α_j, $0 \leq j < Mm$
- For convenience, we number these chunks in reverse order:
  - the first one is $Mm - 1$, the last one 0
- Let $R = w/C$ be the computation-communication ratio
- Let $\gamma_i = \alpha_i w W_{total}$ (the compute time of chunk i)
- Let us write equations that ensure that there is no idle time

No non-initial idle time

- There is no idle time (after the first round) if a worker computes X seconds and the next m communications also take X seconds
  - In that way, a worker finishes computing round j right when its chunk for round $j+1$ has arrived!

$$\forall i \geq m, \quad W + \gamma_i = \frac{1}{R}(\gamma_{i-1} + \gamma_{i-2} + \cdots + \gamma_{i-m}) + mC$$

All workers finish at the same time

- For all workers to finish at the same time, the compute time of the last chunk at a worker should be equal to the time for all remaining communication, and the computation time of the last chunk

  $$\forall 0 \leq i < m, \quad W + \gamma_i = \frac{1}{R}(\gamma_{i-1} + \gamma_{i-2} + \cdots + \gamma_{i-m}) + iC + \gamma_0$$

- To ensure correctness, we also have

  $$\forall i < 0, \quad \gamma_i = 0$$

Infinite series

$$\forall i \geq m, \quad W + \gamma_i = \frac{1}{R}(\gamma_{i-1} + \gamma_{i-2} + \cdots + \gamma_{i-m}) + mC$$
$$\forall 0 \leq i < m, \quad W + \gamma_i = \frac{1}{R}(\gamma_{i-1} + \gamma_{i-2} + \cdots + \gamma_{i-m}) + iC + \gamma_0$$
$$\forall i < 0, \quad \gamma_i = 0$$

- The recursion above corresponds to an infinite γ_i series
- Can be solved using a generating function: $\mathcal{G}(x) = \sum_{i=0}^{\infty} \gamma_i x^i$
- Using the two recursions above, we obtain:

$$\mathcal{G}(x) = \frac{(\gamma_0 - mC)(1 - x^m) + (mC - W) + C\left(\frac{x(1-x^{m-1})}{1-x} - (m-1)x^m\right)}{(1-x) - x(1-x^m)/R}$$

- Using the rational expansion theorem, we obtain the roots of the polynomial denominator, and thus the γ_i values!!

Homogeneous bus, computing M

- Computing the optimal number of rounds is $\mathcal{NP}$-complete in the general case (i.e., non-homogeneity)
- One "brute-force" option is to do an exhaustive search on the number of rounds, searching for the number of rounds that achieves the lowest makespan
  - Potentially exponential time, but in practice likely very doable
- A more elegant approach consists in writing an equation for the makespan and solving an optimization problem
  - Not difficult (based on a Lagrange Multiplier method)
  - Can be extended to heterogeneous platforms

An interesting theoretical result

Theorem

On any bus- or star-platform, with either linear or affine models, a multi-round schedule cannot improve an optimal single-round schedule by more than a factor 2

- How would you prove this result?

An interesting theoretical result: Proof

- Let $\mathcal{S}$ be any optimal multi-round schedule, which uses K rounds, and has makespan T
- We have m workers, and each received a load fraction $\alpha_i(k)$ at round k
- From $\mathcal{S}$, we construct a new schedule $\mathcal{S}'$ that sends in a single message $\sum_{k=1}^{K} \alpha_i(k)$ to workers i
- The master does not communicate more in $\mathcal{S}'$ than in $\mathcal{S}$ (in fact, less with latencies)
- Therefore, not later than time T, all workers have received their load fractions (very coarse upper bound)
- No worker will compute more in $\mathcal{S}'$ than in $\mathcal{S}$
- Therefore, none of them will spend more than T time units to compute in $\mathcal{S}'$
- Conclusion: the makespan of $\mathcal{S}'$ is at most $2T$ $\square$

Outline

So, what do we know?

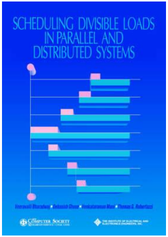
	Bus	Star
Linear	$M = 1$: closed-form optimal: $M = \infty$ given $M < \infty$: closed-form	$M = 1$: closed-form optimal: $M = \infty$ given $M < \infty$: closed-form
Affine	$\mathcal{NP}$-complete (1-round MILP) given M, homogeneous: closed-form optimal M: heuristics	$\mathcal{NP}$-complete (1-round MILP) optimal M: heuristics

- All processors must finish at the same time
- Multi-round buys at most a factor 2 improvement
- Linear models are strange, but latencies make everything difficult (non-divisible!)

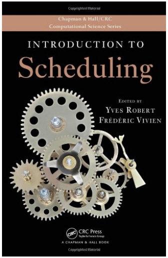
What about sending back results?

- There are essentially no known general results if return messages are to be scheduled
- If returned messages have the same size as the sent messages, it is easy to come up with the best FIFO (same order) and LIFO (reverse order) strategies
- But it is easy to find examples in which optimal is neither FIFO nor LIFO
- Essentially: nobody knows ☺

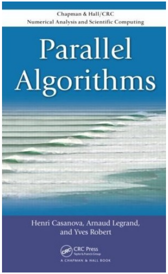
Sources and acknowledgments



V. Bharadwaj
D. Ghose
T. Robertazzi



Y. Robert
F. Vivien



H. Casanova
A. Legrand
Y. Robert

Thanks to Loris Marchal and Yves Robert for some of these slides

☺