A nice little scheduling problem

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Clusters, Clouds, and Data for Scientific Computing
La Maison des Contes (France), September 11-14, 2012

Energy-aware mappings of series-parallel workflows onto chip multiprocessors

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Motivations

- Mapping streaming applications onto parallel platforms: practical applications (image processing, astrophysics, meteorology, neuroscience, ...), but difficult problems (NP-hard)
- Objective: maximize the throughput, i.e., minimize the period of the application
- Energy saving is becoming a crucial problem (economic and environmental reasons)
- M. P. Mills, The internet begins with coal, Environment and Climate News (1999)
- Objective of a mapping: minimize energy consumption while maintaining a given level of performance (bound on period)

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Our contribution

- Applications: most task graphs of streaming applications are series-parallel graphs (SPGs), see for instance the *StreamIt* suite from MIT
- Platforms: Chip MultiProcessors (CMPs)
  $\rightarrow$ $p \times q$ homogeneous cores arranged along a 2D grid
- Trend: increase the number of cores on single chips
- Increasing number of cores rather than processor's complexity: slower growth in power consumption
- This work: energy-aware mappings of SPG streaming applications onto CMPs

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Outline of the talk

1. Framework
   - Application model
   - Platform
   - Mapping strategies
   - Objective functions
2. Complexity results
   - Uni-directional uni-line CMP
   - Bi-directional uni-line CMP
   - Bi-directional square CMP
3. Heuristics
4. Simulations

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Application model

- Series-parallel graph (SPG) streaming application
- Nodes: n application stages
  - w_i: computation requirement of stage S_i
- Edges: precedence constraints
  - $\delta_{i,j}$: volume of communication between S_i and S_j
- G is a SPG if G is a composition of two SPGs
- Elementary SPG:  (two stages $S_1 \rightarrow S_2$)
- Two kind of compositions:

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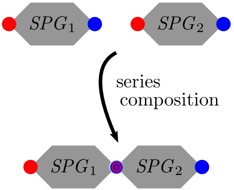

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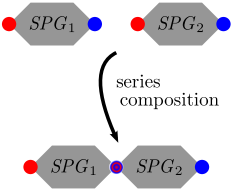
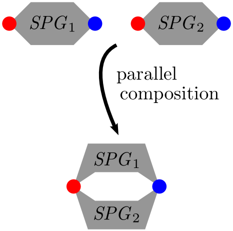
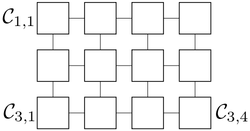




Target platform

- Chip multiprocessor: cores $\mathcal{C}_{u,v}$ on a $p \times q$ grid



- Bidirectional links of bandwidth BW:
- Time $\frac{\delta}{BW}$ to send δ bytes to a neighboring core
- $\mathcal{C}_{u,v}$ running at speed $s_{u,v} \in \{s^{(1)}, \dots, s^{(M)}\}$ (M possible voltage/frequency, leading to different speeds, identical on each core)
- Time $\frac{w_i}{s_{u,v}}$ to compute one data set for stage S_i on core $\mathcal{C}_{u,v}$

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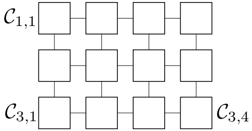

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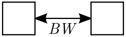

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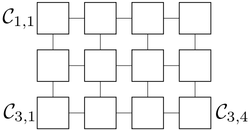

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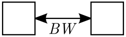

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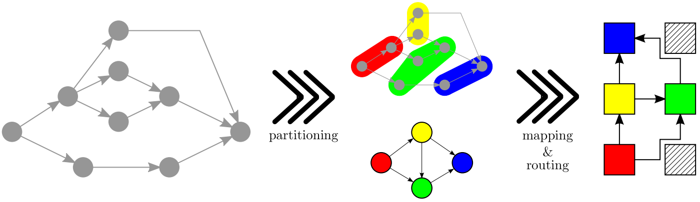
Mapping strategies

- Trade-off between one-to-one and general mappings
  - One-to-one mappings: each stage is mapped on a distinct core; unduly restrictive, high communication costs
  - General mappings: no restriction; arbitrary number of communications between two cores, and NP-complete
  - DAG-partition mappings: first partition the SPG into acyclic clusters, and then perform one-to-one mapping
- Allocation function: $alloc(i) = (u, v)$ if S_i is mapped on $\mathcal{C}_{u,v}$
  Routes to communicate between two cores: $path_{i,j}$



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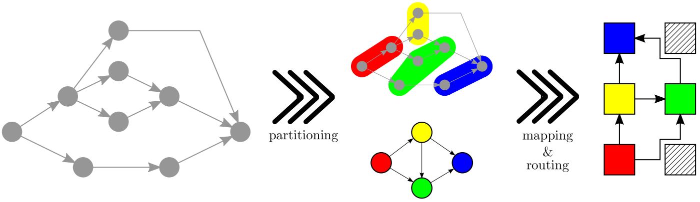

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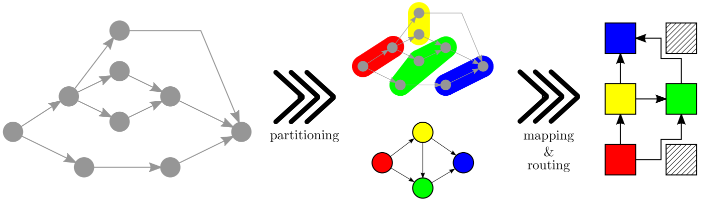

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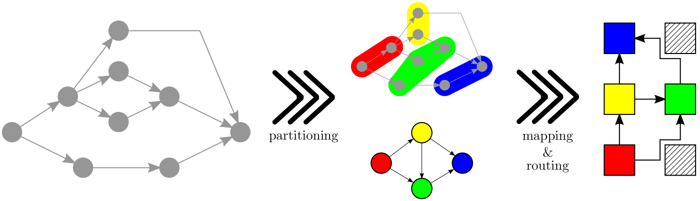

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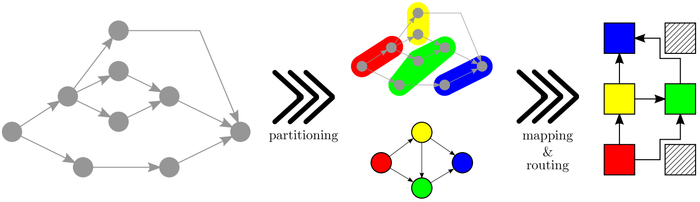

Objective functions: period of the application

- Data sets arrive at regular time intervals: period T
- Given a mapping and an execution speed for each core, check whether the period can be respected, i.e., the cycle-time of each core does not exceed T
- Computations: $w_{u,v} = \sum_{1 \leq i \leq n | alloc(i)=(u,v)} w_i$
  (work assigned to $\mathcal{C}_{u,v}$, running at speed $s_{u,v}$)
  $\rightarrow$ check that $\frac{w_{u,v}}{s_{u,v}} \leq T$
- Communications: $((u' = u+1 \text{ and } v' = v) \text{ or } (u' = u \text{ and } v' = v+1))$
  $b_{(u,v)\leftrightarrow(u',v')} = \sum_{1 \leq i,j \leq n | (u,v)\leftrightarrow(u',v') \in path_{i,j}} \delta_{i,j}$
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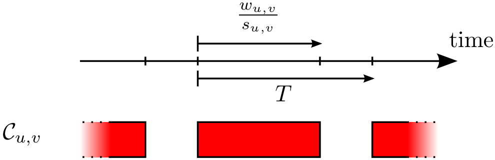
Objective functions: energy consumption

- Energy consumed by computations



$$E^{(\text{comp})} = |\mathcal{A}| \times P_{\text{leak}}^{(\text{comp})} \times T + \sum_{\mathcal{C}_{u,v} \in \mathcal{A}} \frac{w_{u,v}}{s_{u,v}} \times P_{s_{u,v}}^{(\text{comp})},$$

where $\mathcal{A}$ is the set of active cores

- Energy consumed by communications

$$E^{(\text{comm})} = P_{\text{leak}}^{(\text{comm})} \times T + \left(\sum_{u,v} \sum_{u',v'} b_{(u,v) \leftrightarrow (u',v')} \right) \times E^{(\text{bit})}$$

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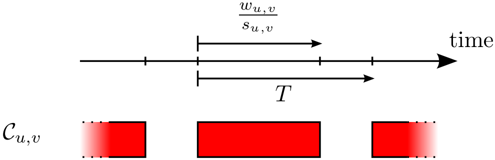

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Optimization problem

MinEnergy(T)

- Given
  - a *(bounded-elevation)* SPG
  - a $p \times q$ CMP
  - a period threshold T
- Find a mapping such that
  - the maximal cycle-time does not exceed T
  - the energy $E = E^{(\mathrm{comp})} + E^{(\mathrm{comm})}$ is minimum

elevation $y_{\max}$: width of the SPG, i.e., max. degree of parallelism

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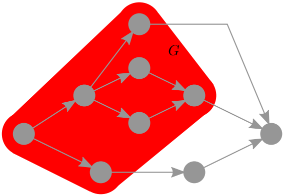
Uni-directional uni-line CMP ($1 \times q$)

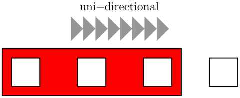
- Polynomial with bounded elevation:

  dynamic programming algorithm

$$\mathcal{E}(G,k) = \min_{G' \subseteq G} \left(\mathcal{E}(G', k-1) \oplus \mathcal{E}^{\mathrm{cal}}(G \setminus G') \right) ,$$

where
- G' is admissible: no more than $n^{y_{\max}}$ such graphs
- outgoing communications of G' do not exceed BW
- energy of communications accounted in the $\oplus$





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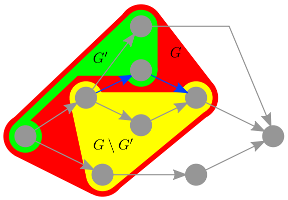

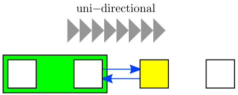

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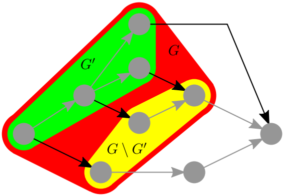

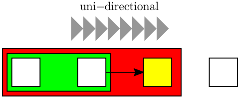

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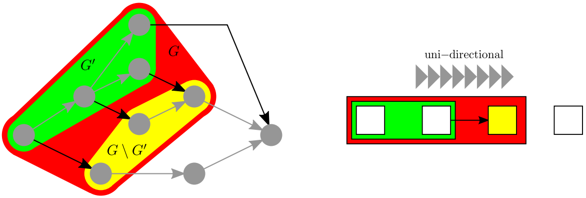
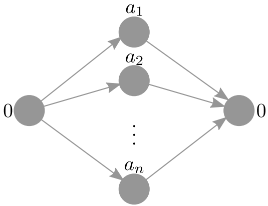
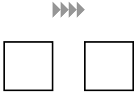


Polynomial: $O(q \times n^{2y_{\max}+1})$

Uni-directional uni-line CMP ($1 \times q$)

- NP-complete with unbounded elevation:
  reduction from 2-PARTITION





Single speed $\frac{\sum_{i=1}^{n} a_i}{2}$

- Previous algorithm: exponential complexity

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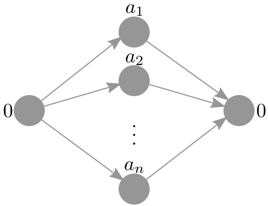

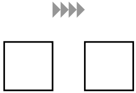

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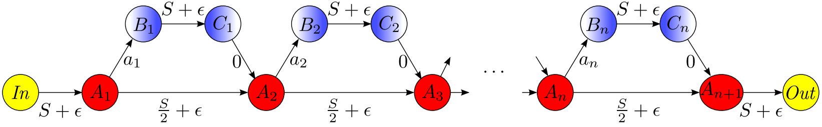
Bi-directional uni-line CMP ($1 \times q$)

- NP-complete with bounded elevation:
  reduction from 2-PARTITION
- We enforce $In, A_1, \ldots, A_{n+1}, Out$ to be mapped consecutively
- 2-partition of the blue nodes on both sides

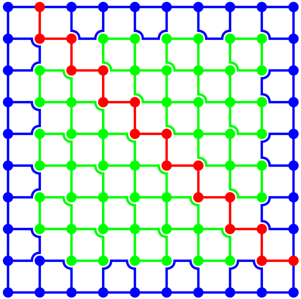


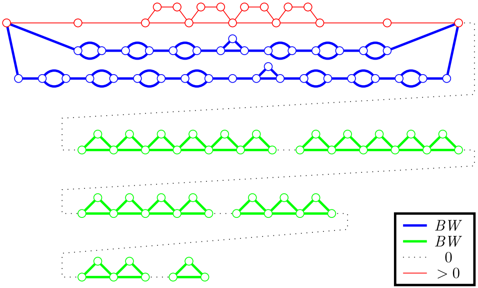
C_{i_1} B_{i_1} C_{i_2} B_{i_2} $\cdots$ C_{i_k} B_{i_k} In A_1 A_2 $\cdots$ A_{n+1} Out $B_{\bar{i}_1}$ $C_{\bar{i}_1}$ $B_{\bar{i}_2}$ $C_{\bar{i}_2}$ $\cdots$ $B_{\bar{i}_{n-k}}$ $C_{\bar{i}_{n-k}}$

$$BW = \frac{3S}{2} + \epsilon$$

Bi-directional square CMP ($p \times p$)

- The previous result implies the NP-completeness for $1 \times q$ CMPs, and hence CMPs of arbitrary shapes ($p \times q$)
- Square: not a direct consequence, but still NP-complete; reuse the uni-line proof by enforcing a line in the square
- Surprisingly involved proof





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   - Application model
   - Platform
   - Mapping strategies
   - Objective functions
2. Complexity results
   - Uni-directional uni-line CMP
   - Bi-directional uni-line CMP
   - Bi-directional square CMP
3. Heuristics
4. Simulations

Heuristics summary

- **Random** heuristic: random speeds for each cores; random assignments preserving a DAG-partition and matching period for computations; comm. always following an XY routing
- **Greedy** heuristic: given a speed s, starting from $C_{1,1}$, process as many stages as possible, partition following stages between right and down cores, iterate on those cores
  Try all possible speed values and keep the best solution
- 2D dynamic programming algorithm, **DPA2D**: map the SPG onto an $x_{max} \times y_{max}$ grid, following labels, and then map the grid onto the CMP thanks to a double nested DP algorithm
- 1D heuristics (2D CMP configured as a snake):
  - **DPA1D**: Optimal solution on uni-directional uni-line CMP
  - **DPA2D1D**: Previous 2D DP heuristic on the snake

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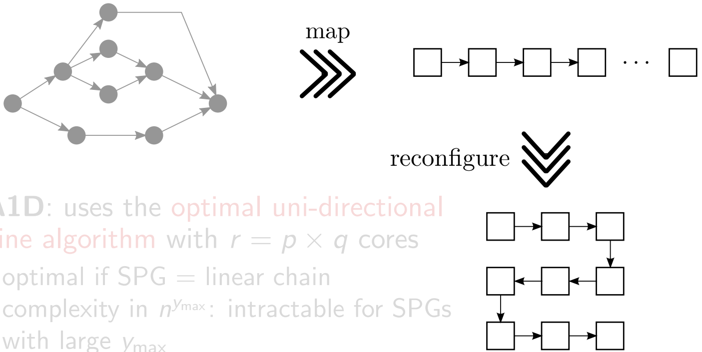

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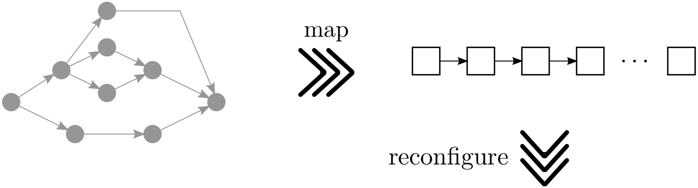

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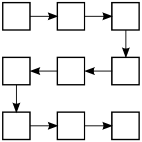

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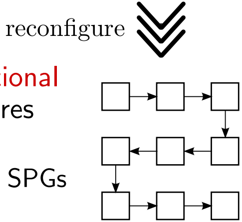

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- Random SPGs
  - Average over 100 applications
  - SPGs with 150 nodes
  - Elevation: from 1 to 30
- Real-life SPGs: the *StreamIt* suite
  - 12 different streaming applications
  - From 8 to 120 nodes
  - Elevation: from 1 to 17
- CMP configuration
  - 4×4 CMP following the Intel Xscale model
  - Five possible speeds per core
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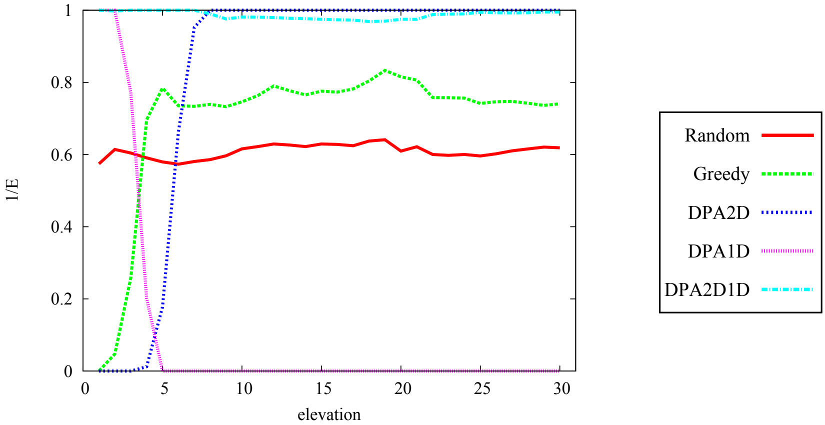
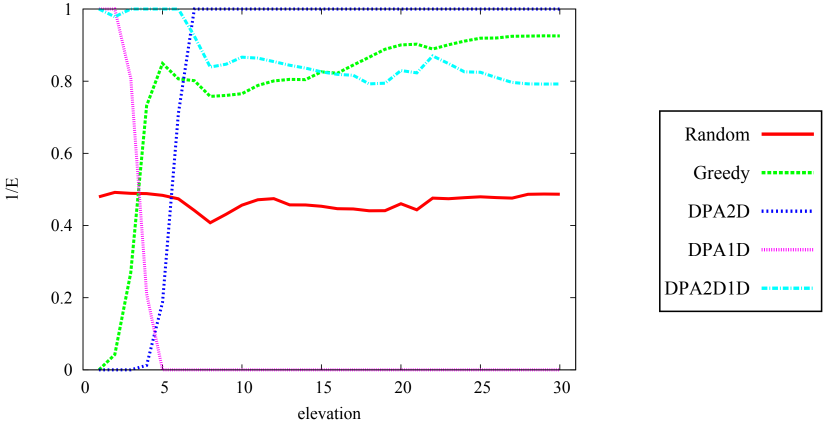
Random SPGs; computation intensive (CCR=10)



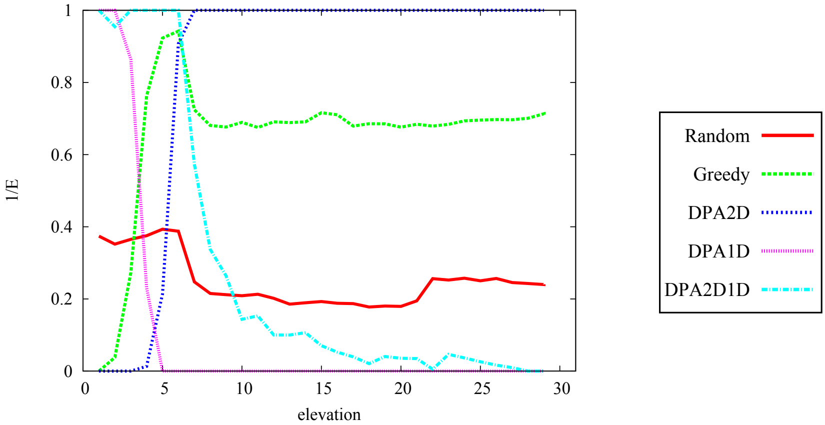
- **DPA1D** best for $1 \leq y_{\max} \leq 3$, then it fails
- **DPA2D** best for $y_{\max} \geq 6$
- **DPA2D1D** always efficient, whatever $y_{\max}$
- **Greedy** intermediate

Random SPGs; balanced (CCR=1)



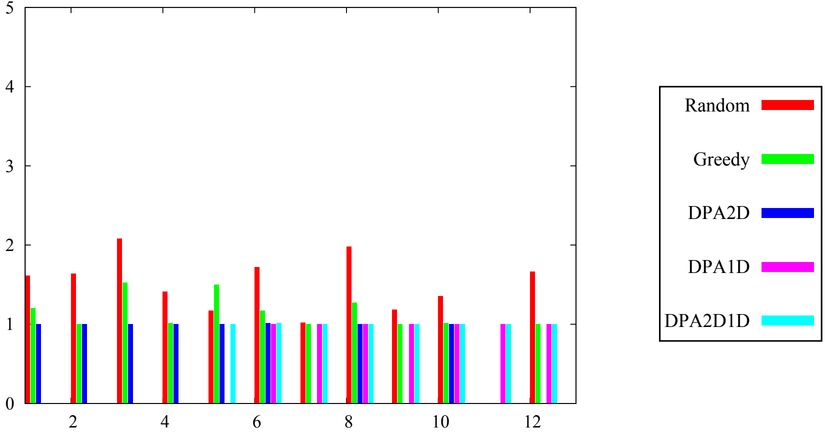
- Almost similar
- **DPA2D1D** is further from the best heuristic: cannot use all communication links

Random SPGs; communication intensive (CCR=0.1)



- **Random** and the 1D heuristics do not perform well for large $y_{\max}$
- **DPA2D** remains the best for large $y_{\max}$

StreamIt; computation intensive (CCR=10)



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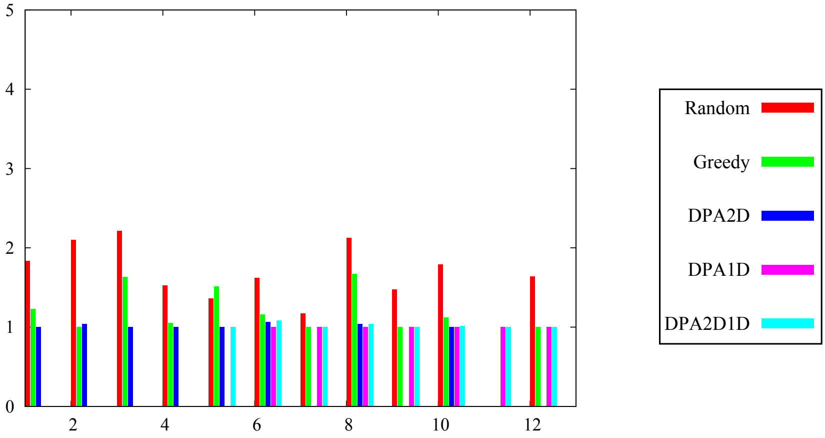

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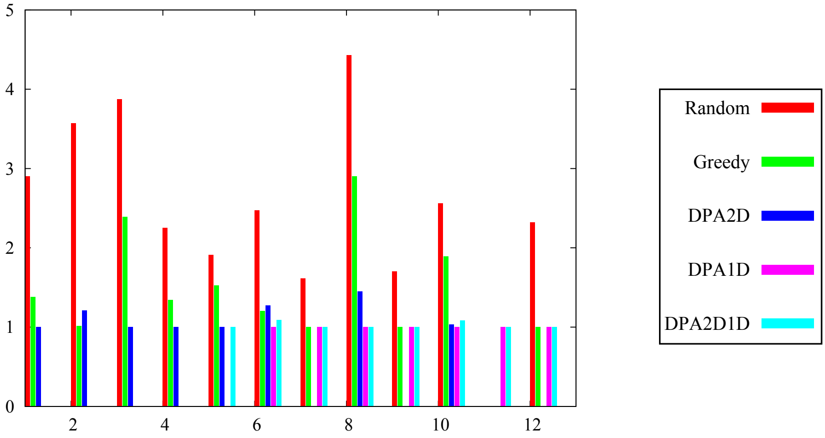

Summary of simulations

- Further simulations on larger applications (up to 200 stages), larger CMPs (6×6), which confirm the results
- Number of failures (out of 1000 instances per CCR value)

CCR	Random	Greedy	DPA2D	DPA1D	DPA2D1D
10	29	28	85	758	1
1	29	28	78	760	3
0.1	300	287	348	670	458

- Execution times: 1ms for **Random** and **Greedy**, 50ms for **DPA2D** and **DPA2D1D**, 10s for **DPA1D**
- **Greedy**: general-purpose heuristic, fast and succeeds on most graphs; **DPA1D**: best for small elevation, optimal with no communication, but very costly; **DPA2D1D**: useful when the elevation gets higher; **DPA2D**: most efficient when communication increases, judiciously handles 2D comms

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On-going and future work

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- Investigate general mappings, and assess the difference with DAG-partition mappings (in theory and in practice)
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Some more material

Application model

- Recursive definition of the label of stage S_i, (x_i, y_i): coordinates along a 2D grid in the recursive construction

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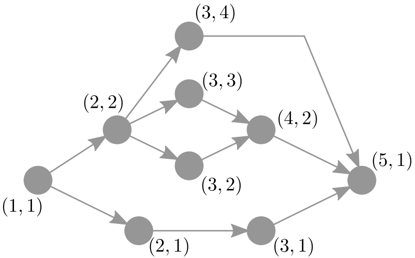

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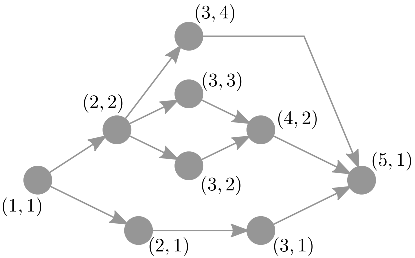

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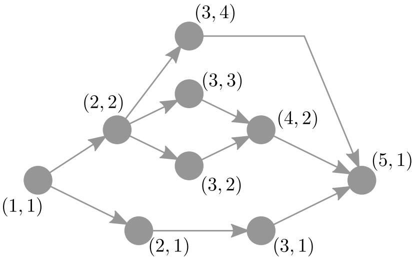

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- $y_{\max}$ is the maximum elevation; special case of bounded-elevation SPGs

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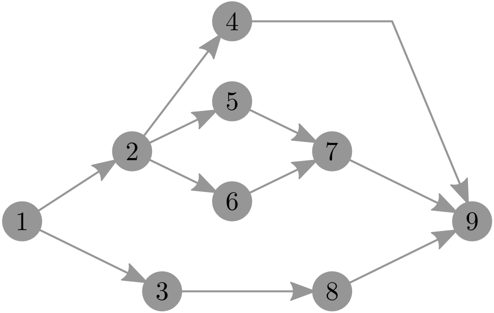

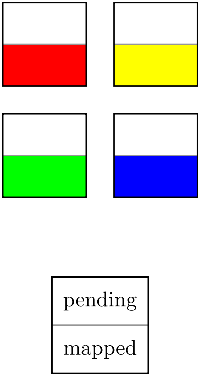

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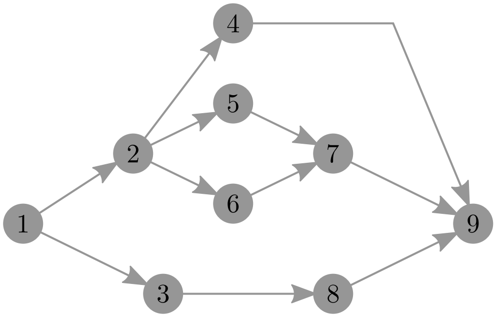

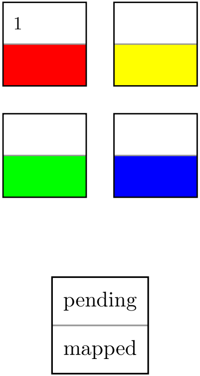

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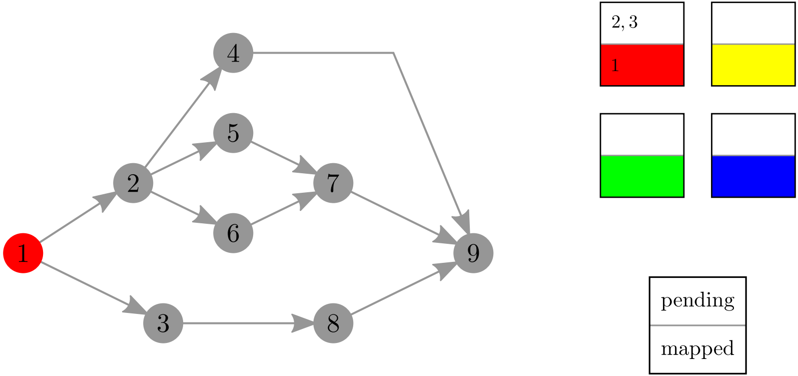

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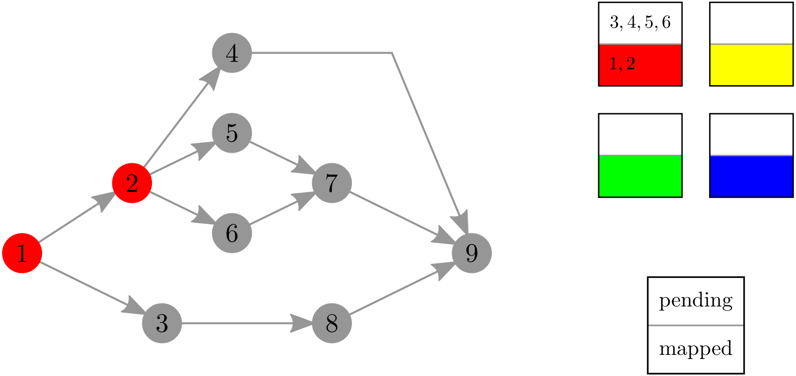

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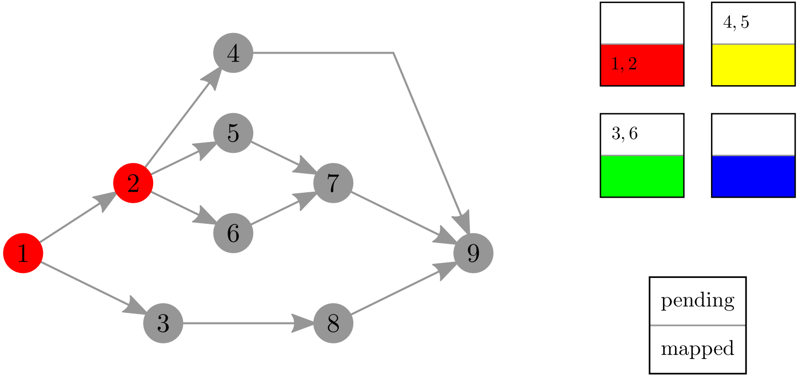

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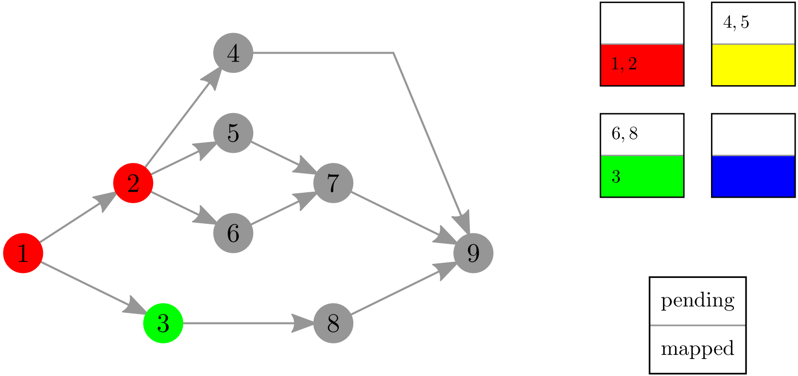

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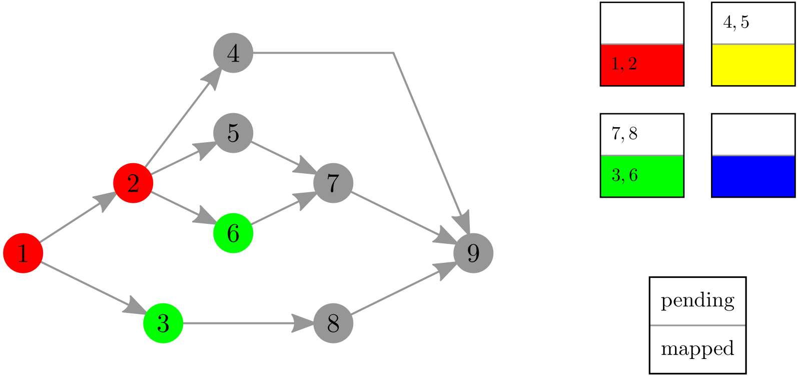

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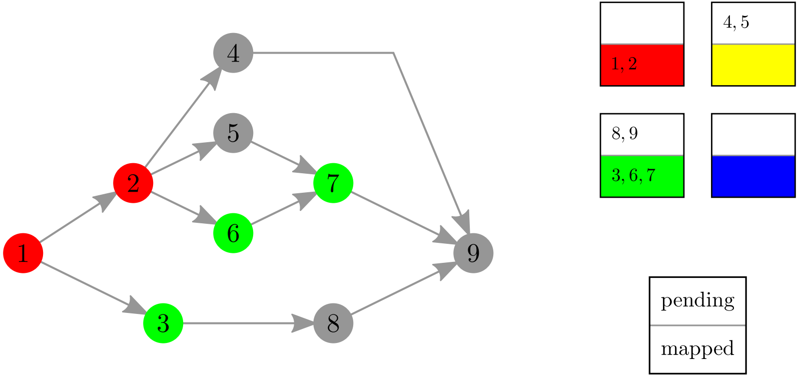

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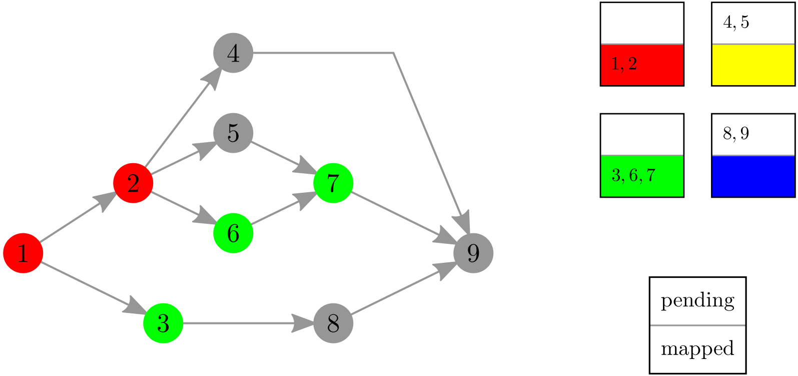

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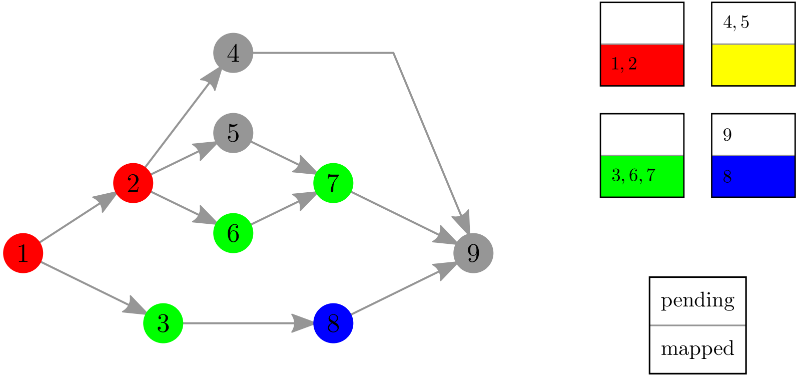

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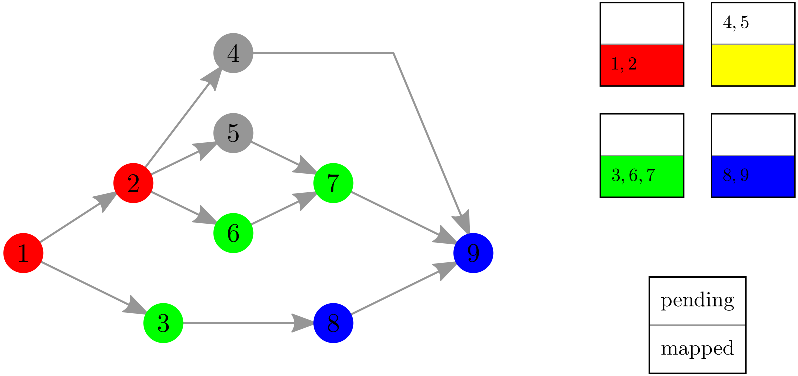

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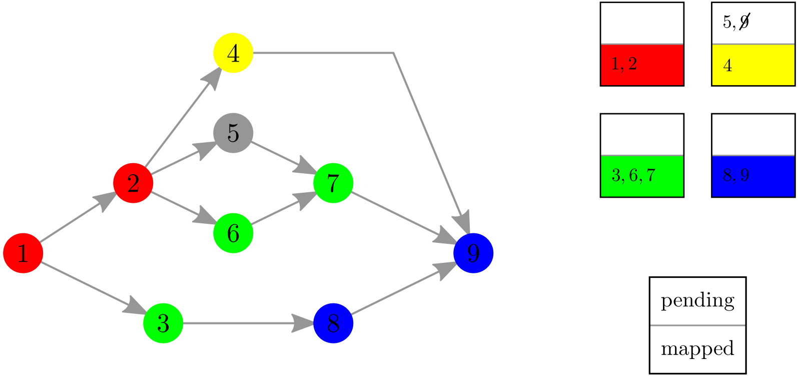

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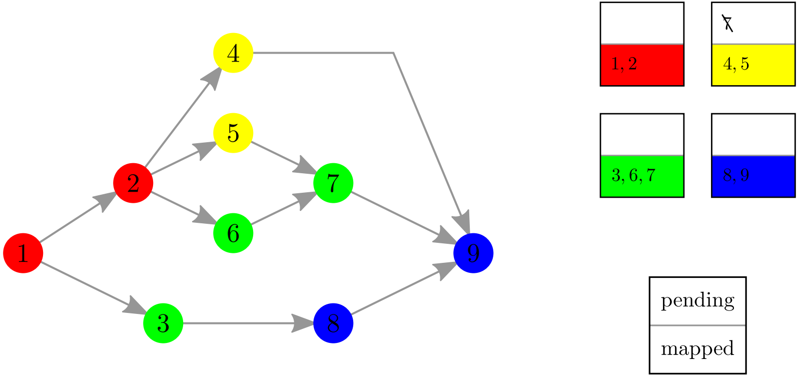

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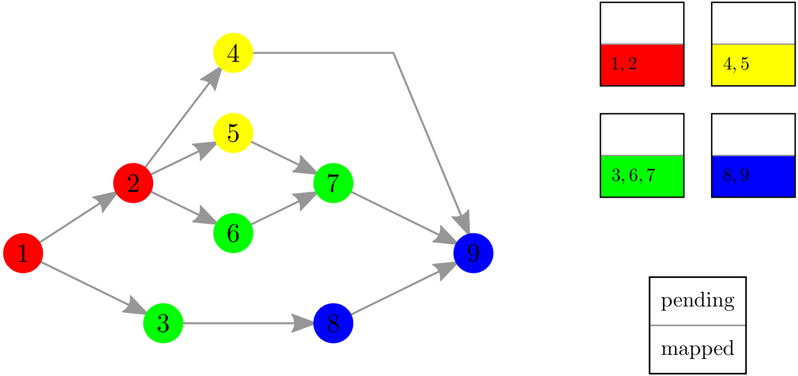
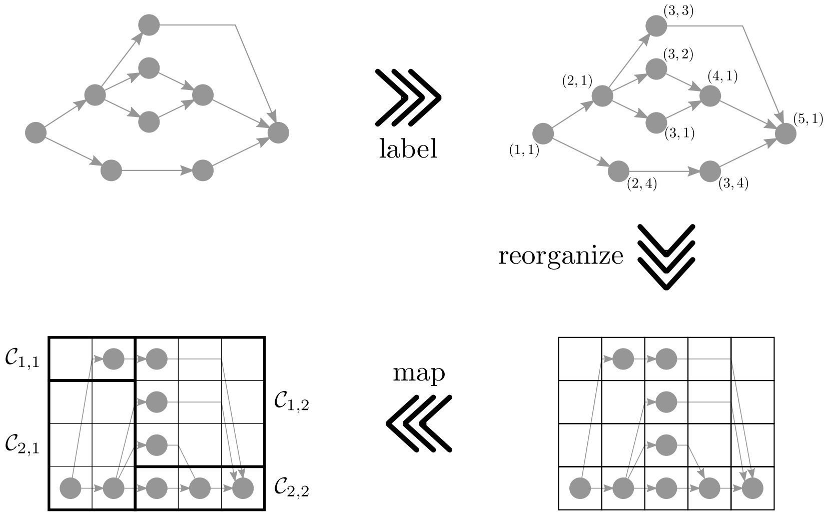


DPA2D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(3, 3)
(3, 2)
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(2, 4)
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reorganize
map
$\mathcal{C}_{1,1}$
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$\mathcal{C}_{2,1}$
$\mathcal{C}_{2,2}$