Co-scheduling algorithms for high-throughput workload execution

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Motivation

- Execution time of HPC applications
  - Can be significantly reduced when using a large number of processors
  - But inefficient resource usage if all resources used for a single application (non-linear decrease of execution time)

- Pool of several applications
  - Co-scheduling algorithms: execute several applications concurrently
  - Increase individual execution time of each application, but
    - (i) improve efficiency of parallelization
    - (ii) reduce total execution time
    - (iii) reduce average response time

- Increase platform yield, and save energy

Framework

- Distributed-memory platform with p identical processors
- Set of n independent tasks (or applications) $T_1, \ldots, T_n$; application T_i can be assigned $\sigma(i) = j$ processors, and
  - p_i is the minimum number of processors required by T_i;
  - $t_{i,j}$ is the execution time of task T_i with j processors;
  - $work(i,j) = j \times t_{i,j}$ is the corresponding work.
- We assume the following for $1 \leq i \leq n$ and $p_i \leq j < p$:

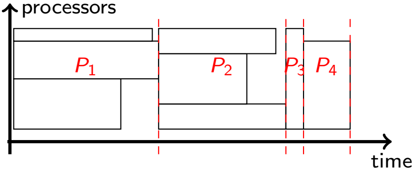
Non increasing execution time: $t_{i,j+1} \leq t_{i,j}$

Non decreasing work: $work(i,j+1) \geq work(i,j)$

Co-schedules

A co-schedule partitions the n tasks into groups (called *packs*):

- All tasks from a given pack start their execution at the same time
- Two tasks from different packs have disjoint execution intervals



A co-schedule with four packs P_1 to P_4

Definition (k-IN-p-COSCHEDULE optimization problem)

Given a fixed constant $k \leq p$, find a co-schedule with at most k tasks per pack that minimizes the execution time.

The most general problem is when $k = p$, but in some frameworks we may have an upper bound $k < p$ on the maximum number of tasks within each pack.

Related work

- *Performance bounds for level-oriented two-dimensional packing algorithms*, Coffman, Garey, Johnson:
  Strip-packing problem, parallel tasks (fixed number of processors), approximation algorithm based on "shelves"
- *Scheduling parallel tasks: Approximation algorithms*, Dutot, Mounié, Trystram:
  Use this model to approximate the moldable model; they studied the p-IN-p-COSCHEDULE for identical moldable tasks (polynomial with DP)
- Widely studied for sequential tasks

Complexity: Polynomial instances

Theorem

The 1-IN-*p*-COSCHEDULE *and* 2-IN-*p*-COSCHEDULE *problems can both be solved in polynomial time.*

Proof.

If there is a batch with exactly tasks T_i and $T_{i'}$, then its execution time is $\min_{j=p_i..p-p_{i'}}\left(\max(t_{i,j}, t_{i',p-j})\right)$.

We then construct the complete weighted graph $G = (V, E)$, where $|V| = n$, and

$$e_{i,i'} = \begin{cases} t_{i,p} & \text{if } i = i' \\ \min_{j=p_i..p-p_{i'}}\left(\max(t_{i,j}, t_{i',p-j})\right) & \text{otherwise} \end{cases}$$

Finally, finding a perfect matching of minimal weight in G leads to the optimal solution for 2-IN-*p*-COSCHEDULE. □

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Finally, finding a perfect matching of minimal weight in G leads to the optimal solution for 2-IN-p-COSCHEDULE. □

Complexity: NP-completeness

Theorem

The 3-IN-p-COSCHEDULE problem is strongly NP-complete.

Proof.

We reduce this problem to 3-PARTITION: Given an integer B and $3n$ integers $a_1, \ldots, a_{3n}$, can we partition the $3n$ integers into n triplets, each of sum B? This problem is strongly NP-hard so we can encode the a_i's and B in unary.

We build instance $\mathcal{I}_2$ of 3-IN-p-COSCHEDULE, with $p = B$ processors, a deadline $D = n$, and $3n$ tasks T_i such that $t_{i,j} = 1 + \frac{1}{a_i}$ if $j < a_i$, $t_{i,j} = 1$ otherwise. *(The $t_{i,j}$'s verify the constraints on work and execution time.)*

Any solution of $\mathcal{I}_2$ has n packs each of cost 1 with exactly 3 tasks in it, and the sum of the weights of these tasks sums up to B. □

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Complexity: NP-completeness

Theorem

For $k \geq 3$, The k-IN-p-COSCHEDULE problem is strongly NP-complete.

Proof.

We reduce these problems to the same instance of the 3-IN-p-COSCHEDULE problem, to which we further add:

- $n(k-3)$ buffer tasks such that $t_{i,j} = \max\left(\frac{B+1}{j}, 1\right)$;
- the number of processors is now $p = B + (k-3)(B+1)$;
- the deadline remains $D = n$.

Again, we need to execute each pack in unit time and at most n packs. The only way to proceed is to execute within each pack $k-3$ buffer tasks on $B+1$ processors. □

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Scheduling a pack of tasks

Theorem

Given k tasks to be scheduled on p processors in a single pack (1-pack-schedule), we can find in time $O(p \log k)$ the schedule that minimizes the cost of the pack.

Greedy algorithm Optimal-1-pack-schedule:

- Initially, each task T_i is assigned its minimum number of processors p_i
- While there remain available processors, assign one to the largest task (with their current processor assignment)

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Optimal solution

Theorem

The following integer linear program characterizes the k-IN-p-COSCHEDULE problem, where the unknown variables are the $x_{i,j,b}$'s (Boolean variables) and the y_b's (rational variables), for $1 \leq i, b \leq n$ and $1 \leq j \leq p$:

$$
\begin{array}{ll}
\text{Minimize} \sum_{b=1}^{n} y_b & \text{subject to} \\
\text{(i)} \ \sum_{j,b} x_{i,j,b} = 1, & 1 \leq i \leq n \\
\text{(ii)} \ \sum_{i,j} x_{i,j,b} \leq k, & 1 \leq b \leq n \\
\text{(iii)} \ \sum_{i,j} j \times x_{i,j,b} \leq p, & 1 \leq b \leq n \\
\text{(iv)} \ x_{i,j,b} \times t_{i,j} \leq y_b, & 1 \leq i, b \leq n, 1 \leq j \leq p
\end{array}
$$

$x_{i,j,b} = 1$ iff T_i is in pack b and executed on j processors

y_b is the execution time of pack b

Approximation algorithm

- 3-approximation algorithm for the problem p-IN-p-COSCHEDULE
- Initialization: task T_i executed on p_i processors
- Greedy procedure MAKE-PACK to create packs (with $k = p$), given $\sigma(i)$ processors for task T_i

procedure MAKE-PACK(n, p, k, σ)
begin
  L: list of tasks sorted in non-increasing execution times $t_{i,\sigma(i)}$;
  while $L \neq \emptyset$ **do**
    Schedule the current task on the first pack with enough available processors and less than k tasks;
    Create a new pack if no existing pack fits;
    Remove the current task from L;
  end
  return *the set of packs*
end

- PACK-APPROX: Iteratively refine the solution, adding a processor to the task with longest execution time

```
procedure PACK-APPROX(T_1, ..., T_n)
begin
    COST = +∞;
    for j = 1 to n do  σ(j) ← p_j ;
    for i = 0 to Σ_j (p − p_j) − 1 do
        Call MAKE-PACK (n, p, p, σ);
        Let COST_i be the cost of the co-schedule;
        if COST_i < COST then  COST ← COST_i;
        Let A_tot(i) = Σ_{j=1}^{n} t_{j,σ(j)} σ(j);
        Let T_{j*} be one task that maximizes t_{j,σ(j)};
        if (A_tot(i) > p × t_{j*,σ(j*)}) or (σ(j*) = p) then
            return COST
        else
            σ(j*) ← σ(j*) + 1
        end
    end
    return COST;
end
```

Theorem

PACK-APPROX *is a 3-approximation algorithm for the* p-IN-p-COSCHEDULE *problem.*

Involved proof, studying the different ways to exit algorithm PACK-APPROX:

- The task with longest execution time is already assigned p processors
- The sum of the work of all tasks $(\sum_{i=1}^{n} t_{i,\sigma(i)}\sigma(i))$ is greater than p times the longest execution time
- Each task has been assigned p processors

Heuristics

In all heuristics (even randoms), once the different packs are chosen, we always run Optimal-1-pack-schedule *on each pack.*

RANDOM-PACK: generates the packs randomly: randomly chooses an integer j between 1 and k, and then randomly selects j tasks to form a pack.

RANDOM-PROC: assigns the number of processors to each task randomly, then calls MAKE-PACK to generate the packs.

PACK-BY-PACK (ε): creates packs that are "well-balanced": the difference between smallest and longest execution times of a pack is small (ratio of $1 + \varepsilon$).

PACK-APPROX: an extension of the approximation algorithm in the case where there are at most k tasks in a pack.

Heuristic variants

Improvement of the heuristics by using up to 9 runs:

- 4 *random* heuristics with either one or nine runs:
  - Random-Pack-1, Random-Pack-9
  - Random-Proc-1, Random-Proc-9
- pack-by-pack (ε) with
  - either one single run with $\varepsilon = 0.5$ (pack-by-pack-1)
  - or 9 runs with $\varepsilon \in \{.1, .2, \ldots, .9\}$ (pack-by-pack-9)
- Only one version of pack-Approx

Further variants: up to 99 runs, or better choice to create packs in pack-by-pack, but only little improvement at the price of a much higher running time

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Workloads

- Workload-I: 10 parallel scientific applications (involving VASP, ABAQUS, LAMMPS, Petsc); execution time observed on a cluster with $p = 16$ processors and 128 cores
- Workload-II: synthetic test suite with 65 tasks for 128 cores ($p = 16$); execution time for problem size m on q cores:

$$t(m,q) = f \times t(m,1) + (1-f)\frac{t(m,1)}{q} + \kappa(m,q)$$

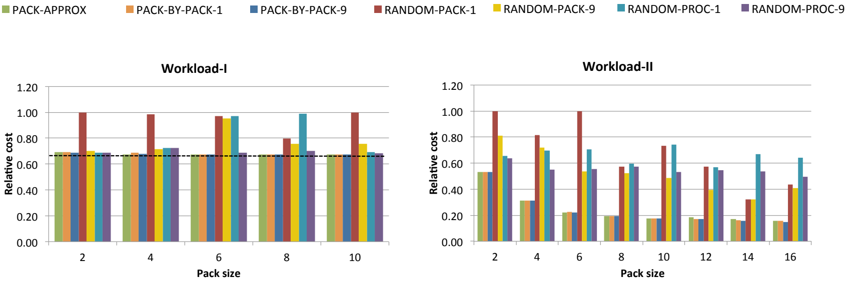
  - f: inherently serial fraction
  - κ: overheads related to synchronization and communication
- Workload-III: similar to Workload-II, but with 260 tasks for 256 cores ($p = 32$)

Assessing the performance of heuristics

- Seven heuristics and three measures:

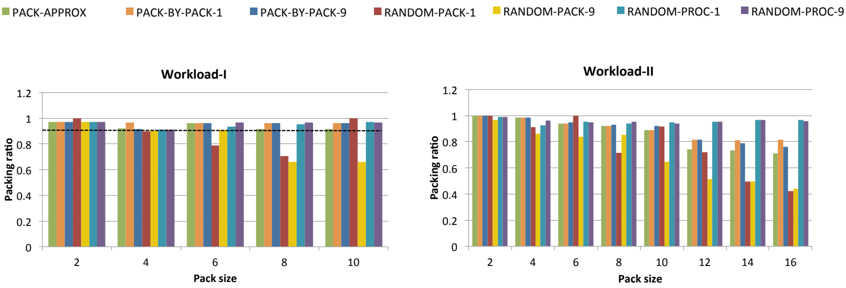
- Relative cost: cost divided by the cost of a schedule with each task scheduled on p processors (schedule used in practice, n-packs-schedule)
- Packing ratio: total work $\sum_{i=1}^{n} t_{i,\sigma(i)} \times \sigma(i)$ divided by p times the cost of the co-schedule; close to 1 if no idle time
- Relative response time: mean response time compared to n-packs-schedule with non-decreasing order of execution time

Results: Relative cost



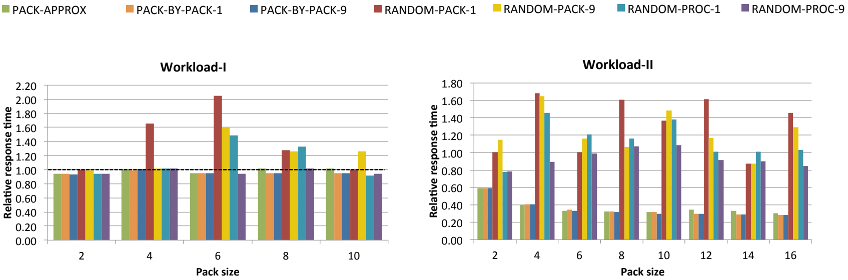
- Horizontal line = optimal co-schedule (exhaustive search for W-I)
- PACK-APPROX and PACK-BY-PACK close to optimal
- Gain of more than 35% compared to *n*-packs-schedule for W-I
- Huge gains for W-II (more than 80%, better for larger values of pack size)

Results: Packing ratio



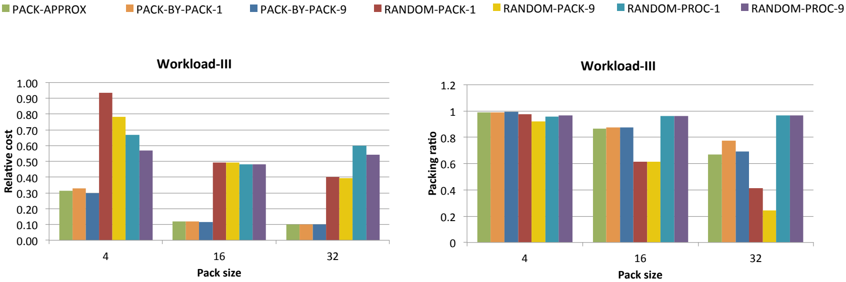
- Packing ratios very close to one for PACK-BY-PACK and PACK-APPROX
- High quality packings

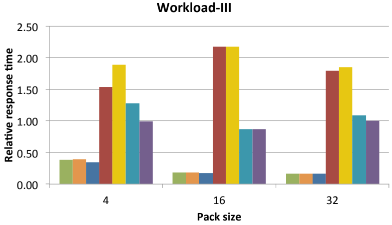
Results: Response time



- Values less than 1: improvements in response times
- For Workload-II and larger values of the pack size, response time gains over 80%
- *k*-IN-*p*-COSCHEDULE attractive from the user perspective

Results: Workload-III





- Scalability trends with 260 tasks on 32 processors
- PACK-APPROX and PACK-BY-PACK are clearly superior

Results: Running times

	Workload-I	Workload-II	Workload-III
PACK-APPROX	0.50	0.30	5.12
PACK-BY-PACK-1	0.03	0.12	0.53
PACK-BY-PACK-9	0.30	1.17	5.07
RANDOM-PACK-1	0.07	0.34	9.30
RANDOM-PACK-9	0.67	2.71	87.25
RANDOM-PROC-1	0.05	0.26	4.49
RANDOM-PROC-9	0.47	2.26	39.54

- Average running times in milliseconds
- All heuristics run within a few ms, even for W-III
- Random heuristics slower (cost of random number generation)
- PACK-BY-PACK-9 comparable with PACK-APPROX

Conclusion

- *Theoretically:* Exhaustive complexity study
  - NP-completeness (need to choose for each task both number of processors and pack)
  - Optimal strategy once the packs are formed
  - Efficient algorithm to partition tasks with pre-assigned resources into packs (3-approximation algorithm for $k = p$)
- *Practically:* Heuristics building upon theoretical study, with very good performance
  - Heuristic of choice: PACK-BY-PACK-9
  - Great improvement compared to existing schedulers (in terms of relative cost)
  - Corresponding savings in system energy cost
  - Measurable benefits in average response time

Future work

- Combine with DVFS technique (dynamic voltage and frequency scaling) to further obtain gains in energy consumption
- Experiment at a larger scale (university computing facilities), where workload attributes do not vary much in time, and energy costs are a limiting factor
- Theoretically, obtain more approximation results