Mapping pipelined applications with replication to increase throughput and reliability

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Motivations

- Mapping pipelined applications onto parallel platforms: practical applications, but difficult challenge
- Both performance (throughput) and reliability objectives: even more difficult!
- Use of replication: mapping an application stage onto more than one processor
  - redundant computations: increase reliability
  - round-robin computations (over consecutive data sets): increase throughput
  - bi-criteria problem: need to trade-off between two kinds of replication

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Main contributions

- Theoretical side:
  assess problem hardness with different mapping rules and platform characteristics

- Practical side:
  heuristics on most general (NP-complete) case,
  exact algorithm based on A*,
  experiments to assess heuristics performance

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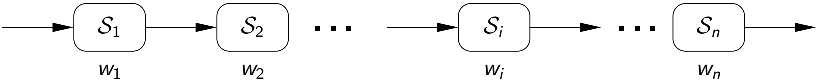
Outline of the talk

Applicative framework



- Pipeline of n stages $\mathcal{S}_1, \ldots, \mathcal{S}_n$
- Stage $\mathcal{S}_i$ performs a number w_i of computations
- Communication costs are negligible in comparison with computation costs

Target platform

- Platform with p processors $P_1, \ldots, P_p$, fully interconnected as a (virtual) clique
- For $1 \leq u \leq p$, processor P_u has speed s_u and failure probability $0 < f_u < 1$
- Failure probability: independent of the duration of the application, meant to run for a long time (cycle-stealing scenario)
- *SpeedHom* platform: identical speeds $s_u = s$ for $1 \leq u \leq p$ (as opposed to *SpeedHet*)
- *FailureHom* platform: identical failure probabilities (as opposed to *FailureHet*)

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- Platform with *p* processors $P_1, \ldots, P_p$, fully interconnected as a (virtual) clique
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Mapping problem

- Interval mapping: consecutive stages mapped together: partition of $[1..n]$ into $m \leq p$ intervals I_j
- I_j mapped onto set of processors A_j, organized into ℓ_j teams
  - processors within a team perform redundant computations (replication for reliability)
  - different teams assigned to same interval execute distinct data sets in a round-robin fashion (replication for performance)
- A processor cannot participate in two different teams
- $\ell = \sum_{j=1}^{m} \ell_j$ is the total number of teams

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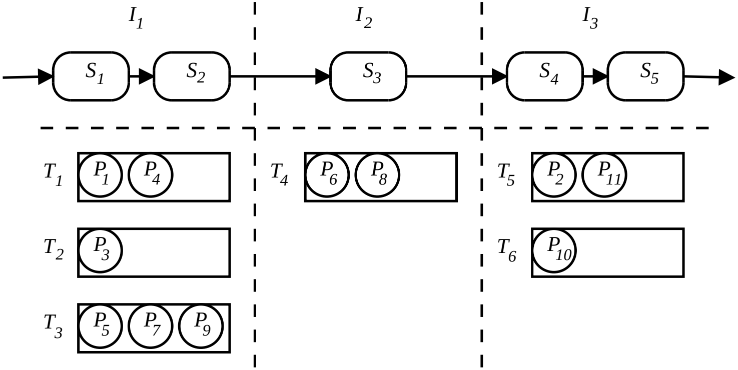
Example of mapping



$n = 5$ stages divided into $m = 3$ intervals

$p = 11$ processors organized in $\ell = 6$ teams

$\ell_1 = 3, \ell_2 = 1, \ell_3 = 2$

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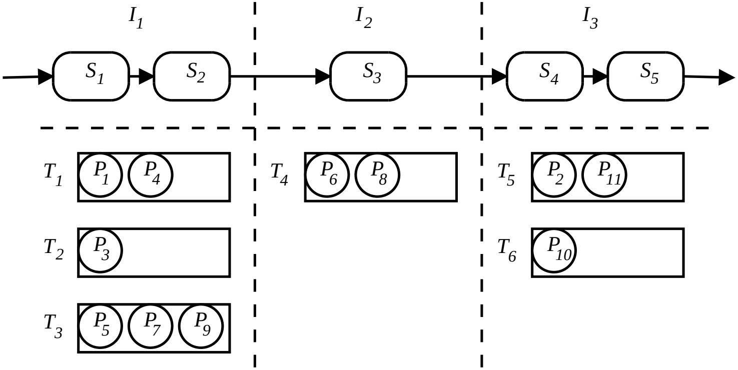

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Objective functions

- Period of the application:

$$\mathcal{P} = \max_{1 \leq j \leq m} \left\{ \frac{\sum_{i \in I_j} w_i}{\ell_j \times \min_{P_u \in A_j} s_u} \right\}$$

Round-robin distribution: each team compute one data set every other ℓ_j ones, computation slowed down by slowest processor for interval

- Failure probability:

$$\mathcal{F} = 1 - \prod_{1 \leq k \leq \ell} (1 - \prod_{P_u \in T_k} f_u)$$

Computation successful if at least one surviving processor per team

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The problem

- Determine the best interval mapping, over all possible partitions into intervals and processor assignments
- Mono-criterion: minimize period or failure probability
- Bi-criteria: (i) given a threshold period, minimize failure probability or (ii) given a threshold failure probability, minimize period

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1. Framework
   - Application
   - Platform
   - Mapping
   - Objective
2. Complexity results
   - Mono-criterion
   - Bi-criteria
   - Approximation results
3. Practical side
   - Heuristics
   - Optimal algorithm using A*
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4. Conclusion

Mono-criterion complexity results

- Failure probability: easy on any kind of platforms: group all stages as a single interval, processed by one single team with all p processors

- Period: one processor per team
  - *SpeedHom* platform: one interval processed by p teams
  - *SpeedHet* platforms: NP-hard in the general case, polynomial if $w_i = w$ for $1 \leq i \leq n$ (see previous work [Algorithmica2010])

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Bi-criteria complexity results

- Preliminary result: for *SpeedHom* platforms, there exists an optimal bi-criteria mapping with one single interval
  - *Proof*: starting from an optimal solution with several intervals, merge intervals, and the single interval is processed by all teams of optimal solution
  - Failure probability remains the same (same teams)
  - New period cannot be greater than optimal period (*SpeedHom* platform)
- Not true on *SpeedHet* platforms:
  example with $w_1 = s_1 = 1$ and $w_2 = s_2 = 2$, $\mathcal{F}^* = 1$
  - period 1 with two intervals
  - period 3/2 with one single interval

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SpeedHom-FailureHom platforms

- *SpeedHom-FailureHom*: Polynomial time algorithm
- Fixed period $\mathcal{P}^*$
  - one single interval with minimum number of teams

$$\ell_{min} = \left\lceil \frac{\sum_{i=1}^{n} w_i}{\mathcal{P}^* \times s} \right\rceil$$

  - greedily assign processors to teams to have balanced teams
  - algorithm in $O(p)$
- Converse problem: fixed $\mathcal{F}^*$
  - one single interval...
  - ...but must try all possible number of teams $1 \leq \ell \leq p$
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With heterogeneous platforms

- *SpeedHet*-*FailureHom* is NP-hard
  because *SpeedHet* is NP-hard for period minimization
- *SpeedHom*-*FailureHet* becomes NP-hard as well:
  balancing processors within teams is combinatorial;
  reduction from 3-PARTITION
- Intermediate result: best reliability always obtained by
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Approximation results

- *SpeedHom*: always optimal with single interval
- *SpeedHet*: period minimization problem (NP-hard)
- The optimal single-interval mapping can be found:
  - sort processors by non-increasing speeds
  - for $1 \leq i \leq p$, compute period using i fastest processors
  - time $O(p \log p)$
- Theorem: single-interval mapping is a n-approximation algorithm for period minimization; this factor cannot be improved
- *Proof sketch*: start from an optimal solution, with $m \leq n$ intervals, and build a single interval solution, with period $\mathcal{P}_1 \leq m \times \mathcal{P}_m$

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Heuristics

- *SpeedHet-FailureHet* platforms
- Minimize $\mathcal{F}$ under a fixed upper period $\mathcal{P}^*$
- Counterpart problem: binary search over $\mathcal{P}^*$

- Two heuristics:
  - OneInterval: stages grouped as a single interval (motivated by complexity results)
  - MultiInterval: solution with multiple intervals (recall that single interval may be far from optimal)

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OneInterval

- One single interval
- Determine number of teams: try all values ℓ between 1 and p
- For a given ℓ, discard processors too slow for period
- Assign processors to teams to minimize failure probability
  - From complexity results: balance reliability across teams
  - NP-hard problem but efficient greedy heuristic: sort processors by non-decreasing failure probability and assign next processor to team with highest failure probability
- Time complexity: $O(p^2 \log p)$

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MultiInterval

- Step 1: create $\min(n, p)$ intervals (one stage per processor, or balance computational load across intervals)
- Step 2: greedily add processors to stages, to minimize maximum ratio of interval computation load to accumulated processor speed
- Step 3: for each interval, use OneInterval to form teams; use previously unallocated processors (too slow for period); increase bound on period for the interval until valid allocation returned
- Step 4: if period bound not achieved for at least one interval, merge interval with largest period with previous or next interval, until bound is achieved
- Step 5: merge intervals with highest failure probability as long as it is beneficial

- Note that OneInterval is called each time we tentatively merge two intervals (steps 4 and 5)
- Time complexity: $O(p^3 \log p)$

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- Step 4: if period bound not achieved for at least one interval, merge interval with largest period with previous or next interval, until bound is achieved
- Step 5: merge intervals with highest failure probability as long as it is beneficial

- Note that OneInterval is called each time we tentatively merge two intervals (steps 4 and 5)
- Time complexity: $O(p^3 \log p)$

A* algorithm

- A* best-first state space search algorithm for small problem instances
- Non-linearity of failure probability: rules out the use of integer linear programming
- Search space: state s is a partial solution (i.e., partial mapping), with underestimated cost value $c(s)$
- Expansion of a partial solution with lowest $c(s)$ value, with a stage or a processor
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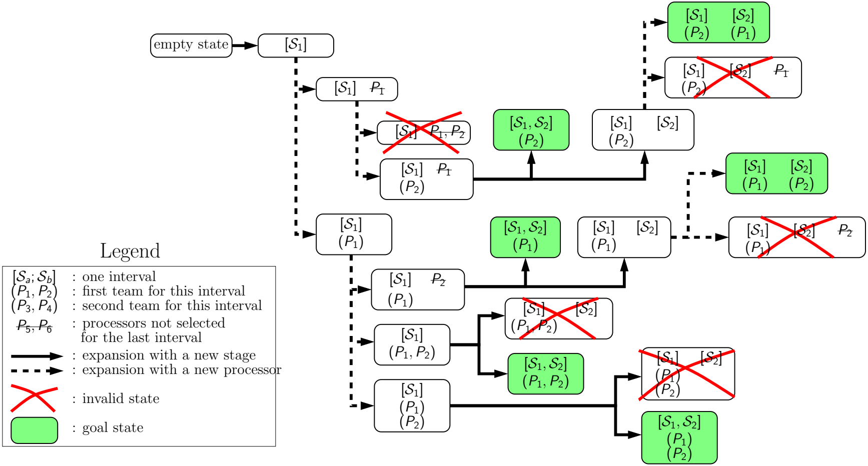
State tree for two stages on two processors



Underestimate cost functions

- Failure probability $\mathcal{F}$
  - Partial mapping: adding team increases failure probability
  - Underestimate: add remaining processors to existing teams
  - NP-hard problem: consider amount of reliability available and distribute it to the existing teams to balance their reliability
- Period $\mathcal{P}$
  - Need to check that partial solution does not exceed the bound: can be computed exactly
  - Second underestimate: optimal period achieved by remaining processors on remaining stages
  - NP-hard problem: consider perfect load balance: $\mathcal{P} \leq \frac{\sum w_i}{\sum s_u}$

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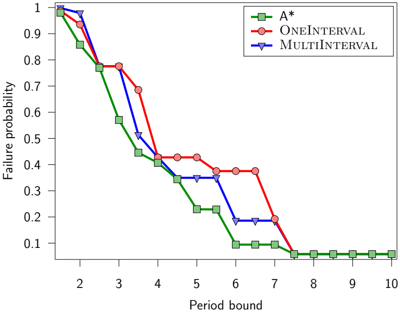
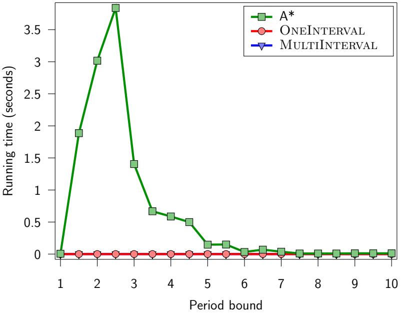
Heuristics vs A*

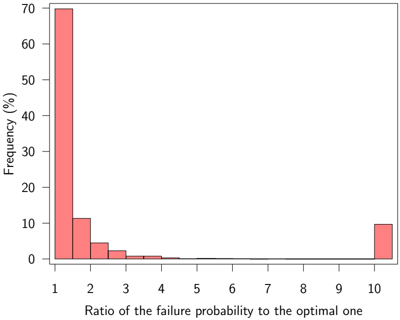
- Randomly generated workload scenarios
- Both heuristics close to optimal solution
- OneInterval is better than MultiInterval in a few cases
- A* much slower, but main limitation is memory

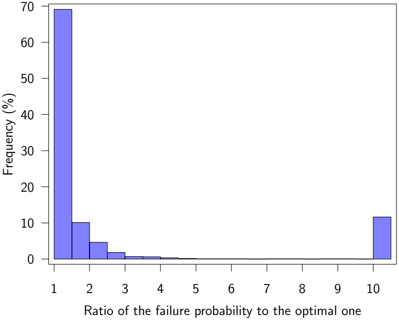




Performance of heuristics

- Distribution of ratio between failure probability obtained by a heuristic (OneInterval in red, MultiInterval in blue) and optimal failure probability (A*) (optimal: ratio 1)
- On average, heuristics 20% above optimal
- Ratio 10: cases in which heuristics find no solution (≈ 10%)





Larger scenarios

- ONEINTERVAL better in 61% of the cases
- MULTIINTERVAL better in 20% of the cases
- On average, failure probability of ONEINTERVAL 2% above MULTIINTERVAL
- Comparison of ONEINTERVAL with optimal single-interval solution (easy to compute with A*): in average, 0.05% above optimal, and 5% in the worst case

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Outline of the talk

1. Framework
   - Application
   - Platform
   - Mapping
   - Objective
2. Complexity results
   - Mono-criterion
   - Bi-criteria
   - Approximation results
3. Practical side
   - Heuristics
   - Optimal algorithm using A*
   - Evaluation results
4. Conclusion

Conclusion and future work

- Exhaustive complexity study
  - polynomial time algorithm for *SpeedHom-FailureHom* platforms
  - NP-completeness with one level of heterogeneity
  - approximation results to compare single interval solution with any other solution
- Practical solution to the problem
  - efficient heuristics (inspired by theoretical study) for *SpeedHet-FailureHet* platforms
  - A* algorithm with non-trivial underestimate functions
  - experimental results: very good behaviour of heuristics
- Future work
  - further approximation results
  - enhanced multiple interval heuristics
  - improved A* techniques

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