Resilient and energy-aware scheduling algorithms

Anne Benoit

LIP, Ecole Normale Supérieure de Lyon, France

Anne.Benoit@ens-lyon.fr

http://graal.ens-lyon.fr/~abenoit/

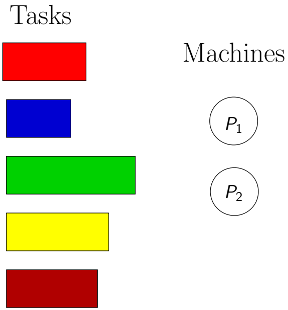
4th GDR RSD and ASF Winter School on
Distributed Systems and Networks, February 2019

Motivation

Scheduling: Allocate resources to applications to optimize some performance metrics

- Resources: Large-scale distributed systems with millions of components
- Applications: Parallel applications, expressed as a set of tasks, or divisible application with some work to complete
- Performance metrics: Of course we are concerned with the performance of the applications, but also with resilience and energy consumption

Classical scheduling problems

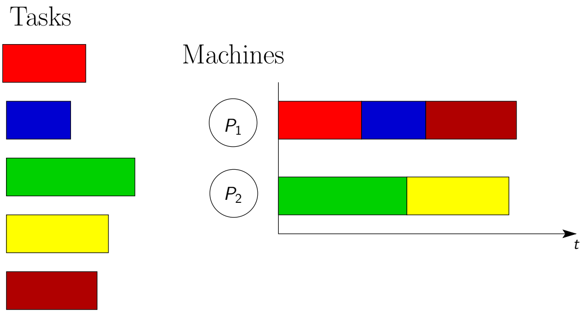


Objectives:

- Minimizing total execution time (C_{max})
- Minimizing weighted sum of execution times $\sum_i w_i C_i$

Results: NP-completeness, algorithms, approximation algorithms, (in-)approximation bounds

Classical scheduling problems

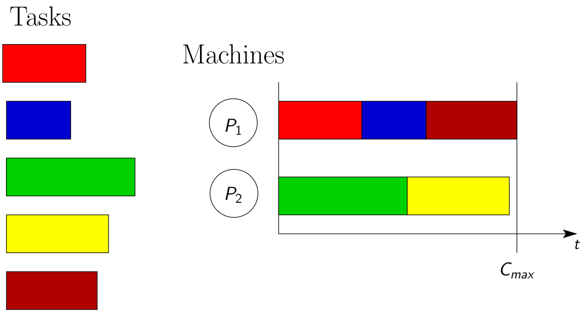


Objectives:

- Minimizing total execution time (C_{max})
- Minimizing weighted sum of execution times $\sum_i w_i C_i$

Results: NP-completeness, algorithms, approximation algorithms, (in-)approximation bounds

Classical scheduling problems

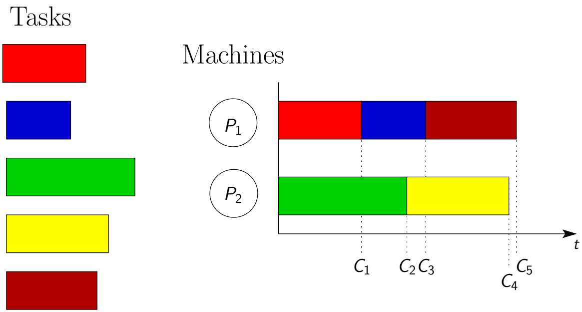


Objectives:

- Minimizing total execution time (C_{max})
- Minimizing weighted sum of execution times $\sum_i w_i C_i$

Results: NP-completeness, algorithms, approximation algorithms, (in-)approximation bounds

Classical scheduling problems

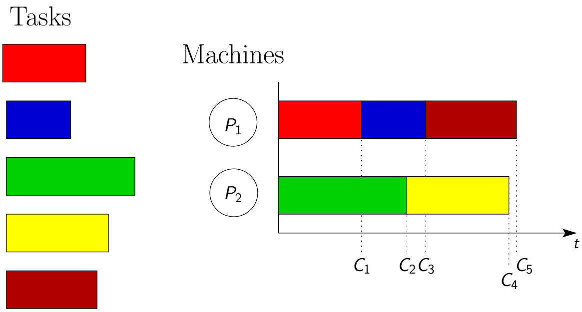


Objectives:

- Minimizing total execution time (C_{max})
- Minimizing weighted sum of execution times $\sum_i w_i C_i$

Results: NP-completeness, algorithms, approximation algorithms, (in-)approximation bounds

Classical scheduling problems



Objectives:

- Minimizing total execution time (C_{max})
- Minimizing weighted sum of execution times $\sum_i w_i C_i$

Results: NP-completeness, algorithms, approximation algorithms, (in-)approximation bounds

Dealing with failures

- Consider one processor (e.g. in your laptop)
  - Mean Time Between Failures (MTBF) = 100 years
  - (Almost) no failures in practice ☺

  Why bother about failures?

- **Theorem:** The MTBF decreases linearly with the number of processors! With 36500 processors:
  - MTBF = 1 day
  - A failure every day on average!

A large simulation can run for weeks, hence it will face failures ☹

Dealing with failures

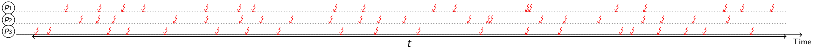
- Consider one processor (e.g. in your laptop)
  - Mean Time Between Failures (MTBF) = 100 years
  - (Almost) no failures in practice ☺

  Why bother about failures?

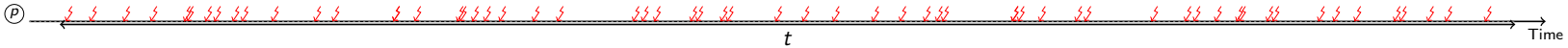
- **Theorem:** The MTBF decreases linearly with the number of processors! With 36500 processors:
  - MTBF = 1 day
  - A failure every day on average!

A large simulation can run for weeks, hence it will face failures ☹

Intuition



If three processors have around 20 faults during a time t ($\mu = \frac{t}{20}$)...

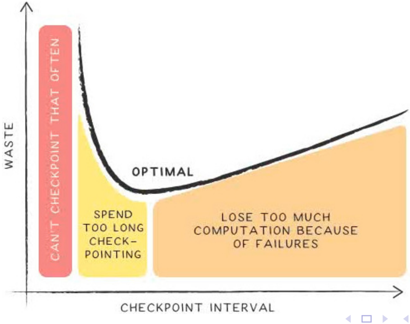


...during the same time, the platform has around 60 faults ($\mu_p = \frac{t}{60}$)

So, how to deal with failures?

Failures usually handled by adding redundancy:

- Replicate the work (for instance, use only half of the processors, and the other half is used to redo the same computation)
- Checkpoint the application: Periodically save the state of the application on stable storage, so that we can restart in case of failure without loosing everything



Another crucial issue: Energy consumption

"The internet begins with coal"

- Nowadays: more than 90 billion kilowatt-hours of electricity a year; requires 34 giant (500 megawatt) coal-powered plants, and produces huge CO_2 emissions
- Explosion of artificial intelligence; AI is hungry for processing power! Need to double data centers in next four years
  $\rightarrow$ how to get enough power?
- Failures: Redundant work consumes even more energy

Energy and power awareness $\leadsto$ crucial for both environmental and economical reasons

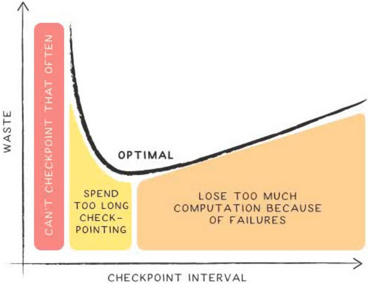
Outline

1. Checkpointing for resilience
   - How to cope with errors?
   - Optimization objective and optimal period
   - Optimal period when accounting for energy consumption
2. Combining checkpoint with replication
   - Replication analysis
   - Simulations
3. Back to task scheduling
4. A different re-execution speed can help
   - Model, optimization problem, optimal solution
   - Simulations
   - Extensions: both fail-stop and silent errors
5. Summary and need for trade-offs

Introduction to resilience

- Fail-stop errors:
  - Component failures (node, network, power, ...)
  - Application fails and data is lost
- Silent data corruptions:
  - Bit flip (Disk, RAM, Cache, Bus, ...)
  - Detection is not immediate, and we may get wrong results

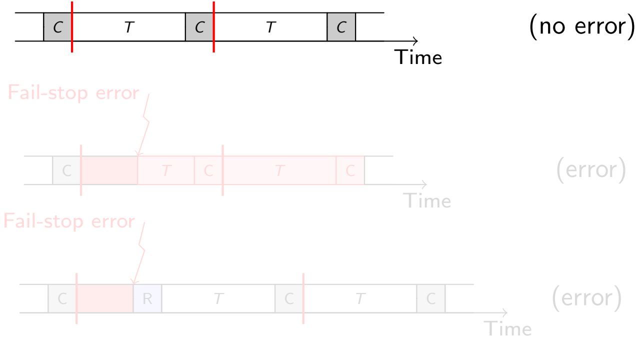
How often should we checkpoint to minimize the waste, i.e., the time lost because of resilience techniques and failures?



Outline

Coping with fail-stop errors

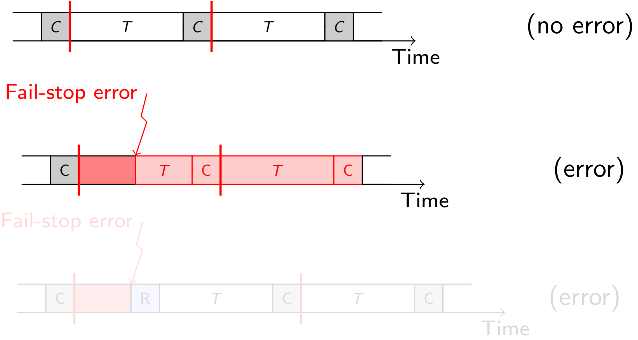
Periodic checkpoint, rollback, and recovery:



- Coordinated checkpointing (the platform is a giant macro-processor)
- Assume instantaneous interruption and detection.
- Rollback to last checkpoint and re-execute.

Coping with fail-stop errors

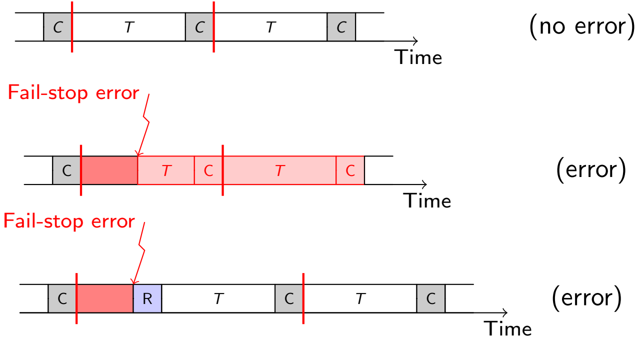
Periodic checkpoint, rollback, and recovery:



- Coordinated checkpointing (the platform is a giant macro-processor)
- Assume instantaneous interruption and detection.
- Rollback to last checkpoint and re-execute.

Coping with fail-stop errors

Periodic checkpoint, rollback, and recovery:



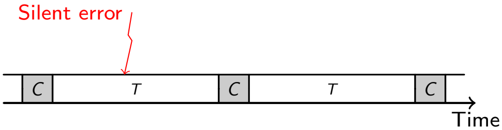
- Coordinated checkpointing (the platform is a giant macro-processor)
- Assume instantaneous interruption and detection.
- Rollback to last checkpoint and re-execute.

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

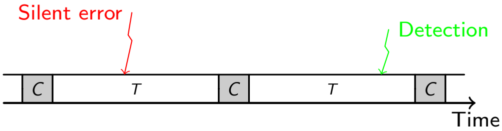
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

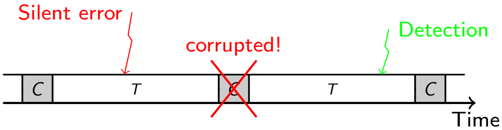
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

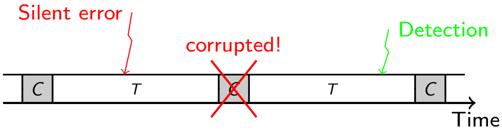
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

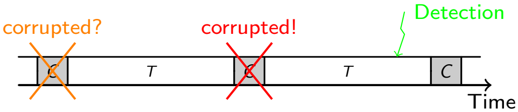
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

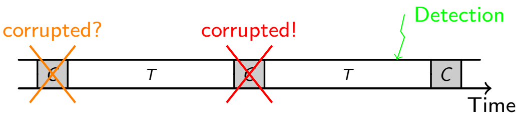
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

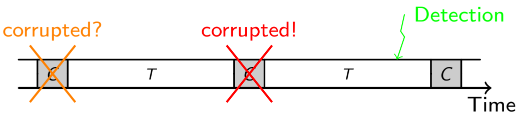
Need an active method to detect silent errors!

Coping with silent errors

Silent error = detection latency

Error is detected only when corrupted data is activated

Same approach?



Keep multiple checkpoints?

Which checkpoint to recover from?

Need an active method to detect silent errors!

Methods for detecting silent errors

General-purpose approaches

- Replication [*Fiala et al. 2012*] or triple modular redundancy and voting [*Lyons and Vanderkulk 1962*]

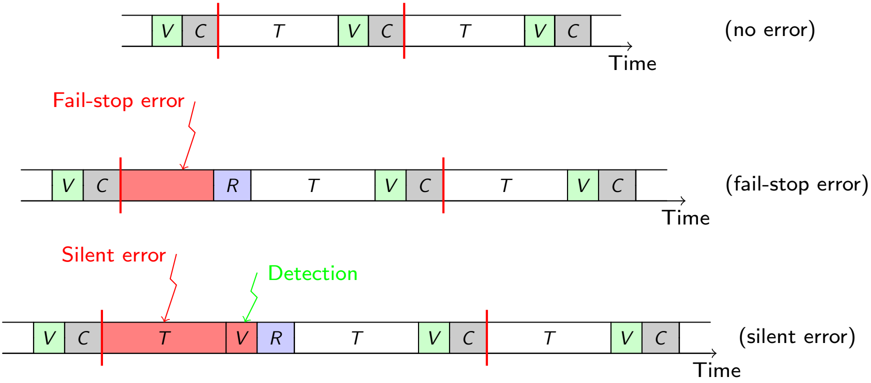
Application-specific approaches

- Algorithm-based fault tolerance (ABFT): checksums in dense matrices Limited to one error detection and/or correction in practice [*Huang and Abraham 1984*]
- Partial differential equations (PDE): use lower-order scheme as verification mechanism [*Benson, Schmit and Schreiber 2014*]
- Generalized minimal residual method (GMRES): inner-outer iterations [*Hoemmen and Heroux 2011*]
- Preconditioned conjugate gradients (PCG): orthogonalization check every k iterations, re-orthogonalization if problem detected [*Sao and Vuduc 2013, Chen 2013*]

Data-analytics approaches

- Dynamic monitoring of HPC datasets based on physical laws (e.g., temperature limit, speed limit) and space or temporal proximity [*Bautista-Gomez and Cappello 2014*]
- Time-series prediction, spatial multivariate interpolation [*Di et al. 2014*]

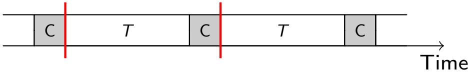
Coping with fail-stop and silent errors



What is the optimal checkpointing period?

Outline

Optimization objective (1/2)



- T is the **pattern length** (time without failures)
- C is the checkpoint cost
- $\mathbb{E}(T)$ is the expected execution time of the pattern

By definition, the overhead of the pattern is defined as:

$$\mathbb{H}(T) = \frac{\mathbb{E}(T)}{T} - 1$$

The overhead measures the fraction of **extra time** due to:

- Checkpoints
- Recoveries and re-executions (failures)

The goal is to minimize the quantity: $\mathbb{H}(T)$

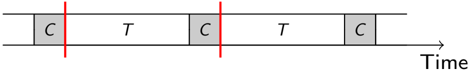
Optimization objective (2/2)

- Goal: Find the **optimal pattern length** T^*, so that the overhead is minimized
- Overhead: $\mathbb{H}(T) = \frac{\mathbb{E}(T)}{T} - 1$

1. Compute expected execution time $\mathbb{E}(T)$ (exact formula)
2. Compute overhead $\mathbb{H}(T)$ (first-order approximation)
3. Derive optimal T^*: fail-stop errors
4. Derive optimal T^*: silent errors
5. Derive optimal T^*: both

1. Expected execution time $\mathbb{E}(T)$

- T: Pattern length
- C: Checkpoint time
- R: Recovery time
- $\lambda^f = \frac{1}{\mu^f}$: Fail-stop error rate



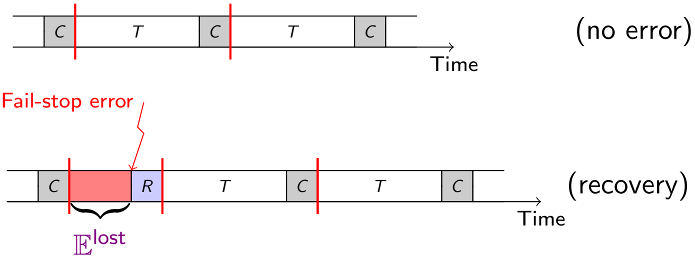
(no error)

$$\mathbb{E}(T) = \mathbb{P}_{no-error}\,(T + C)$$

$$+$$

1. Expected execution time $\mathbb{E}(T)$

- T: Pattern length
- C: Checkpoint time
- R: Recovery time
- $\lambda^f = \frac{1}{\mu^f}$: Fail-stop error rate



$$\mathbb{E}(T) = \mathbb{P}_{no-error}\,(T + C) + \mathbb{P}_{error}\left(\mathbb{E}^{\text{lost}} + R + \mathbb{E}(T)\right)$$

1. Expected execution time $\mathbb{E}(T)$

Assume that failures follow an **exponential distribution** $\text{Exp}(\lambda^f)$

- Independent errors (memoryless property)

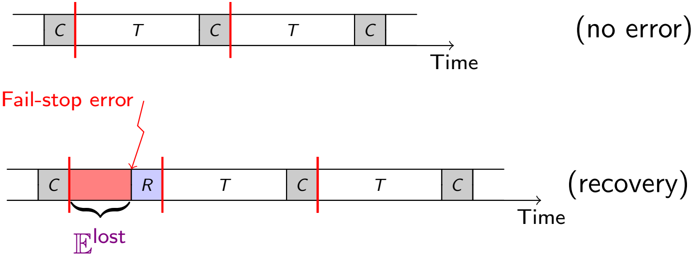
There is at least one error before time t with probability:

$$\mathbb{P}(X \leq t) = 1 - e^{-\lambda^f t} \quad \text{(cdf)}$$

Probability of failure / no-failure

- $\mathbb{P}_{error} = 1 - e^{-\lambda^f T}$
- $\mathbb{P}_{no-error} = e^{-\lambda^f T}$

1. Expected execution time $\mathbb{E}(T)$



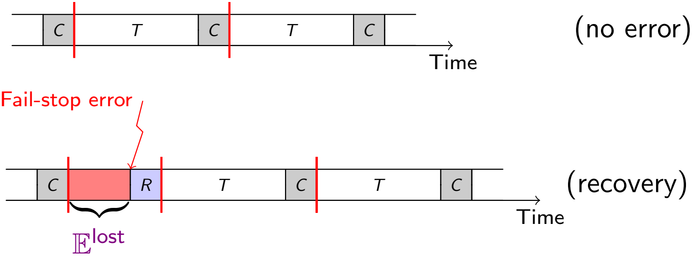
$$\mathbb{E}(T) = e^{-\lambda^f T}(T + C) + (1 - e^{-\lambda^f T})\left(\mathbb{E}^{\text{lost}} + R + \mathbb{E}(T)\right)$$
$$= T + C + (e^{\lambda^f T} - 1)\left(\mathbb{E}^{\text{lost}} + R\right)$$

$\mathbb{E}^{\text{lost}}$ is the time lost when the failure strikes:

$$\mathbb{E}^{\text{lost}} = \int_0^\infty t\mathbb{P}(X = t | X < T)dt = \frac{1}{\lambda^f} - \frac{T}{e^{\lambda^f T} - 1} = \frac{T}{2} + o(\lambda^f T)$$

- **We lose half the pattern upon failure (in expectation)!**

1. Expected execution time $\mathbb{E}(T)$



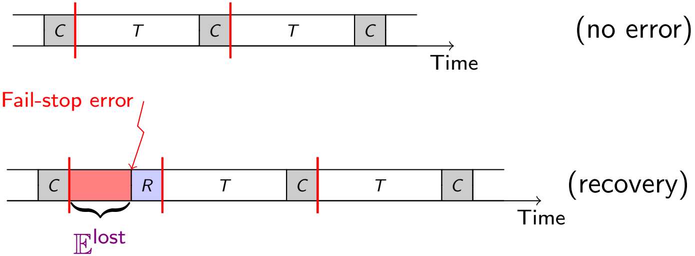
$$\mathbb{E}(T) = e^{-\lambda^f T}(T+C) + (1 - e^{-\lambda^f T})\left(\mathbb{E}^{\text{lost}} + R + \mathbb{E}(T)\right)$$
$$= T + C + (e^{\lambda^f T} - 1)\left(\mathbb{E}^{\text{lost}} + R\right)$$

$\mathbb{E}^{\text{lost}}$ is the time lost when the failure strikes:

$$\mathbb{E}^{\text{lost}} = \int_0^\infty t\mathbb{P}(X = t | X < T)dt = \frac{1}{\lambda^f} - \frac{T}{e^{\lambda^f T} - 1} = \frac{T}{2} + o(\lambda^f T)$$

- **We lose half the pattern upon failure (in expectation)!**

2. Compute overhead $\mathbb{H}(T)$



We use Taylor series to approximate $e^{-\lambda^f T}$ up to first-order terms:

$$e^{-\lambda^f T} = 1 - \lambda^f T + o(\lambda^f T)$$

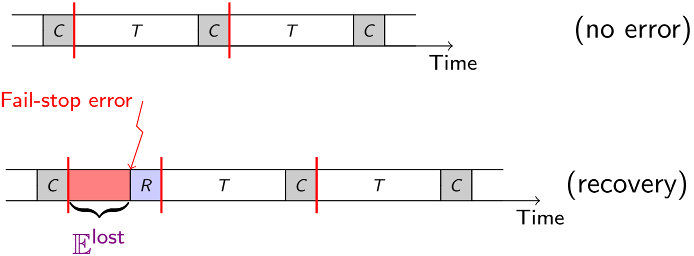
Works well provided that $\lambda^f << T, C, R$

$$\mathbb{E}(T) = T + C + \lambda^f T \left(\frac{T}{2} + R\right) + o(\lambda^f T)$$

Finally, we get the overhead of the pattern:

$$\mathbb{H}(T) = \frac{C}{T} + \lambda^f \frac{T}{2} + o(\lambda^f T)$$

2. Compute overhead $\mathbb{H}(T)$



We use Taylor series to approximate $e^{-\lambda^f T}$ up to first-order terms:

$$e^{-\lambda^f T} = 1 - \lambda^f T + o(\lambda^f T)$$

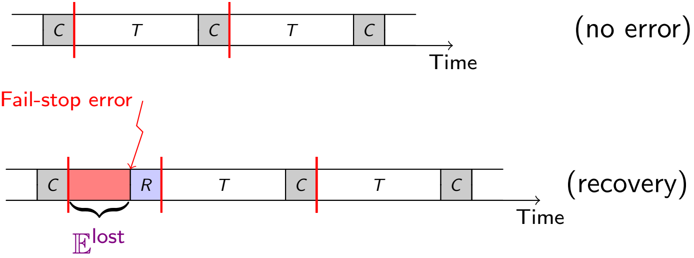
Works well provided that $\lambda^f << T, C, R$

$$\mathbb{E}(T) = T + C + \lambda^f T\left(\frac{T}{2} + R\right) + o(\lambda^f T)$$

Finally, we get the overhead of the pattern:

$$\mathbb{H}(T) = \frac{C}{T} + \lambda^f \frac{T}{2} + o(\lambda^f T)$$

3. Derive optimal T^*: Fail-stop errors



$$\mathbb{H}(T) = \frac{C}{T} + \lambda^f \frac{T}{2} + o(\lambda^f T)$$

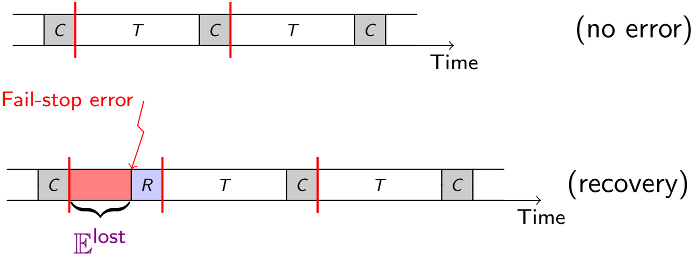
We solve:

$$\frac{\partial \mathbb{H}(T)}{\partial T} = -\frac{C}{T^2} + \frac{\lambda^f}{2} = 0$$

Finally, we retrieve:

$$T^* = \sqrt{\frac{2C}{\lambda^f}} = \sqrt{2\mu^f C}$$

3. Derive optimal T^*: Fail-stop errors



$$\mathbb{H}(T) = \frac{C}{T} + \lambda^f \frac{T}{2} + o(\lambda^f T)$$

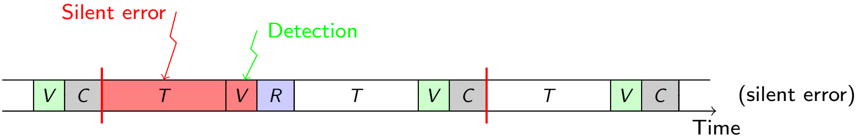
We solve:

$$\frac{\partial \mathbb{H}(T)}{\partial T} = -\frac{C}{T^2} + \frac{\lambda^f}{2} = 0$$

Finally, we retrieve:

$$T^* = \sqrt{\frac{2C}{\lambda^f}} = \sqrt{2\mu^f C}$$

4. Derive optimal T^*: Silent errors



Similar to fail-stop except:

- $\lambda^f \to \lambda^s$
- $\mathbb{E}^{\text{lost}} = T$
- V: verification time

Using the same approach:

$$\mathbb{H}(T) = \frac{C+V}{T} + \underbrace{\lambda^s T}_{silent} + o(\lambda^s T)$$

5. Derive optimal T^*: Both errors

$$\mathbb{H}(T) = \frac{C+V}{T} + \underbrace{\lambda^f \frac{T}{2}}_{fail-stop} + \underbrace{\lambda^s T}_{silent} + o(\lambda T)$$

First-order approximations [Young 1974, Daly 2006, AB et al. 2016]

	Fail-stop errors	Silent errors	Both errors
Pattern	$T+C$	$T+V+C$	$T+V+C$
Optimal T^*	$\sqrt{\frac{C}{\frac{\lambda^f}{2}}}$	$\sqrt{\frac{V+C}{\lambda^s}}$	$\sqrt{\frac{V+C}{\lambda^s+\frac{\lambda^f}{2}}}$
Overhead $\mathbb{H}^*$	$2\sqrt{\frac{\lambda^f}{2}C}$	$2\sqrt{\lambda^s(V+C)}$	$2\sqrt{\left(\lambda^s+\frac{\lambda^f}{2}\right)(V+C)}$

Is this optimal for energy consumption?

5. Derive optimal T^*: Both errors

$$\mathbb{H}(T) = \frac{C+V}{T} + \underbrace{\lambda^f \frac{T}{2}}_{fail-stop} + \underbrace{\lambda^s T}_{silent} + o(\lambda T)$$

First-order approximations [Young 1974, Daly 2006, AB et al. 2016]

	Fail-stop errors	Silent errors	Both errors
Pattern	$T + C$	$T + V + C$	$T + V + C$
Optimal T^*	$\sqrt{\frac{C}{\frac{\lambda^f}{2}}}$	$\sqrt{\frac{V+C}{\lambda^s}}$	$\sqrt{\frac{V+C}{\lambda^s+\frac{\lambda^f}{2}}}$
Overhead $\mathbb{H}^*$	$2\sqrt{\frac{\lambda^f}{2}C}$	$2\sqrt{\lambda^s(V+C)}$	$2\sqrt{\left(\lambda^s+\frac{\lambda^f}{2}\right)(V+C)}$

Is this optimal for energy consumption?

Outline

Energy model (1/2)

- Modern processors equipped with dynamic voltage and frequency scaling (DVFS) capability
- Power consumption of processing unit is $P_{idle} + \kappa\sigma^3$, where $\kappa > 0$ and σ is the processing speed
- Error rate: May also depend on processing speed
  - $\lambda(\sigma)$ follows a U-shaped curve
  - increases exponentially with decreased processing speed σ
  - increases also with increased speed because of high temperature

Energy model (2/2)

- Total power consumption depends on:
  - P_{idle}: static power dissipated when platform is on (even idle)
  - $P_{cpu}(\sigma)$: dynamic power spent by operating CPU at speed σ
  - P_{io}: dynamic power spent by I/O transfers (checkpoints and recoveries)
- Computation and verification: power depends upon σ (total time $T_{cpu}(\sigma)$)
- Checkpointing and recovering: I/O transfers (total time T_{io})
- Total energy consumption:

$$Energy(\sigma) = T_{cpu}(\sigma)(P_{idle} + P_{cpu}(\sigma)) + T_{io}(P_{idle} + P_{io})$$

  - Checkpoint: $E^C = C(P_{idle} + P_{io})$
  - Recover: $E^R = R(P_{idle} + P_{io})$
  - Verify at speed σ: $E^V(\sigma) = V(\sigma)(P_{idle} + P_{cpu}(\sigma))$

Bi-criteria problem

Linear combination of execution time and energy consumption:

$$a \cdot Time + b \cdot Energy$$

Theorem

Application subject to both fail-stop and silent errors

Minimize $a \cdot Time + b \cdot Energy$

The optimal checkpointing period is $T^*(\sigma) = \sqrt{\frac{2(V(\sigma)+C_e(\sigma))}{\lambda^f(\sigma)+2\lambda^s(\sigma)}}$,

where $C_e(\sigma) = \frac{a+b(P_{idle}+P_{io})}{a+b(P_{idle}+P_{cpu}(\sigma))} C$

Similar optimal period as without energy, but account for new parameters!

$$T^* = \sqrt{\frac{2(V+C)}{\lambda^f+2\lambda^s}}$$

Bi-criteria problem

Linear combination of execution time and energy consumption:

$$a \cdot Time + b \cdot Energy$$

Theorem

Application subject to both fail-stop and silent errors
Minimize $a \cdot Time + b \cdot Energy$
The optimal checkpointing period is $T^*(\sigma) = \sqrt{\frac{2(V(\sigma)+C_e(\sigma))}{\lambda^f(\sigma)+2\lambda^s(\sigma)}}$,
where $C_e(\sigma) = \frac{a+b(P_{idle}+P_{io})}{a+b(P_{idle}+P_{cpu}(\sigma))} C$

Similar optimal period as without energy, but account for new parameters!

$$T^* = \sqrt{\frac{2(V+C)}{\lambda^f + 2\lambda^s}}$$

Outline

When Amdahl meets Young/Daly

Error-free speedup with P processors and α sequential fraction:

Amdahl's Law: $S(P) = \frac{1}{\alpha + \frac{1-\alpha}{P}}$

- Bounded above by $1/\alpha$
- Strictly increasing function of P

Allocating more processors on an error-prone platform?

- Higher error-free speedup ☺
- More errors/faults ☹
  - More frequent checkpointing ☹
    - More resilience overhead ☹

We can compute optimal processor allocation
and checkpointing interval!

How is replication used?

On a Q-processor platform, application is replicated n times:

- **Duplication**: each replica has $P = Q/2$ processors
- **Triplication**: each replica has $P = Q/3$ processors
- **General case**: each replica has $P = Q/n$ processors

Having more replicas on an error-prone platform?

- Lower error-free speedup ☹
- More resilient ☺
  - Smaller checkpointing frequency ☺
    - Less resilience overhead ☺

Optimal replication level, processor allocation per replica, and checkpointing interval?

How is replication used?

On a Q-processor platform, application is replicated n times:

- **Duplication**: each replica has $P = Q/2$ processors
- **Triplication**: each replica has $P = Q/3$ processors
- **General case**: each replica has $P = Q/n$ processors

Having more replicas on an error-prone platform?

- Lower error-free speedup ☹
- More resilient ☺
  - Smaller checkpointing frequency ☺
    - Less resilience overhead ☺

Optimal replication level, processor allocation per replica, and checkpointing interval?

How is replication used?

On a Q-processor platform, application is replicated n times:

- **Duplication**: each replica has $P = Q/2$ processors
- **Triplication**: each replica has $P = Q/3$ processors
- **General case**: each replica has $P = Q/n$ processors

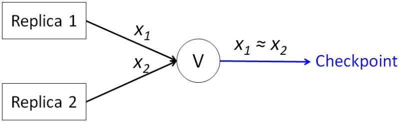
Having more replicas on an error-prone platform?

- Lower error-free speedup ☹
- More resilient ☺
  - Smaller checkpointing frequency ☺
    - Less resilience overhead ☺

Optimal replication level, processor allocation per replica, and checkpointing interval?

Why is replication useful?

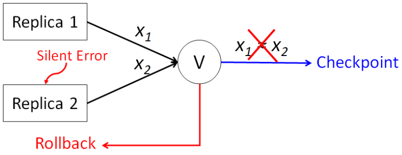
- **Error detection (duplication)**:



- **Error correction (triplication)**:

Why is replication useful?

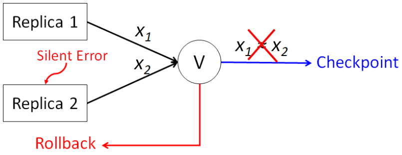
- **Error detection (duplication)**:



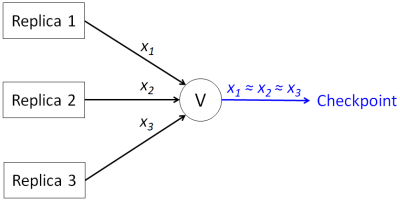
- Error correction (triplication):

Why is replication useful?

- **Error detection (duplication)**:

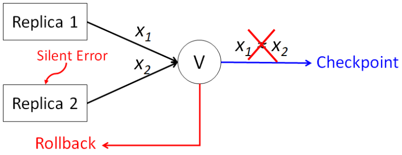


- **Error correction (triplication)**:

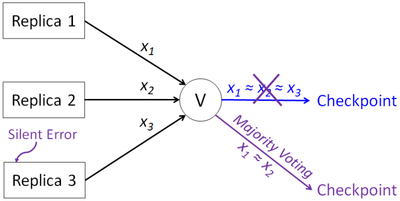


Why is replication useful?

- **Error detection (duplication)**:

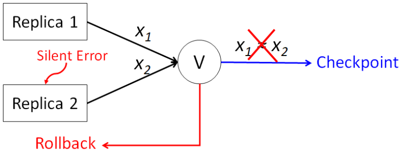


- **Error correction (triplication)**:

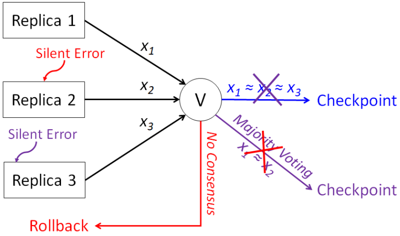


Why is replication useful?

- **Error detection (duplication)**:



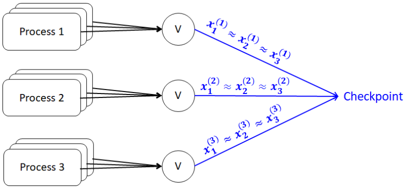
- **Error correction (triplication)**:



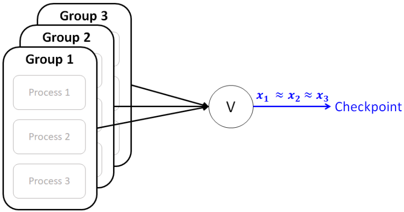
Outline

Two replication modes

- **Process replication**:

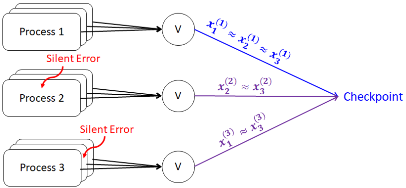


- **Group replication**:

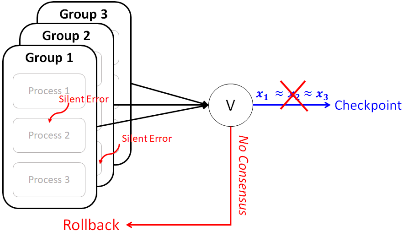


Two replication modes

- **Process replication**:



- **Group replication**:



Probability of failure

Independent process error distribution:

- Exponential $Exp(\lambda)$, $\lambda = 1/\mu$ (Memoryless)
- *Error probability of one process during T time of computation:*

$$\mathbb{P}(T) = 1 - e^{-\lambda T}$$

Process triplication:

- *Failure probability of any triplicated process:*

$$\mathbb{P}_3^{\text{prc}}(T,1) = \binom{3}{2}\Big(1 - \mathbb{P}(T)\Big)\mathbb{P}(T)^2 + \mathbb{P}(T)^3$$
$$= 3e^{-\lambda T}\left(1 - e^{-\lambda T}\right)^2 + \left(1 - e^{-\lambda T}\right)^3 = 1 - 3e^{-2\lambda T} + 2e^{-3\lambda T}$$

- *Failure probability of P-process application:*

$$\mathbb{P}_3^{\text{prc}}(T,P) = 1 - \mathbb{P}(\text{"No process fails"})$$
$$= 1 - \left(1 - \mathbb{P}_3^{\text{prc}}(T,1)\right)^P = 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P$$

Probability of failure

Independent process error distribution:

- Exponential $Exp(\lambda)$, $\lambda = 1/\mu$ (Memoryless)
- *Error probability of one process during T time of computation:*

$$\mathbb{P}(T) = 1 - e^{-\lambda T}$$

Process triplication:

- *Failure probability of any triplicated process:*

$$\mathbb{P}_3^{\text{prc}}(T,1) = \binom{3}{2}\Big(1 - \mathbb{P}(T)\Big)\mathbb{P}(T)^2 + \mathbb{P}(T)^3$$
$$= 3e^{-\lambda T}\left(1 - e^{-\lambda T}\right)^2 + \left(1 - e^{-\lambda T}\right)^3 = 1 - 3e^{-2\lambda T} + 2e^{-3\lambda T}$$

- *Failure probability of P-process application:*

$$\mathbb{P}_3^{\text{prc}}(T,P) = 1 - \mathbb{P}(\text{"No process fails"})$$
$$= 1 - \left(1 - \mathbb{P}_3^{\text{prc}}(T,1)\right)^P = 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P$$

Probability of failure

Independent process error distribution:

- Exponential $Exp(\lambda)$, $\lambda = 1/\mu$ (Memoryless)
- *Error probability of one process during T time of computation:*

$$\mathbb{P}(T) = 1 - e^{-\lambda T}$$

Process triplication:

- *Failure probability of any triplicated process:*

$$\mathbb{P}_3^{\text{prc}}(T,1) = \binom{3}{2}\Big(1 - \mathbb{P}(T)\Big)\mathbb{P}(T)^2 + \mathbb{P}(T)^3$$
$$= 3e^{-\lambda T}\left(1 - e^{-\lambda T}\right)^2 + \left(1 - e^{-\lambda T}\right)^3 = 1 - 3e^{-2\lambda T} + 2e^{-3\lambda T}$$

- *Failure probability of P-process application:*

$$\mathbb{P}_3^{\text{prc}}(T,P) = 1 - \mathbb{P}(\text{"No process fails"})$$
$$= 1 - (1 - \mathbb{P}_3^{\text{prc}}(T,1))^P = 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P$$

Probability of failure

Group triplication:

- *Failure probability of any P-process group:*

$$\mathbb{P}_1^{\text{grp}}(T,P) = 1 - \mathbb{P}(\text{“No process in group fails”}) = 1 - \left(1 - \mathbb{P}(T)\right)^P = 1 - e^{-\lambda P T}$$

- *Failure probability of three-group application:*

$$\mathbb{P}_3^{\text{grp}}(T,P) = \binom{3}{2}\left(1 - \mathbb{P}_1^{\text{grp}}(T,1)\right)\mathbb{P}_1^{\text{grp}}(T,1)^2 + \mathbb{P}_1^{\text{grp}}(T,1)^3$$
$$= 3e^{-\lambda PT}\left(1 - e^{-\lambda PT}\right)^2 + \left(1 - e^{-\lambda PT}\right)^3$$
$$= 1 - 3e^{-2\lambda PT} + 2e^{-3\lambda PT}$$
$$> 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P = \mathbb{P}_3^{\text{prc}}(T,P)$$

What about duplication? (any error kills both cases)

$$\mathbb{P}_2^{\text{prc}}(T,P) = \mathbb{P}_2^{\text{grp}}(T,P) = 1 - e^{-2\lambda PT}$$

Probability of failure

Group triplication:

- *Failure probability of any P-process group:*

$$\mathbb{P}_1^{\text{grp}}(T,P) = 1 - \mathbb{P}(\text{“No process in group fails”}) = 1 - \left(1-\mathbb{P}(T)\right)^P = 1 - e^{-\lambda P T}$$

- *Failure probability of three-group application:*

$$\mathbb{P}_3^{\text{grp}}(T,P) = \binom{3}{2}\left(1-\mathbb{P}_1^{\text{grp}}(T,1)\right)\mathbb{P}_1^{\text{grp}}(T,1)^2 + \mathbb{P}_1^{\text{grp}}(T,1)^3$$
$$= 3e^{-\lambda PT}\left(1-e^{-\lambda PT}\right)^2 + \left(1-e^{-\lambda PT}\right)^3$$
$$= 1 - 3e^{-2\lambda PT} + 2e^{-3\lambda PT}$$
$$> 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P = \mathbb{P}_3^{\text{prc}}(T,P)$$

What about duplication? (any error kills both cases)

$$\mathbb{P}_2^{\text{prc}}(T,P) = \mathbb{P}_2^{\text{grp}}(T,P) = 1 - e^{-2\lambda PT}$$

Probability of failure

Group triplication:

- *Failure probability of any P-process group:*

$$\mathbb{P}_1^{\text{grp}}(T,P) = 1 - \mathbb{P}(\text{“No process in group fails”}) = 1 - \left(1-\mathbb{P}(T)\right)^P = 1 - e^{-\lambda PT}$$

- *Failure probability of three-group application:*

$$\mathbb{P}_3^{\text{grp}}(T,P) = \binom{3}{2}\left(1-\mathbb{P}_1^{\text{grp}}(T,1)\right)\mathbb{P}_1^{\text{grp}}(T,1)^2 + \mathbb{P}_1^{\text{grp}}(T,1)^3$$

$$= 3e^{-\lambda PT}\left(1-e^{-\lambda PT}\right)^2 + \left(1-e^{-\lambda PT}\right)^3$$

$$= 1 - 3e^{-2\lambda PT} + 2e^{-3\lambda PT}$$

$$> 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P = \mathbb{P}_3^{\text{prc}}(T,P)$$

What about duplication? (any error kills both cases)

$$\mathbb{P}_2^{\text{prc}}(T,P) = \mathbb{P}_2^{\text{grp}}(T,P) = 1 - e^{-2\lambda PT}$$

Probability of failure

Group triplication:

- *Failure probability of any P-process group:*

$$\mathbb{P}_1^{\text{grp}}(T,P) = 1 - \mathbb{P}(\text{“No process in group fails”}) = 1 - \left(1 - \mathbb{P}(T)\right)^P = 1 - e^{-\lambda PT}$$

- *Failure probability of three-group application:*

$$\begin{aligned}\mathbb{P}_3^{\text{grp}}(T,P) &= \binom{3}{2}\left(1 - \mathbb{P}_1^{\text{grp}}(T,1)\right)\mathbb{P}_1^{\text{grp}}(T,1)^2 + \mathbb{P}_1^{\text{grp}}(T,1)^3 \\ &= 3e^{-\lambda PT}\left(1 - e^{-\lambda PT}\right)^2 + \left(1 - e^{-\lambda PT}\right)^3 \\ &= 1 - 3e^{-2\lambda PT} + 2e^{-3\lambda PT} \\ &> 1 - \left(3e^{-2\lambda T} - 2e^{-3\lambda T}\right)^P = \mathbb{P}_3^{\text{prc}}(T,P)\end{aligned}$$

What about duplication? (any error kills both cases)

$$\mathbb{P}_2^{\text{prc}}(T,P) = \mathbb{P}_2^{\text{grp}}(T,P) = 1 - e^{-2\lambda PT}$$

Two observations

Observation 1 (Implementation)

- **Process replication** is more resilient than group replication (assuming same overhead)
- **Group replication** is easier to implement by treating an application as a blackbox

Observation 2 (Analysis)

Following two scenarios are equivalent w.r.t. failure probability:

- **Group replication** with n replicas, where each replica has P processes and each process has error rate λ
- **Process replication** with one process, which has error rate λP and which is replicated n times

Benefit of analysis: $\text{Group}(n, P, \lambda) \rightarrow \text{Process}(n, 1, \lambda P)$

Two observations

Observation 1 (Implementation)

- **Process replication** is more resilient than group replication (assuming same overhead)
- **Group replication** is easier to implement by treating an application as a blackbox

Observation 2 (Analysis)

Following two scenarios are equivalent w.r.t. failure probability:

- **Group replication** with n replicas, where each replica has P processes and each process has error rate λ
- **Process replication** with one process, which has error rate λP and which is replicated n times

Benefit of analysis: Group$(n, P, \lambda) \rightarrow$ Process$(n, 1, \lambda P)$

Analysis steps

Maximize error-aware speedup

$$\mathbb{S}_n(T,P) = \frac{S(P)}{\mathbb{E}_n(T,P)/T}$$

1. Derive failure probability $\mathbb{P}_n^{\text{prc}}(T,P)$ or $\mathbb{P}_n^{\text{grp}}(T,P)$ — exact
2. Compute expected execution time $\mathbb{E}_n(T,P)$ — exact
3. Compute first-order approx. of error-aware speedup $\mathbb{S}_n(T,P)$
4. Derive optimal T_{opt}, P_{opt} and get $\mathbb{S}_n(T_{\text{opt}},P_{\text{opt}})$
5. Choose right replication level n

Analytical results

Duplication:
On a platform with Q processors and checkpointing cost C, the optimal resilience parameters for *process/group duplication* are:

$$P_{\text{opt}} = \min\left\{\frac{Q}{2}, \left(\frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{C\lambda}\right)^{\frac{1}{3}}\right\}$$

$$T_{\text{opt}} = \left(\frac{C}{2\lambda P_{\text{opt}}}\right)^{\frac{1}{2}}$$

$$\mathbb{S}_{\text{opt}} = \frac{S(P_{\text{opt}})}{1 + 2\left(2\lambda C P_{\text{opt}}\right)^{\frac{1}{2}}}$$

Triplication & (n, k)-replication (k-out-of-n replica consensus):
similar results but different for process and group, less practical for $n > 3$

- For $\alpha > 0$, not necessarily use up all available Q processors
- Checkpointing interval T_{opt} nicely extends Young/Daly's result
- Error-aware speedup $\mathbb{S}_{\text{opt}}$ minimally affected for small λ

Analytical results

Duplication:
On a platform with Q processors and checkpointing cost C, the optimal resilience parameters for *process/group duplication* are:

$$P_{\text{opt}} = \min\left\{\frac{Q}{2}, \left(\frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{C\lambda}\right)^{\frac{1}{3}}\right\}$$

$$T_{\text{opt}} = \left(\frac{C}{2\lambda P_{\text{opt}}}\right)^{\frac{1}{2}}$$

$$\mathbb{S}_{\text{opt}} = \frac{S(P_{\text{opt}})}{1 + 2\left(2\lambda C P_{\text{opt}}\right)^{\frac{1}{2}}}$$

Triplication & (n, k)-replication (k-out-of-n replica consensus):
similar results but different for process and group, less practical for $n > 3$

- For $\alpha > 0$, not necessarily use up all available Q processors
- Checkpointing interval T_{opt} nicely extends Young/Daly's result
- Error-aware speedup $\mathbb{S}_{\text{opt}}$ minimally affected for small λ

Analytical results

Duplication:
On a platform with Q processors and checkpointing cost C, the optimal resilience parameters for *process/group duplication* are:

$$P_{\text{opt}} = \min \left\{ \frac{Q}{2}, \left(\frac{1}{2} \left(\frac{1-\alpha}{\alpha} \right)^2 \frac{1}{C\lambda} \right)^{\frac{1}{3}} \right\}$$

$$T_{\text{opt}} = \left(\frac{C}{2\lambda P_{\text{opt}}} \right)^{\frac{1}{2}}$$

$$\mathbb{S}_{\text{opt}} = \frac{S(P_{\text{opt}})}{1 + 2\left(2\lambda C P_{\text{opt}}\right)^{\frac{1}{2}}}$$

Triplication & (n, k)-replication (k-out-of-n replica consensus):
similar results but different for process and group, less practical for $n > 3$

- For $\alpha > 0$, not necessarily use up all available Q processors
- Checkpointing interval T_{opt} nicely extends Young/Daly's result
- Error-aware speedup $\mathbb{S}_{\text{opt}}$ minimally affected for small λ

Results comparison

For fully parallel jobs, i.e., $\alpha = 0$ (similar for $\alpha > 0$)

- Duplication v.s. Process triplication

Duplication	Process triplication	
$P_{opt} = \frac{Q}{2}$	$P_{opt} = \frac{Q}{3}$	(Processors ↓)
$T_{opt} = \sqrt{\frac{C}{\lambda Q}}$	$T_{opt} = \sqrt[3]{\frac{C}{2\lambda^2 Q}}$	(Chkpt interval ↑)
$\mathbb{S}_{opt} = \frac{Q/2}{1 + 2\sqrt{\lambda C Q}}$	$\mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\left(\frac{\lambda C}{2}\right)^2 Q}}$	(Exp. speedup??)

- Process triplication v.s. Group triplication

Process triplication	Group triplication	
$P_{opt} = \frac{Q}{3}$	$P_{opt} = \frac{Q}{3}$	(Processors =)
$T_{opt} = \sqrt[3]{\frac{C}{2\lambda^2 Q}}$	$T_{opt} = \sqrt[3]{\frac{3C}{2(\lambda Q)^2}}$	(Chkpt interval ↓)
$\mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\left(\frac{\lambda C}{2}\right)^2 Q}}$	$\mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\frac{1}{3}\left(\frac{\lambda C Q}{2}\right)^2}}$	(Exp. speedup ↓)

Results comparison

For fully parallel jobs, i.e., $\alpha = 0$ (similar for $\alpha > 0$)

- Duplication v.s. Process triplication

$$P_{opt} = \frac{Q}{2} \qquad P_{opt} = \frac{Q}{3} \qquad \text{(Processors} \downarrow\text{)}$$

$$T_{opt} = \sqrt{\frac{C}{\lambda Q}} \qquad T_{opt} = \sqrt[3]{\frac{C}{2\lambda^2 Q}} \qquad \text{(Chkpt interval} \uparrow\text{)}$$

$$\mathbb{S}_{opt} = \frac{Q/2}{1 + 2\sqrt{\lambda C Q}} \qquad \mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\left(\frac{\lambda C}{2}\right)^2 Q}} \qquad \text{(Exp. speedup??)}$$

- Process triplication v.s. Group triplication

$$P_{opt} = \frac{Q}{3} \qquad P_{opt} = \frac{Q}{3} \qquad \text{(Processors} =\text{)}$$

$$T_{opt} = \sqrt[3]{\frac{C}{2\lambda^2 Q}} \qquad T_{opt} = \sqrt[3]{\frac{3C}{2(\lambda Q)^2}} \qquad \text{(Chkpt interval} \downarrow\text{)}$$

$$\mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\left(\frac{\lambda C}{2}\right)^2 Q}} \qquad \mathbb{S}_{opt} = \frac{Q/3}{1 + 3\sqrt[3]{\frac{1}{3}\left(\frac{\lambda C Q}{2}\right)^2}} \qquad \text{(Exp. speedup} \downarrow\text{)}$$

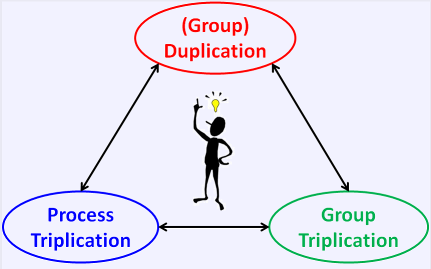
Results comparison

For fully parallel jobs, i.e., $\alpha = 0$ (similar for $\alpha > 0$)

- Duplication v.s. Process triplication

Choosing right mode & level of replication

Based on analytical results, app. output structure and system/language support



Outline

Simulations

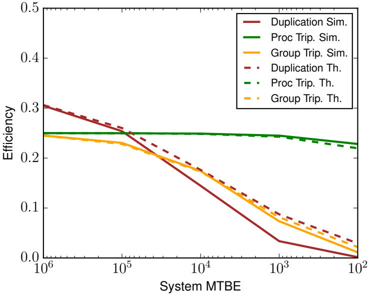
Consider a platform with $Q = 10^6$, and study

$$\textit{Efficiency} = \frac{\mathbb{S}_{\text{opt}}}{Q}$$

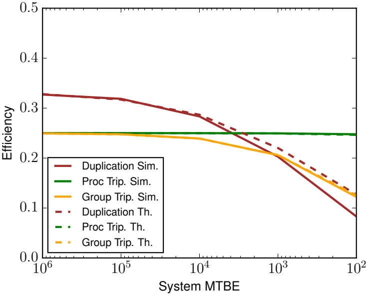
- Impact of MTBE and checkpointing cost C
- Impact of sequential fraction α
- Impact of number of processes P

Impact of MTBE and checkpointing cost

$\alpha = 10^{-6}$



(a) $C = 1800s$

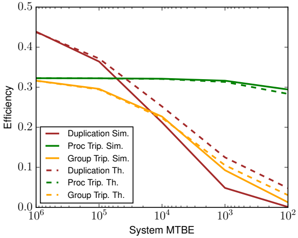


(b) $C = 60s$

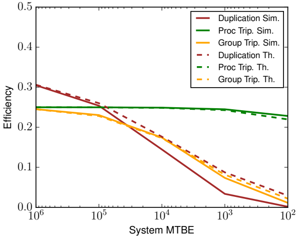
- First-order accurate except for duplication (where P is larger) and with small MTBE
- Duplication can be sufficient for large MTBE, especially for small checkpointing cost

Impact of sequential fraction

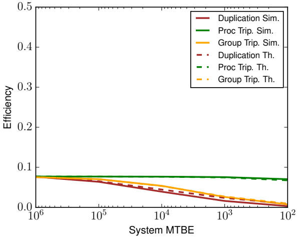
$C = 1800s$



(c) $\alpha = 10^{-7}$



(d) $\alpha = 10^{-6}$

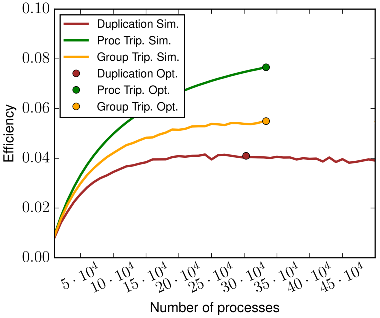


(e) $\alpha = 10^{-5}$

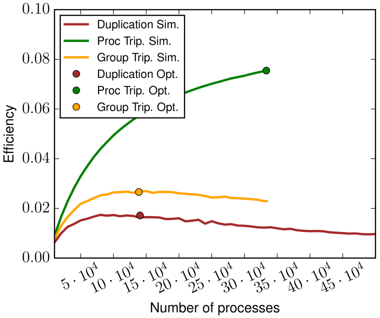
- Increased α reduces efficiency
- Increased α increases minimum MTBE for which duplication is sufficient

Impact of number of processes

$$\alpha = 10^{-5}, C = 1800s$$



(f) $MTBE = 10^4$



(g) $MTBE = 10^3$

- Efficiency/speedup not strictly increasing with P
- First-order P_{opt} close to actual optimum

What to remember

- "Replication + checkpointing" as a general-purpose fault-tolerance protocol for detecting/correcting silent errors in HPC
- Process replication is more resilient than group replication, but group replication is easier to implement
- Analytical solution for $P_{\text{opt}}, T_{\text{opt}}, \textit{and } \mathbb{S}_{\text{opt}}$ and for choosing right replication mode and level

Outline

Chains of tasks

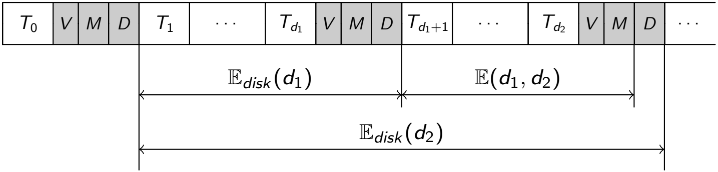
- High-performance computing (HPC) application: chain of tasks $T_1 \rightarrow T_2 \rightarrow \cdots \rightarrow T_n$
- Parallel tasks executed on the whole platform
- For instance: tightly-coupled computational kernels, image processing applications, ...
- Goal: efficient execution, i.e., minimize total execution time
- Checkpoints can only be done after a task has completed

Chains of tasks

- High-performance computing (HPC) application: chain of tasks $T_1 \rightarrow T_2 \rightarrow \cdots \rightarrow T_n$
- Parallel tasks executed on the whole platform
- For instance: tightly-coupled computational kernels, image processing applications, ...
- Goal: efficient execution, i.e., minimize total execution time
- Checkpoints can only be done after a task has completed

Dynamic programming algorithm without replication

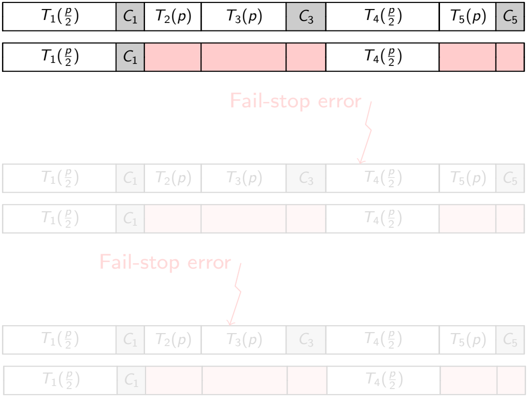
Possibility to add verification, memory checkpoint and disk checkpoint at the end of a task



$$\mathbb{E}_{disk}(d_2) = \min_{0 \le d_1 < d_2} \{\mathbb{E}_{disk}(d_1) + \mathbb{E}(d_1, d_2) + C_D\}$$

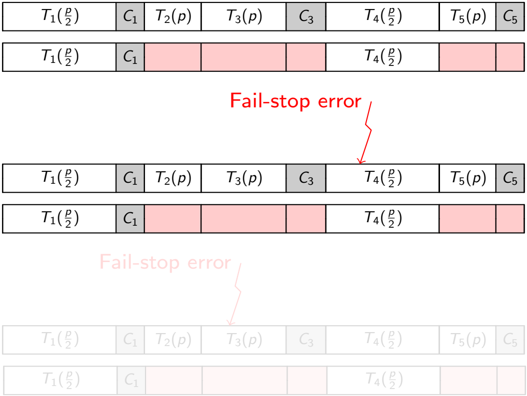
- Initialization: $\mathbb{E}_{disk}(0) = 0$
- Objective: Compute $\mathbb{E}_{disk}(n)$
- Compute $\mathbb{E}_{disk}(0), \mathbb{E}_{disk}(1), \mathbb{E}_{disk}(2), \ldots, \mathbb{E}_{disk}(n)$ in that order
- Complexity: $O(n^2)$

Coping with fail-stop errors with replication



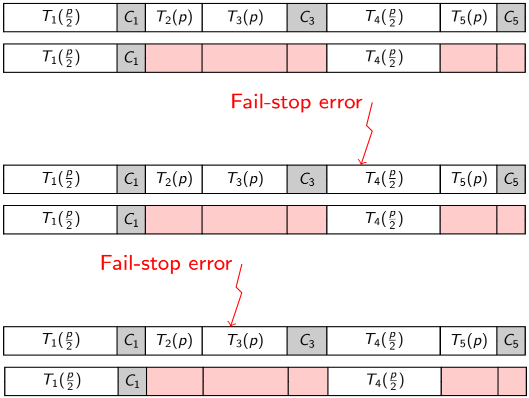
- The whole platform is used at all time, some tasks are replicated
- If failure hits a replicated task, no need to rollback
- Otherwise, rollback to last checkpoint and re-execute

Coping with fail-stop errors with replication



- The whole platform is used at all time, some tasks are replicated
- If failure hits a replicated task, no need to rollback
- Otherwise, rollback to last checkpoint and re-execute

Coping with fail-stop errors with replication



- The whole platform is used at all time, some tasks are replicated
- If failure hits a replicated task, no need to rollback
- Otherwise, rollback to last checkpoint and re-execute

Dynamic programming algorithm with replication

- Recursively computes expectation of optimal time required to execute tasks T_1 to T_i and then checkpoint T_i
- Distinguish whether T_i is replicated or not

- $T_{opt}^{rep}(i)$: knowing that T_i is replicated
- $T_{opt}^{norep}(i)$: knowing that T_i is not replicated

- Solution: $\min\left\{T_{opt}^{rep}(n) + C_n^{rep}, T_{opt}^{norep}(n) + C_n^{norep}\right\}$

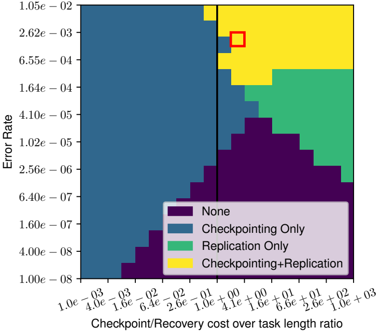
Computing $T_{opt}^{rep}(j)$: j is replicated

$$T_{opt}^{rep}(j) = \min_{1 \le i < j} \begin{cases} T_{opt}^{rep}(i) + C_i^{rep} + T_{NC}^{rep,rep}(i+1,j), \\ T_{opt}^{rep}(i) + C_i^{rep} + T_{NC}^{norep,rep}(i+1,j), \\ T_{opt}^{norep}(i) + C_i^{norep} + T_{NC}^{rep,rep}(i+1,j), \\ T_{opt}^{norep}(i) + C_i^{norep} + T_{NC}^{norep,rep}(i+1,j), \\ R_1^{rep} + T_{NC}^{rep,rep}(1,j), \\ R_1^{norep} + T_{NC}^{norep,rep}(1,j) \end{cases}$$

- T_i: last checkpointed task before T_j
- T_i can be replicated or not, T_{i+1} can be replicated or not
- $T_{NC}^{A,B}$: no intermediate checkpoint, first/last task replicated or not, previous task checkpointed: complicated formula but done in constant time
- Similar equation for $T_{opt}^{norep}(j)$
- Overall complexity: $O(n^2)$

Comparison to checkpoint only

- With identical tasks
- Reports occ. of checkpoints and replicas in optimal solution
- Checkpointing cost $\leq$ task length $\Rightarrow$ no replication



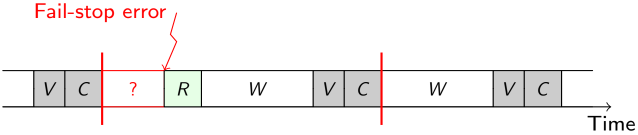
Summary

- Goal: Minimize execution time of linear workflows
- Decide which task to checkpoint and/or replicate
- Sophisticated dynamic programming algorithms: optimal solutions
- Even when accounting for energy: decide at which speed to execute each task
- Even with k different levels of checkpoints and partial verifications: algorithm in $O(n^{k+5})$
- Simulations: With replication, gain over checkpoint-only approach is quite significant, when checkpoint is costly and error rate is high

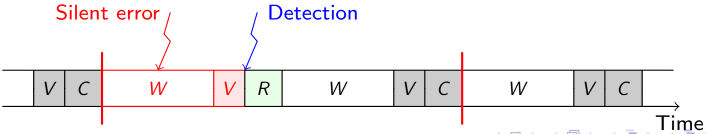
Outline

Silent vs fail-stop errors

- C: time to checkpoint; V: time to verify; R: time to recover; λ: error rate (platform MTBF $\mu = 1/\lambda$)
- Optimal checkpointing period W for fail-stop errors (Young/Daly): $W = \sqrt{2C\mu}$ $\quad (V = 0)$



- Silent errors: $W = \sqrt{(V+C)\mu}$
  ($C \to V + C$; missing factor 2)



Back to energy consumption

- Need to reduce energy consumption of future platforms
- Popular technique: dynamic voltage and frequency scaling (DVFS)
- Lower speed $\rightarrow$ energy savings: when computing at speed σ, power proportional to σ^3 and execution time proportional to $1/\sigma$ $\rightarrow$ (dynamic) energy proportional to σ^2
- Also account for static energy: trade-offs to be found
- Realistic approach: minimize energy consumption while guaranteeing a performance bound
- $\Rightarrow$ At which speed should we execute the workload?

Back to energy consumption

- Need to reduce energy consumption of future platforms
- Popular technique: dynamic voltage and frequency scaling (DVFS)
- Lower speed $\rightarrow$ energy savings: when computing at speed σ, power proportional to σ^3 and execution time proportional to $1/\sigma \rightarrow$ (dynamic) energy proportional to σ^2
- Also account for static energy: trade-offs to be found
- Realistic approach: minimize energy consumption while guaranteeing a performance bound
- $\Rightarrow$ At which speed should we execute the workload?

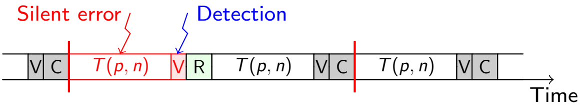
Outline

Framework

- Divisible-load applications
- Subject to silent data corruption
- Checkpoint/restart strategy: periodic patterns that repeat over time
- Verified checkpoints
- Is it better to use two different speeds rather than only one? What are the optimal checkpointing period and optimal execution speeds?

Model

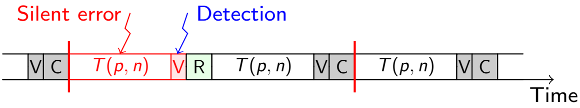
- Set of speeds $S = \{s_1, \ldots, s_K\}$: $\sigma_1 \in S$ speed for first execution, $\sigma_2 \in S$ speed for re-executions
- Silent errors: exponential distribution of rate λ
- Verification: V units of work; Checkpointing: time C; Recovery: time R
- P_{idle} and P_{io} constant; and $P_{\text{cpu}}(\sigma) = \kappa\sigma^3$
- Energy for W units of work at speed σ: $\frac{W}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a verification at speed σ: $\frac{V}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a checkpoint: $C(P_{\text{idle}} + P_{\text{io}})$
  Energy of a recovery: $R(P_{\text{idle}} + P_{\text{io}})$



With a silent error

Model

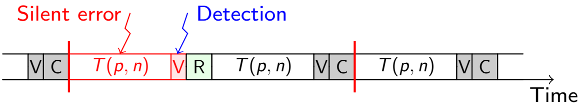
- Set of speeds $S = \{s_1, \ldots, s_K\}$: $\sigma_1 \in S$ speed for first execution, $\sigma_2 \in S$ speed for re-executions
- Silent errors: exponential distribution of rate λ
- Verification: V units of work; Checkpointing: time C; Recovery: time R
- P_{idle} and P_{io} constant; and $P_{\text{cpu}}(\sigma) = \kappa\sigma^3$
- Energy for W units of work at speed σ: $\frac{W}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a verification at speed σ: $\frac{V}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a checkpoint: $C(P_{\text{idle}} + P_{\text{io}})$
  Energy of a recovery: $R(P_{\text{idle}} + P_{\text{io}})$



With a silent error

Model

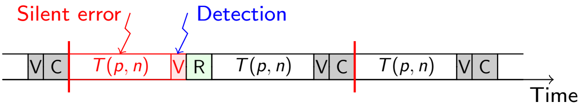
- Set of speeds $S = \{s_1, \ldots, s_K\}$: $\sigma_1 \in S$ speed for first execution, $\sigma_2 \in S$ speed for re-executions
- Silent errors: exponential distribution of rate λ
- Verification: V units of work; Checkpointing: time C; Recovery: time R
- P_{idle} and P_{io} constant; and $P_{\text{cpu}}(\sigma) = \kappa\sigma^3$
- Energy for W units of work at speed σ: $\frac{W}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a verification at speed σ: $\frac{V}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a checkpoint: $C(P_{\text{idle}} + P_{\text{io}})$
  Energy of a recovery: $R(P_{\text{idle}} + P_{\text{io}})$



With a silent error

Model

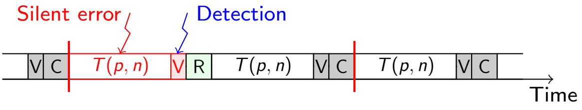
- Set of speeds $S = \{s_1, \dots, s_K\}$: $\sigma_1 \in S$ speed for first execution, $\sigma_2 \in S$ speed for re-executions
- Silent errors: exponential distribution of rate λ
- Verification: V units of work; Checkpointing: time C; Recovery: time R
- P_{idle} and P_{io} constant; and $P_{cpu}(\sigma) = \kappa\sigma^3$
- Energy for W units of work at speed σ: $\frac{W}{\sigma}(P_{idle} + \kappa\sigma^3)$
  Energy of a verification at speed σ: $\frac{V}{\sigma}(P_{idle} + \kappa\sigma^3)$
  Energy of a checkpoint: $C(P_{idle} + P_{io})$
  Energy of a recovery: $R(P_{idle} + P_{io})$



With a silent error

Model

- Set of speeds $S = \{s_1, \ldots, s_K\}$: $\sigma_1 \in S$ speed for first execution, $\sigma_2 \in S$ speed for re-executions
- Silent errors: exponential distribution of rate λ
- Verification: V units of work; Checkpointing: time C; Recovery: time R
- P_{idle} and P_{io} constant; and $P_{\text{cpu}}(\sigma) = \kappa\sigma^3$
- Energy for W units of work at speed σ: $\frac{W}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a verification at speed σ: $\frac{V}{\sigma}(P_{\text{idle}} + \kappa\sigma^3)$
  Energy of a checkpoint: $C(P_{\text{idle}} + P_{\text{io}})$
  Energy of a recovery: $R(P_{\text{idle}} + P_{\text{io}})$



With a silent error

Problem

Optimization problem BiCrit:

$$\text{Minimize } \frac{\mathcal{E}(W,\sigma_1,\sigma_2)}{W} \text{ s.t. } \frac{\mathcal{T}(W,\sigma_1,\sigma_2)}{W} \leq \rho,$$

- $\mathcal{E}(W,\sigma_1,\sigma_2)$ is the expected energy consumed to execute W units of work at speed σ_1, with eventual re-executions at speed σ_2
- $\mathcal{T}(W,\sigma_1,\sigma_2)$ is the expected execution time to execute W units of work at speed σ_1, with eventual re-executions at speed σ_2
- ρ is a performance bound, or admissible degradation factor

Computing expected execution time

Proposition (1)

For the BiCrit *problem with a single speed,*

$$\mathcal{T}(W,\sigma,\sigma) = C + e^{\frac{\lambda W}{\sigma}}\left(\frac{W+V}{\sigma}\right) + \left(e^{\frac{\lambda W}{\sigma}} - 1\right)R$$

Proposition (2)

For the BiCrit *problem,*

$$\mathcal{T}(W,\sigma_1,\sigma_2) = C + \frac{W+V}{\sigma_1} + \left(1 - e^{-\frac{\lambda W}{\sigma_1}}\right)e^{\frac{\lambda W}{\sigma_2}}\left(R + \frac{W+V}{\sigma_2}\right)$$

Proof of Proposition 1

Proof.

The recursive equation to compute $\mathcal{T}(W,\sigma,\sigma)$ writes:

$$\mathcal{T}(W,\sigma,\sigma) = \frac{W+V}{\sigma} + p(W/\sigma)\left(R + \mathcal{T}(W,\sigma,\sigma)\right) + (1 - p(W/\sigma))C,$$

where $p(W/\sigma) = 1 - e^{-\frac{\lambda W}{\sigma}}$. The reasoning is as follows:

- We always execute W units of work followed by the verification, in time $\frac{W+V}{\sigma}$;
- With probability $p(W/\sigma)$, a silent error occurred and is detected, in which case we recover and start anew;
- Otherwise, with probability $1 - p(W/\sigma)$, we simply checkpoint after a successful execution.

Solving this equation leads to the expected execution time. □

Proof of Proposition 2

Proof.

The recursive equation to compute $\mathcal{T}(W, \sigma_1, \sigma_2)$ writes:

$$\mathcal{T}(W, \sigma_1, \sigma_2) = \frac{W + V}{\sigma_1} + p(W/\sigma_1)\left(R + \mathcal{T}(W, \sigma_2, \sigma_2)\right) + (1 - p(W/\sigma_1))C,$$

where $p(W/\sigma_1) = 1 - e^{-\frac{\lambda W}{\sigma_1}}$. The reasoning is as follows:

- We always execute W units of work followed by the verification, in time $\frac{W+V}{\sigma_1}$;
- With probability $p(W/\sigma_1)$, a silent error occurred and is detected, in which case we recover and start anew at speed σ_2;
- Otherwise, with probability $1 - p(W/\sigma_1)$, we simply checkpoint after a successful execution.

Solving this equation leads to the expected execution time. □

Computing expected energy consumption

Proposition

For the BiCrit *problem,*

$$\mathcal{E}(W,\sigma_1,\sigma_2) = \left(C + \left(1 - e^{-\frac{\lambda W}{\sigma_1}}\right) e^{\frac{\lambda W}{\sigma_2}} R\right)(P_{\text{io}} + P_{\text{idle}}) + \frac{W+V}{\sigma_1}(\kappa\sigma_1^3 + P_{\text{idle}}) + \frac{W+V}{\sigma_2}(1 - e^{-\frac{\lambda W}{\sigma_1}})e^{\frac{\lambda W}{\sigma_2}}(\kappa\sigma_2^3 + P_{\text{idle}})$$

Power spent during checkpoint or recovery: $P_{\text{io}} + P_{\text{idle}}$; power spent during computation and verification at speed σ: $P_{\text{cpu}}(\sigma) + P_{\text{idle}} = \kappa\sigma^3 + P_{\text{idle}}$. From Proposition 2, we get the expression of $\mathcal{E}(W,\sigma_1,\sigma_2)$.

Finding optimal pattern length (1)

To get closed-form expression for optimal value of W, use of first-order approximations, using Taylor expansion $e^{\lambda W} = 1 + \lambda W + O(\lambda^2 W^2)$:

$$\frac{\mathcal{T}(W,\sigma_1,\sigma_2)}{W} = \frac{1}{\sigma_1} + \frac{\lambda W}{\sigma_1 \sigma_2} + \frac{\lambda R}{\sigma_1} + \frac{\lambda V}{\sigma_1 \sigma_2} + \frac{C + V/\sigma_1}{W} + O(\lambda^2 W) \quad (1)$$

$$\begin{aligned}\frac{\mathcal{E}(W,\sigma_1,\sigma_2)}{W} &= \frac{\kappa\sigma_1^3 + P_{\text{idle}}}{\sigma_1} + \frac{\lambda W}{\sigma_1\sigma_2}(\kappa\sigma_2^3 + P_{\text{idle}}) \\ &+ \frac{\lambda R}{\sigma_1}(P_{\text{io}} + P_{\text{idle}}) + \frac{\lambda V}{\sigma_1\sigma_2}(\kappa\sigma_1^3 + P_{\text{idle}}) \\ &+ \frac{C(P_{\text{io}} + P_{\text{idle}}) + V(\kappa\sigma_1^3 + P_{\text{idle}})/\sigma_1}{W} + O(\lambda^2 W)\end{aligned} \quad (2)$$

Finding optimal pattern length (2)

Theorem

Given σ_1, σ_2 and ρ, consider the equation $aW^2 + bW + c = 0$, where $a = \frac{\lambda}{\sigma_1\sigma_2}$, $b = \frac{1}{\sigma_1} + \lambda\left(\frac{R}{\sigma_1} + \frac{V}{\sigma_1\sigma_2}\right) - \rho$ and $c = C + \frac{V}{\sigma_1}$.

- *If there is no positive solution to the equation, i.e., $b > -2\sqrt{ac}$, then* BiCrit *has no solution.*
- *Otherwise, let W_1 and W_2 be the two solutions of the equation with $W_1 \leq W_2$ (at least W_2 is positive and possibly $W_1 = W_2$). Then, the optimal pattern size is*

$$W_{\text{opt}} = \min(\max(W_1, W_e), W_2), \tag{3}$$

where $$W_e = \sqrt{\frac{C(P_{\text{io}} + P_{\text{idle}}) + \frac{V}{\sigma_1}(\kappa\sigma_1^3 + P_{\text{idle}})}{\frac{\lambda}{\sigma_1\sigma_2}(\kappa\sigma_2^3 + P_{\text{idle}})}}. \tag{4}$$

Finding optimal pattern length (3)

Proof.

Neglecting lower-order terms, Equation (2) is minimized when $W = W_e$ given by Equation (4).

Two cases:

- ρ is too small $\Rightarrow$ no solution
- $W_2 > 0$:
  - $W_e < W_1$
  - $W_1 \leq W_e \leq W_2$
  - $W_e > W_2$

Using that the energy overhead is a convex function, we get the result (W_{opt} is in the interval $[W_1, W_2]$) □

Finding optimal speed pair

- Speed pair (s_i, s_j), with $1 \leq i, j \leq K$: $\rho_{i,j}$ is the minimum performance bound for which the BiCrit problem with $\sigma_1 = s_i$ and $\sigma_2 = s_j$ admits a solution
- For each speed pair, compute W_1, W_2 the roots of $aW^2 + bW + c$; discard pairs with $\rho < \rho_{i,j}$
- For each remaining speed pair (σ_1, σ_2), compute W_{opt} and associated energy overhead
- Select speed pair (σ_1^*, σ_2^*) that minimizes energy overhead
- Time $O(K^2)$, where K is the number of available speeds, usually a small constant

Outline

Simulation setup

- Platform parameters, based on real platforms

Platform	λ	$C = R$	V
Hera	3.38e-6	300s	15.4
Atlas	7.78e-6	439s	9.1
Coastal	2.01e-6	1051s	4.5
Coastal SSD	2.01e-6	2500s	180.0

- Power parameters, determined by the processor used

Processor	Normalized speeds	$P(\sigma)$ (mW)
Intel Xscale	$0.15, 0.4, 0.6, 0.8, 1$	$1550\sigma^3 + 60$
Transmeta Crusoe	$0.45, 0.6, 0.8, 0.9, 1$	$5756\sigma^3 + 4.4$

- Default values: P_{io} equivalent to power used when running at lowest speed; $\rho = 3$

Simulation results, using Hera/XScale configuration

A different re-execution speed does help!

And all speed pairs can be optimal solutions (depending on ρ)!

σ_1	**Best** σ_2	W_{opt}	$\frac{\mathcal{E}(W_{opt},\sigma_1,\sigma_2)}{W_{opt}}$
0.15	0.4	1711	466
0.4	**0.4**	2764	416
0.6	0.4	3639	674
0.8	0.4	4627	1082
1	0.4	5742	1625

$\rho = 8$

σ_1	**Best** σ_2	W_{opt}	$\frac{\mathcal{E}(W_{opt},\sigma_1,\sigma_2)}{W_{opt}}$
0.15	-	-	-
0.4	**0.4**	2764	416
0.6	0.4	3639	674
0.8	0.4	4627	1082
1	0.4	5742	1625

$\rho = 3$

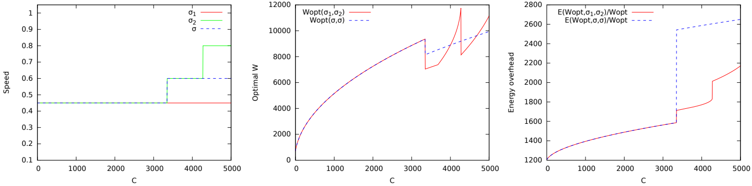
σ_1	**Best** σ_2	W_{opt}	$\frac{\mathcal{E}(W_{opt},\sigma_1,\sigma_2)}{W_{opt}}$
0.15	-	-	-
0.4	-	-	-
0.6	**0.8**	4251	690
0.8	0.4	4627	1082
1	0.4	5742	1625

$\rho = 1.775$

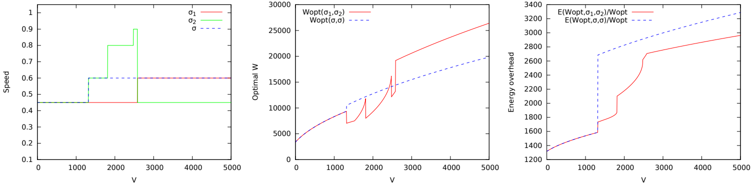
σ_1	**Best** σ_2	W_{opt}	$\frac{\mathcal{E}(W_{opt},\sigma_1,\sigma_2)}{W_{opt}}$
0.15	-	-	-
0.4	-	-	-
0.6	-	-	-
0.8	**0.4**	4627	1082
1	0.4	5742	1625

$\rho = 1.4$

Simulations - Impact of the parameters (1)



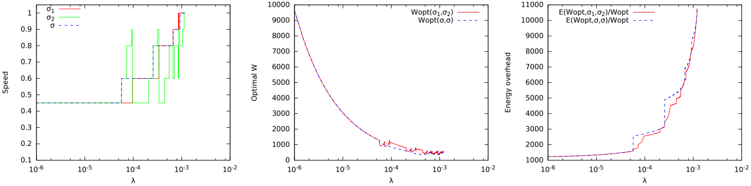
Opt. solution (speed pair, pattern size, and energy overhead) as a function of the checkpointing time c in Atlas/Crusoe configuration.



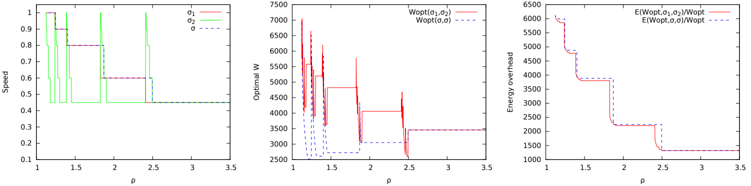
Opt. solution (speed pair, pattern size, and energy overhead) as a function of the verification time v in Atlas/Crusoe configuration.

Dotted line: one single speed; up to 35% improvement with two speeds

Simulations - Impact of the parameters (2)



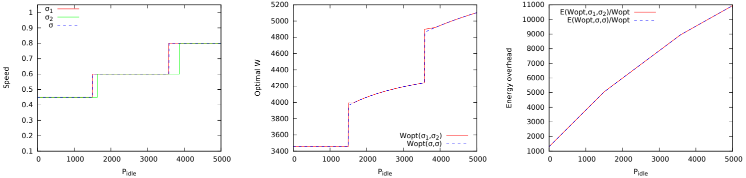
Opt. solution (speed pair, pattern size, and energy overhead) as a function of the error rate λ in Atlas/Crusoe configuration.



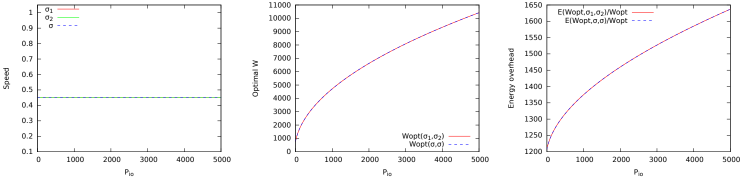
Opt. solution (speed pair, pattern size, and energy overhead) as a function of the performance bound ρ in Atlas/Crusoe configuration.

Two speeds: checkpoint less frequently and provide energy savings

Simulations - Impact of the parameters (3)



Optimal solution (speed pair, pattern size, and energy overhead) as a function of the idle power P_{idle} in Atlas/Crusoe configuration.



Optimal solution (speed pair, pattern size, and energy overhead) as a function of the I/O power P_{io} in Atlas/Crusoe configuration.

Increase of W and E with P_{idle} and P_{io}; P_{io} has no impact on speeds

Outline

Extensions: With fail-stop errors

- f: proportion of fail-stop errors
- s: proportion of silent errors

Proposition (3)

With fail-stop and silent errors,

$$\frac{\mathcal{T}(W,\sigma_1,\sigma_2)}{W} = \cdots + \left(\frac{(f+s)}{\sigma_1\sigma_2} - \frac{f}{2\sigma_1^2}\right)\lambda W + O(\lambda^2 W). \quad (5)$$

$$\frac{\mathcal{E}(W,\sigma_1,\sigma_2)}{W} = \cdots + \left(\frac{(f+s)(\kappa\sigma_2^3 + P_{\text{idle}})}{\sigma_1\sigma_2} - \frac{f(\kappa\sigma_1^3 + P_{\text{idle}})}{2\sigma_1^2}\right)\lambda W + O(\lambda^2 W) \quad (6)$$

Limit of the first-order approximation

For BiCrit, the first-order approximation leads to a solution iff

$$\left(2\left(1+\frac{s}{f}\right)\right)^{-1/2} < \frac{\sigma_2}{\sigma_1} < 2\left(1+\frac{s}{f}\right)$$

Use second-order approximation? Open problem in the general case!

Interesting case

Theorem

When considering only fail-stop errors with rate λ*, the optimal pattern size* W *to minimize the time overhead* $\frac{\mathcal{T}(W,\sigma,2\sigma)}{W}$ *is*

$$W_{\text{opt}} = \sqrt[3]{\frac{12C}{\lambda^2}}\sigma$$

- Young/Daly's formula: $W_{\text{opt}} = \sqrt{2C/\lambda}\sigma = O(\lambda^{-1/2})$
- Here: $W_{\text{opt}} = O(\lambda^{-2/3})$

Conclusion

- A different re-execution speed indeed helps saving energy while satisfying a performance constraint
- Silent errors: extension of Young/Daly formula → general closed-form solution to get optimal speed pair and optimal checkpointing period (first-order)
- Extensive simulations: up to 35% energy savings, any speed pair can be optimal

- BiCrit still open for general case with both silent and fail-stop errors
- Interesting case with fail-stop errors and double re-execution speed: $O(\lambda^{-2/3})$ vs $O(\lambda^{-1/2})$
- New methods needed to capture the general case

Outline

Summary and need for trade-offs

- Two major challenges for Exascale systems:
  - Resilience: need to handle failures
  - Energy: need to reduce energy consumption
- The main objective is often performance, such as execution time, but other criteria must be accounted for
- Many models for which we have the answer:
  - Optimal checkpointing period, with fail-stop / silent errors
  - Use of replication to detect and correct silent errors
  - When to checkpoint, replicate and verify for a chain of tasks?
  - Use a different re-execution speed after a failure
- Still a lot of challenges to address, and techniques to be developed for many kinds of high-performance applications, making trade-offs between performance, reliability, and energy consumption

Summary and need for trade-offs

- Two major challenges for Exascale systems:
  - Resilience: need to handle failures
  - Energy: need to reduce energy consumption
- The main objective is often performance, such as execution time, but other criteria must be accounted for
- Many models for which we have the answer:
  - Optimal checkpointing period, with fail-stop / silent errors
  - Use of replication to detect and correct silent errors
  - When to checkpoint, replicate and verify for a chain of tasks?
  - Use a different re-execution speed after a failure
- Still a lot of challenges to address, and techniques to be developed for many kinds of high-performance applications, making trade-offs between performance, reliability, and energy consumption

Summary and need for trade-offs

- Two major challenges for Exascale systems:
  - Resilience: need to handle failures
  - Energy: need to reduce energy consumption
- The main objective is often performance, such as execution time, but other criteria must be accounted for
- Many models for which we have the answer:
  - Optimal checkpointing period, with fail-stop / silent errors
  - Use of replication to detect and correct silent errors
  - When to checkpoint, replicate and verify for a chain of tasks?
  - Use a different re-execution speed after a failure
- Still a lot of challenges to address, and techniques to be developed for many kinds of high-performance applications, making trade-offs between performance, reliability, and energy consumption

Thanks...

- ... to my co-authors
  - Valentin Le Fèvre, Aurélien Cavelan, Hongyang Sun
  - Yves Robert
  - Franck Cappello, Padma Raghavan, Florina M. Ciorba
- ... and to the Winter School organizers for their kind invitation!
- A few references:

  - A. Benoit, A. Cavelan, Y. Robert, H. Sun. Assessing General-Purpose Algorithms to Cope with Fail-Stop and Silent Errors. TOPC, 2016
  - A. Benoit, A. Cavelan, F. Cappello, P. Raghavan, Y. Robert, H. Sun. Identifying the right replication level to detect and correct silent errors at scale. FTXS/HPDC, 2017.
  - A. Benoit, A. Cavelan, Y. Robert and H. Sun. Multi-level checkpointing and silent error detection for linear workflows. JoCS, 2017.
  - A. Benoit, A. Cavelan, F. Ciorba, V. Le Fèvre, Y. Robert. Combining checkpointing and replication for reliable execution of linear workflows with fail-stop and silent errors. IJNC, 2019.
  - A. Benoit, A. Cavelan, V. Le Fèvre, Y. Robert, H. Sun. A different re-execution speed can help. PASA/ICPP, 2016.

Thanks...

- ... to my co-authors
  - Valentin Le Fèvre, Aurélien Cavelan, Hongyang Sun
  - Yves Robert
  - Franck Cappello, Padma Raghavan, Florina M. Ciorba
- ... and to the Winter School organizers for their kind invitation!

- A few references:

  - A. Benoit, A. Cavelan, Y. Robert, H. Sun. Assessing General-Purpose Algorithms to Cope with Fail-Stop and Silent Errors. TOPC, 2016
  - A. Benoit, A. Cavelan, F. Cappello, P. Raghavan, Y. Robert, H. Sun. Identifying the right replication level to detect and correct silent errors at scale. FTXS/HPDC, 2017.
  - A. Benoit, A. Cavelan, Y. Robert and H. Sun. Multi-level checkpointing and silent error detection for linear workflows. JoCS, 2017.
  - A. Benoit, A. Cavelan, F. Ciorba, V. Le Fèvre, Y. Robert. Combining checkpointing and replication for reliable execution of linear workflows with fail-stop and silent errors. IJNC, 2019.
  - A. Benoit, A. Cavelan, V. Le Fèvre, Y. Robert, H. Sun. A different re-execution speed can help. PASA/ICPP, 2016.