Scheduling for Reliability: Complexity and Algorithms

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Context

Increasing need of computing power:

- simulations (weather, nuclear, ...)
- data processing

A solution to increase the computing power: parallelization

- split a computation into many smaller ones
- execute each small task on one computer

More computers → high probability of failure

Some areas are extremely sensitive to failures:

- critical transportations (airplanes,...)
- nuclear power plants

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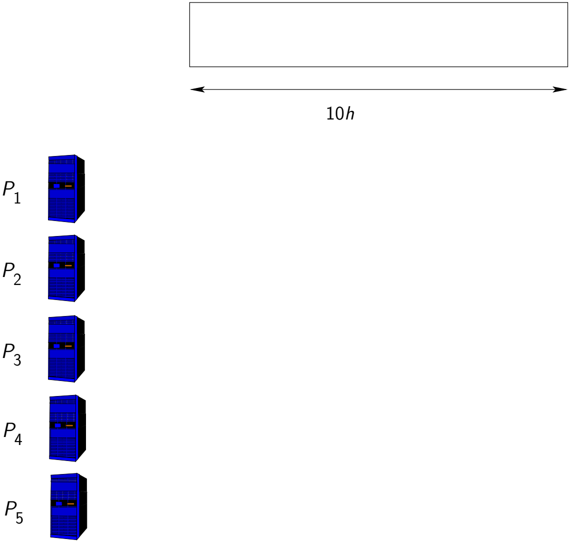

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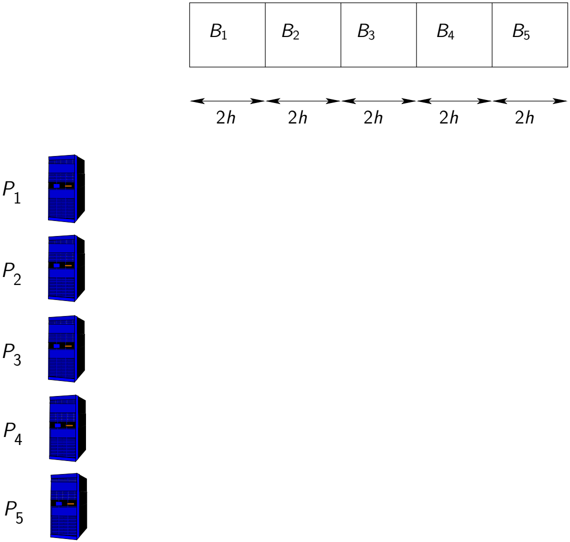

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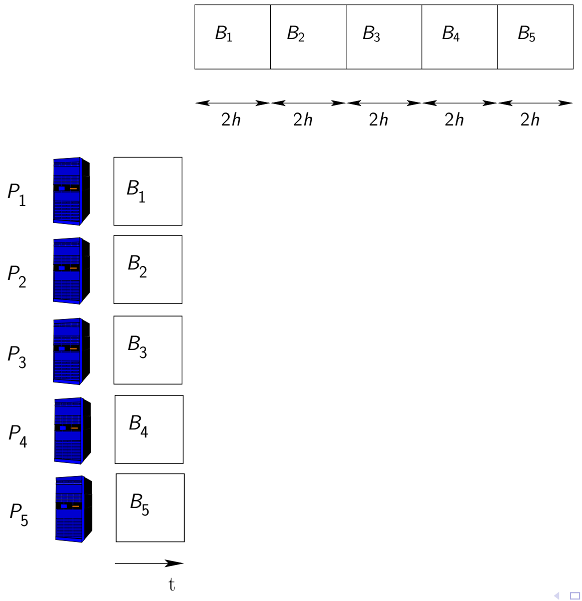
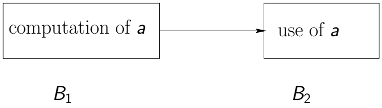


Precedence constraints and DAGs



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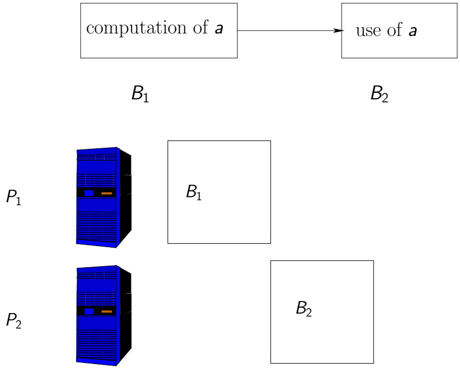

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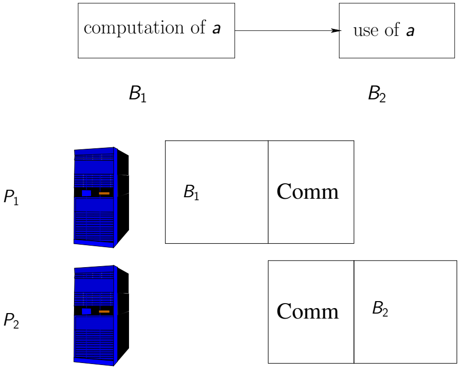
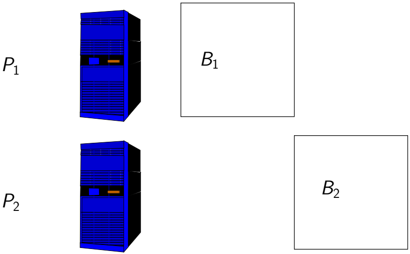


Objectives

- Latency



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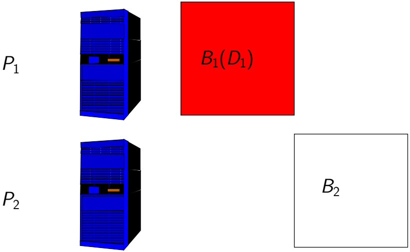

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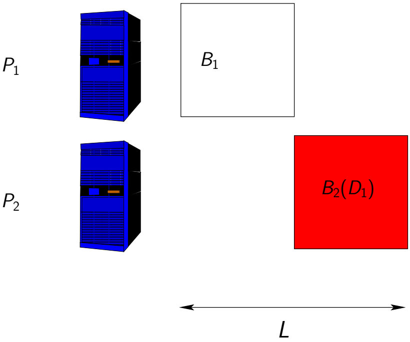

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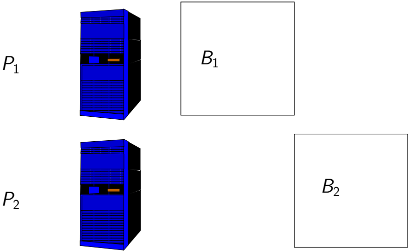
- Period



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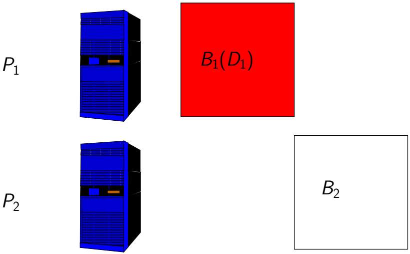

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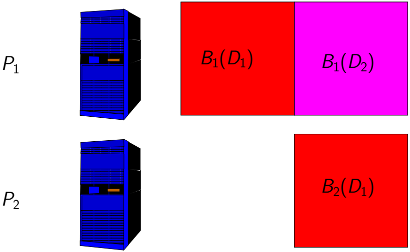

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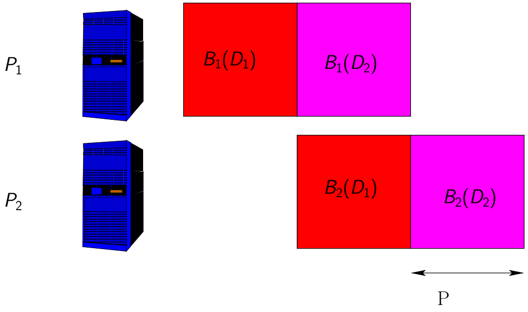

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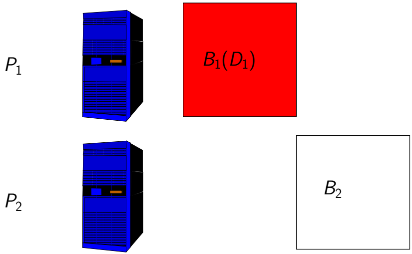
- Reliability



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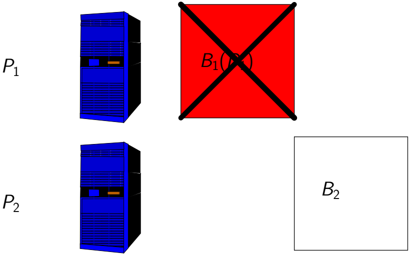

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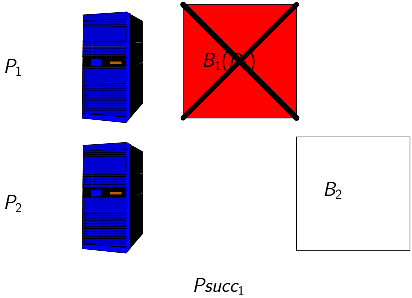

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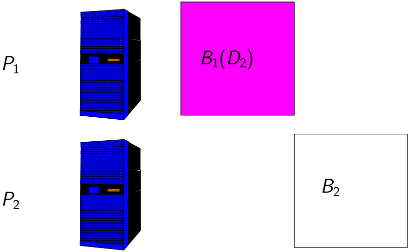

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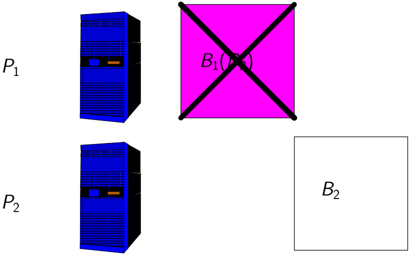

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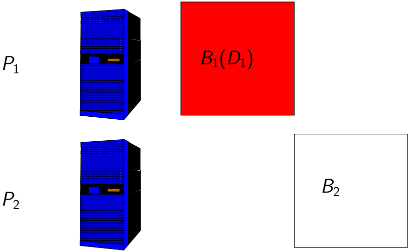

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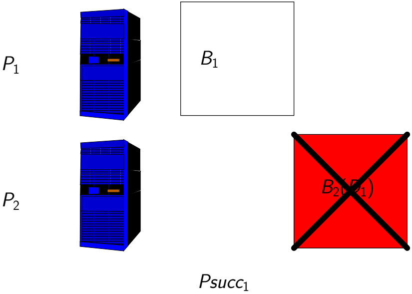

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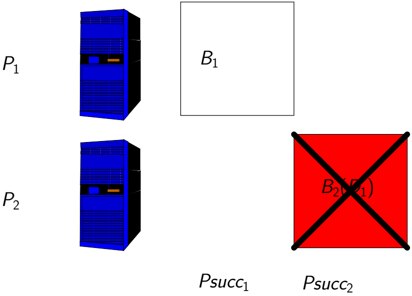

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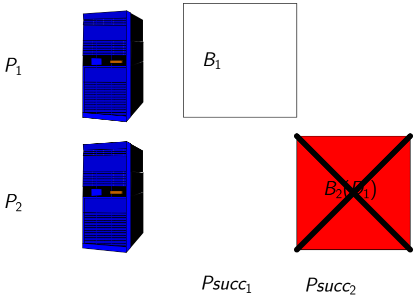

$$Psucc = Psucc_1 \times Psucc_2$$

Reliability of processors

Computers are subject to failures:

- transient failures
- fail-stop failures

Methods used to increase the reliability:

- Replication
- Checkpointing
- Migration

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Roadmap:

- efficiently allocate tasks to processors
- design fast algorithms for these problems
- assess the complexity of these problems

For polynomial problems:

- provide optimal polynomial algorithms

For NP-complete problems:

- prove NP-completeness
- design optimal (exponential) algorithms (ILP,...)
- design approximation algorithms
- find bounds on approximation ratio
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Plan

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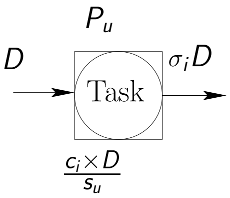
Application Model

Problem under study:

- streaming applications
- filtering tasks with selectivity σ_i and cost c_i (data bases, web services,...)
- servers with speed s_u
- possibility to add dependencies
- one-to-one mapping

Objective:

- minimize period
- minimize latency



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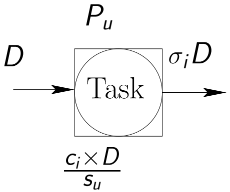

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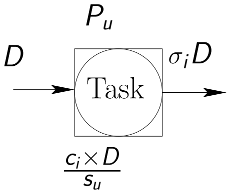
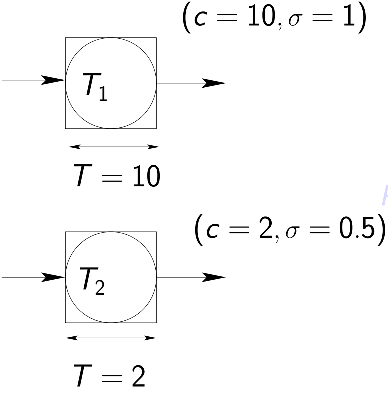


Example

Instance with two independent tasks and two identical processors:

- $\sigma_1 = 1, \; c_1 = 10$
- $\sigma_2 = 0.5, \; c_2 = 2$
- $s = 1$
- $D = 1$



$P = 10, \; L = 10$

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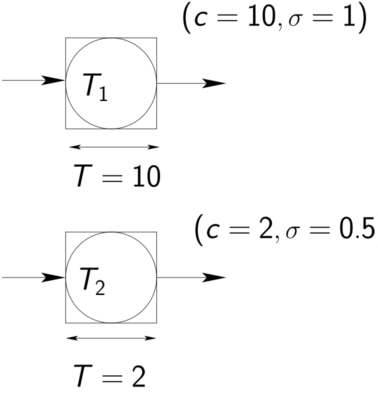

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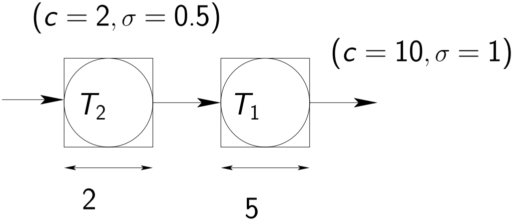

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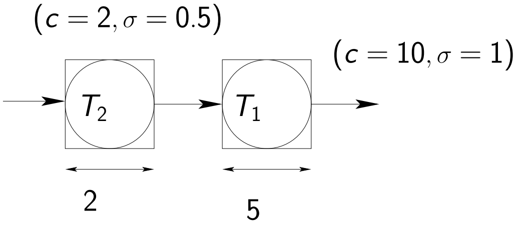

$P = 5,\ L = 7$

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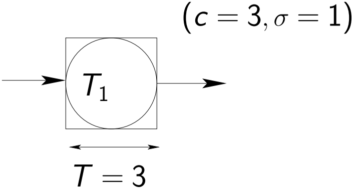

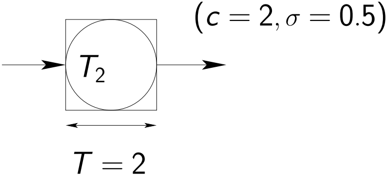

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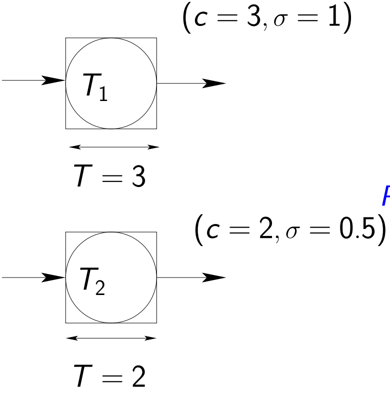

$P = 3,\ L = 3$

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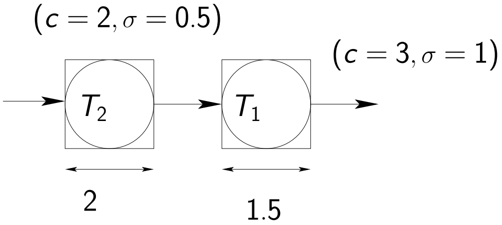

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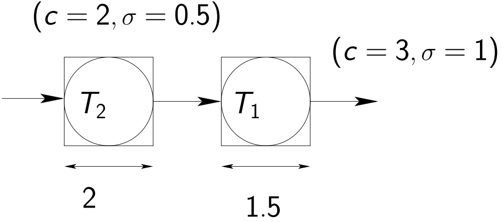
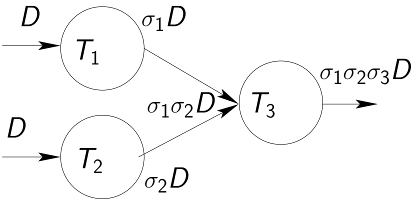


$P = 2$, $L = 3.5$

Example



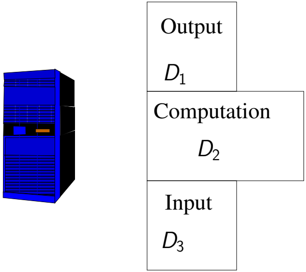
Combining selectivities

$$\mathcal{P} = \max\left(\frac{c_1}{s_1}, \frac{c_2}{s_2}, \frac{\sigma_1\sigma_2 c_3}{s_3}\right)$$

$$\mathcal{L} = \max\left(\frac{c_1}{s_1}, \frac{c_2}{s_2}\right) + \frac{\sigma_1\sigma_2 c_3}{s_3}$$

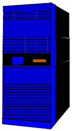
Communication models

- OVERLAP: overlap between communications and computations



Communication models

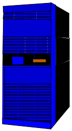
- OVERLAP: overlap between communications and computations
- INORDER: no overlap and FIFO execution of data sets by processors



Input	Computation	Output
D_1	D_1	D_1

Communication models

- OVERLAP: overlap between communications and computations
- INORDER: no overlap and FIFO execution of data sets by processors
- OUTORDER: no overlap and any possible executions order



Computation	Input	Output
D_1	D_2	D_1

General problem

Instance description:

- set of tasks
- dependence graph of these tasks
- set of processors
- communication model
- objective

The schedule:

- a plan (possibility to add dependencies)
- an allocation to processors (if they are heterogeneous)
- the execution times of computations and communications

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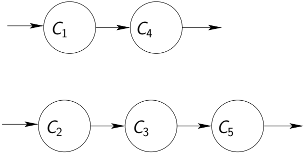

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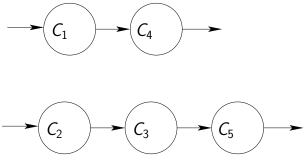

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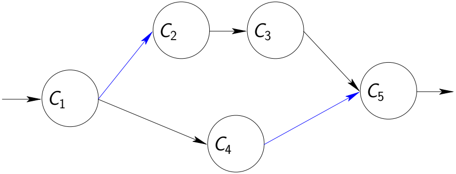

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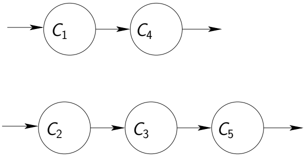

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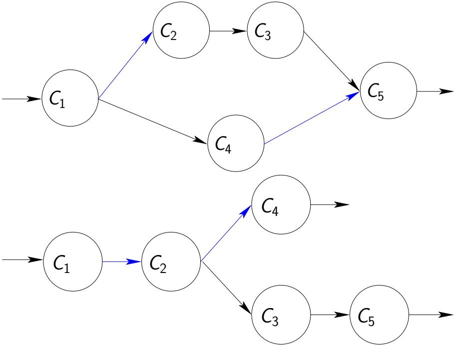

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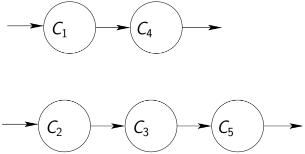

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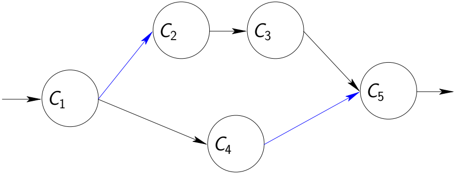

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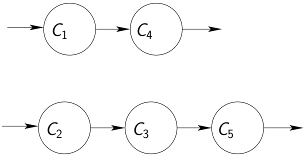
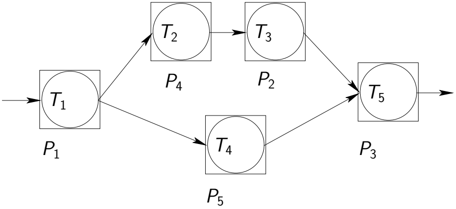


The allocation:



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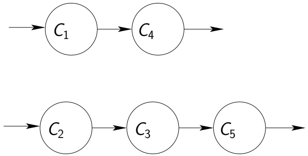
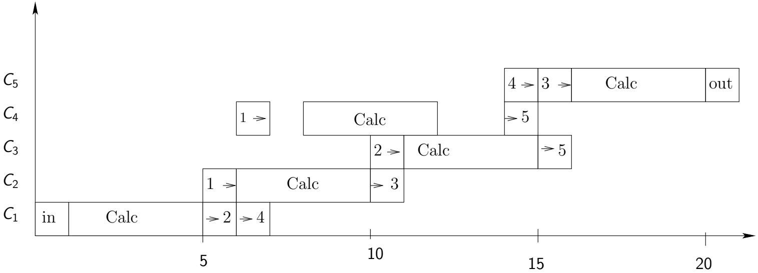


Execution times:



Complexity results

	Period	Latency
Hom. without comm.	Polynomial	Polynomial
Het. without comm.	Polynomial	Polynomial
Hom. with comm.	OVERLAP: Polynomial Other models: NP-hard	NP-hard

Complexity results for a given mapping

Complexity results

	Period	Latency
Hom. without comm.	Polynomial	Polynomial
Het. without comm.	NP-hard Inapproximable	NP-hard Inapproximable
Hom. with comm.	NP-hard	NP-hard

Complexity results for computing the optimal mapping

Conclusion for this study

Theoretical results

- a complete set of complexity results
- some approximation results
- integer linear program for some problems

Simulation results

- heuristics for the model with no communication costs
- simulations

Perspectives

- heuristics and simulations including communication costs
- approximation results for all NP-complete problems
- extend the model to replication

Outline

1. Introduction
2. Scheduling filtering applications (overview)
3. Reliability of pipelined real-time systems (overview)
4. Scheduling on volatile ressources
5. Conclusion and perspectives

Pipelined real-time systems

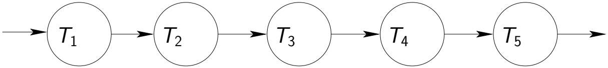
Real-time systems:

- jobs released
- deadline for each job

Pipelined real-time systems:

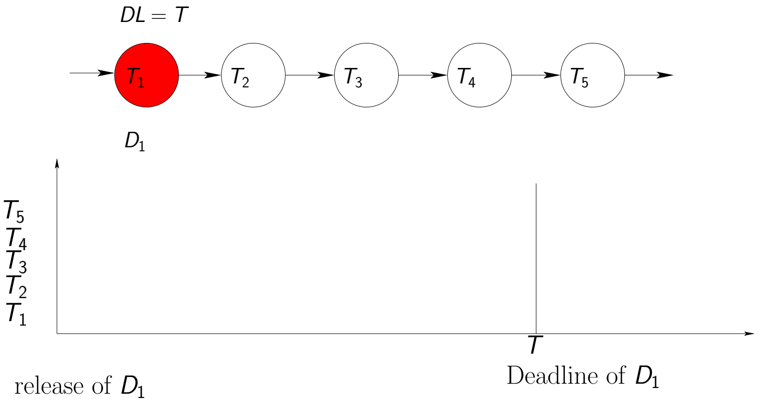
- a chain of tasks
- data sets are periodically released
- a deadline for each data set
- deadlines are periodic

Example



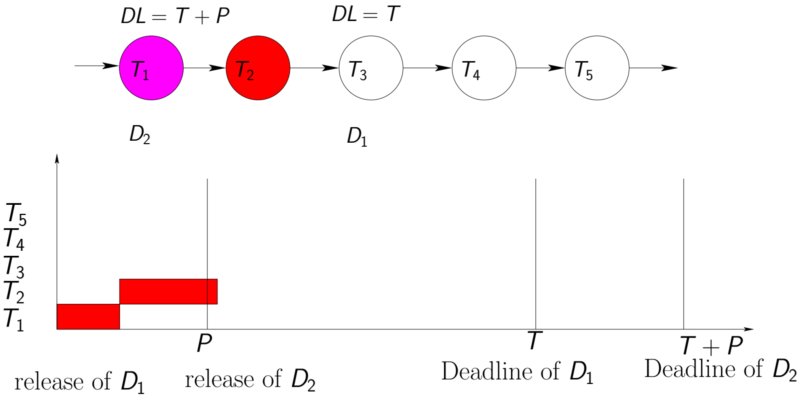
Example

$t = 0$



Example

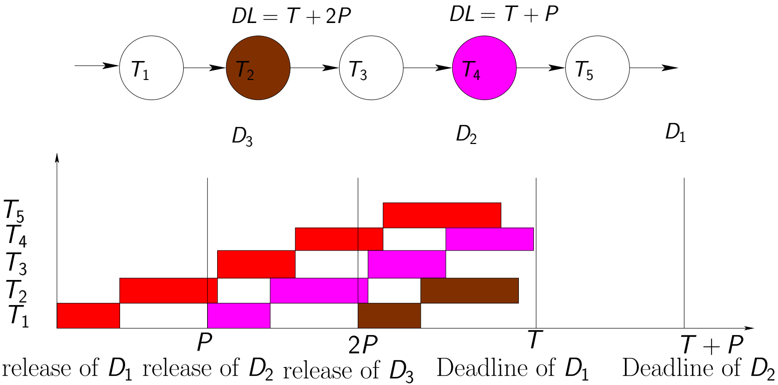
$t = P$



$DL = T + P$
$DL = T$
T_1
T_2
T_3
T_4
T_5
D_2
D_1
T_5
T_4
T_3
T_2
T_1
P
T
$T + P$
release of D_1
release of D_2
Deadline of D_1
Deadline of D_2

Example

$t = T$



$DL = T + 2P$
$DL = T + P$
T_1
T_2
T_3
T_4
T_5
D_3
D_2
D_1
T_5
T_4
T_3
T_2
T_1
P
$2P$
T
$T + P$
release of D_1
release of D_2
release of D_3
Deadline of D_1
Deadline of D_2

General model

Application model:

A chain of n tasks. Task T_k is characterized by:

- cost c_k
- output data size o_k (we suppose $o_n = 0$)

Platform:

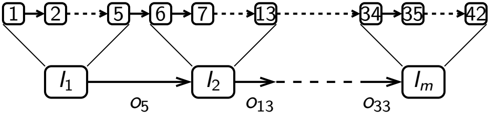
n processors. Processor P_u is characterized by:

- speed s_u
- failure rate per time unit λ_u

A processor can simultaneously execute some task and communicate data.

Interval mapping

The chain of tasks is divided into m intervals.



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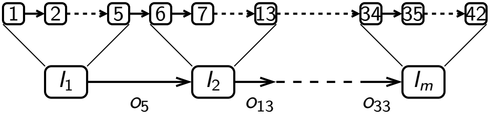
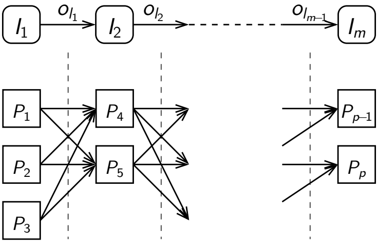
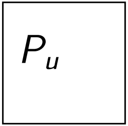


The intervals are replicated on several processors.
Each processor executes only one interval.
Bound $\mathcal{K}$ on the number of replication of an interval.



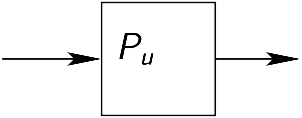
Failure model



Probability of success for computation of an interval I_i on P_u:

$$r_{u,i} = e^{-\lambda_u \times \frac{c_i}{s_u}}$$

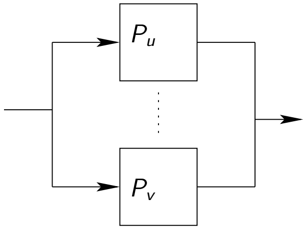
Failure model



Probability of success of an interval I_i of size W_i on P_u including communications:

$$r_{u,I_i} = r_{comm,i-1} \times e^{-\lambda_u \times \frac{c_i}{s_u}} \times r_{comm,i}$$

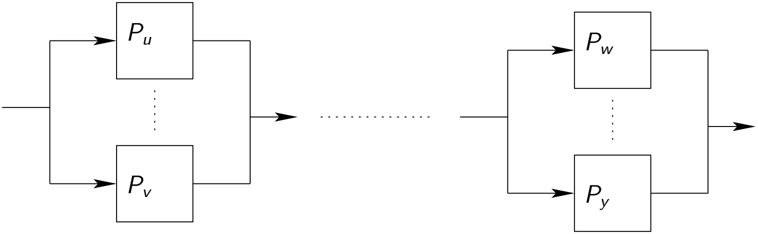
Failure model



Reliability of interval I_i on the set of processors $\mathcal{P}_i$ including communications:

$$1 - \prod_{P_u \in \mathcal{P}_i} (1 - r_{comm,i-1} \times r_{u,I_i} \times r_{comm,i})$$

Failure model



Reliability of a schedule:

$$r = \prod_{i=1}^{t} \left(1 - \prod_{P_u \in \mathcal{P}_i} (1 - r_{comm,i-1} \times r_{u,I_i} \times r_{comm,i}) \right)$$

Complexity results

	mono-criteria	bi-criteria	three-criteria
homogeneous case	period: Poly latency: Poly reliability: Poly	reliability-latency: NP-c reliability-period: Poly latency-period: Poly	NP-c
heterogeneous case	reliability: NP-c period: NP-c latency: Poly	NP-c	NP-c

Conclusion for this study

Theoretical results

- Realistic scenario for a classical model
- A complete theoretical study

Simulation results

- Heuristics for period and latency optimization
- Simulations

Perspectives

- More realistic probability distributions

Outline

Dealing with volatile resources

- Deploy applications on desktop grids (SETI@home,...)
- Iterative applications [Bahi07,Heddaya94]
- Resource availability: *UP*/ *DOWN* processes

The goal: **on-line** policies for resource selection:
- Which resources to enroll?
- How to compare configurations?

Dealing with volatile resources

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Main assumptions

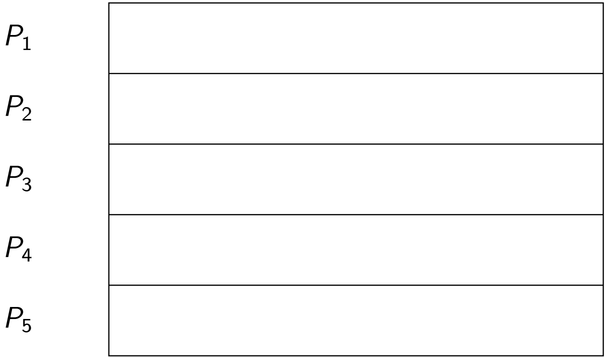
Problem

- Iterative application
- Master-worker paradigm
- Synchronization after each iteration
- Volatile platforms: transient failures & preemption
- Heterogeneous processors
- Limited available bandwidth from master to workers

Objective: Maximize expected number of iterations executed within limited time

Example of iterative application

- 5 tasks
- 5 processors



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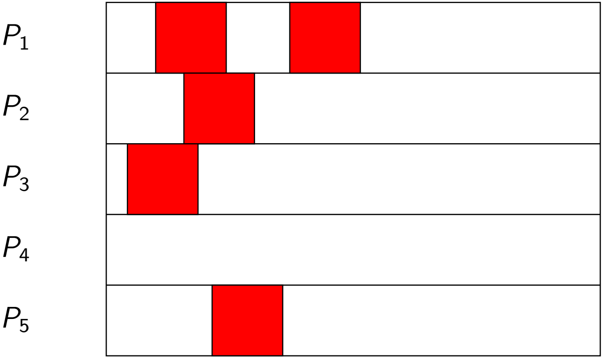

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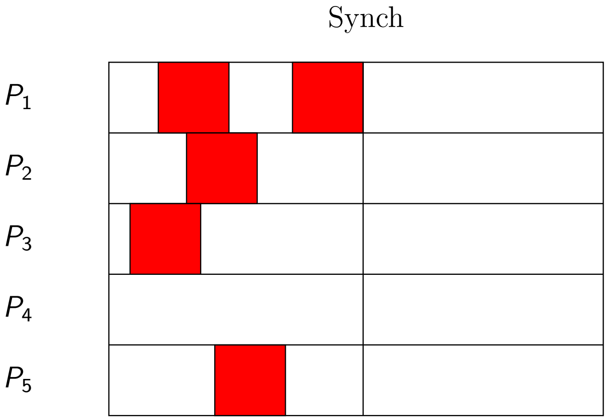

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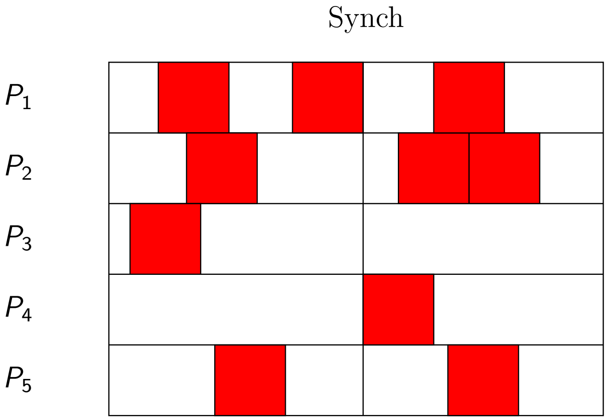

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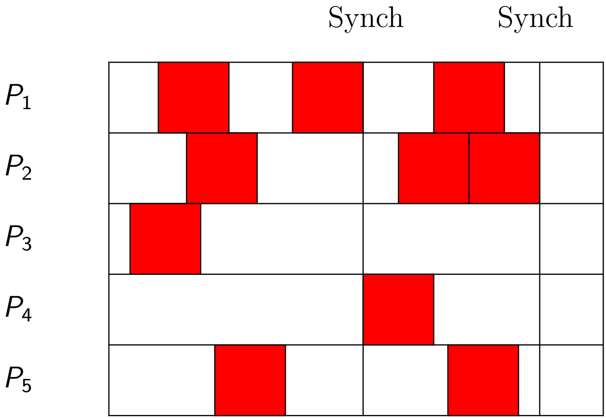


A realistic model (1/2)

Application

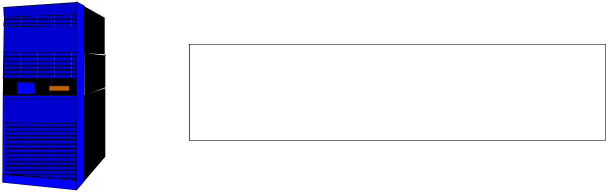
- Successive iterations
- Synchronization after each iteration
- For each iteration, m same-size tasks
- Two scenarios

TIGHTLY-COUPLED	continuously interacting tasks
INDEPENDENT	independent tasks

- Same program of size V_{prog} for all iterations
- Data set of size V_{data} for each task

Independent vs Tightly-Coupled scenarios

- 5 tasks
- 1 processor
- $T_{prog} = T_{data} = 0$



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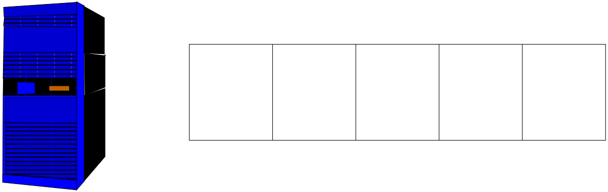
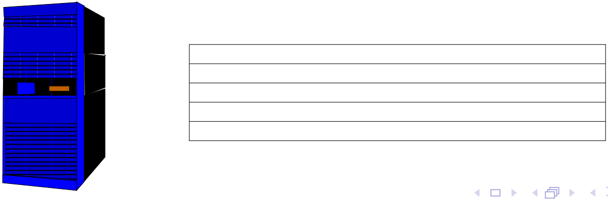


Tightly-Coupled:



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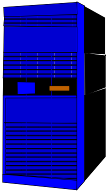

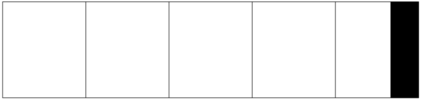

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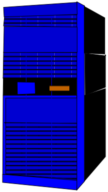

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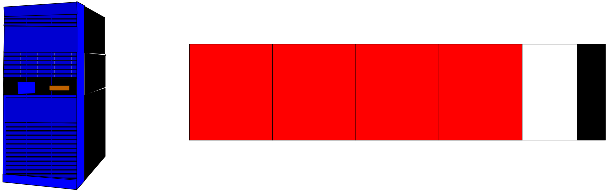

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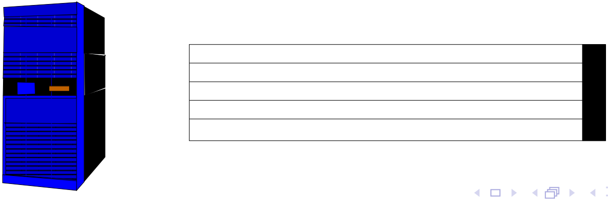

Independent vs Tightly-Coupled scenarios

- 5 tasks
- 1 processor
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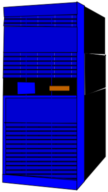

INDEPENDENT vs TIGHTLY-COUPLED scenarios

- 5 tasks
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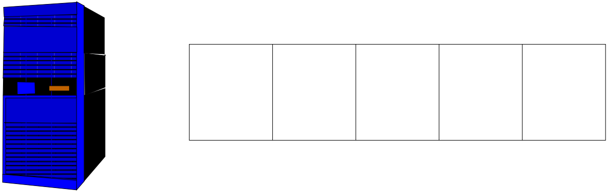
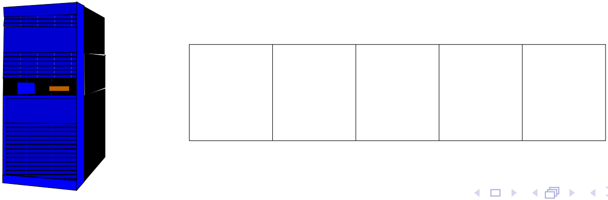
- $T_{prog} = W = 0$

INDEPENDENT:



TIGHTLY-COUPLED:



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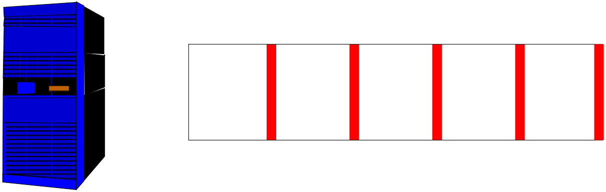

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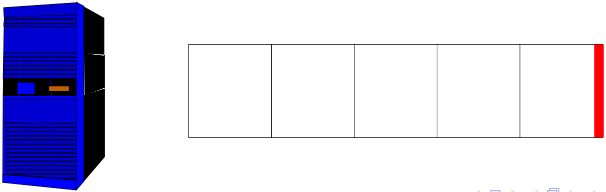

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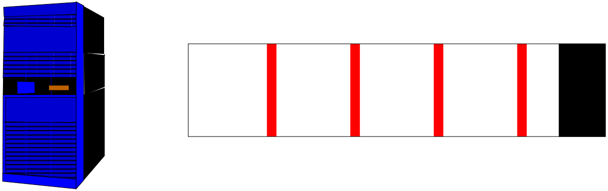

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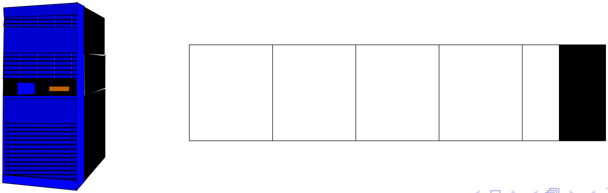
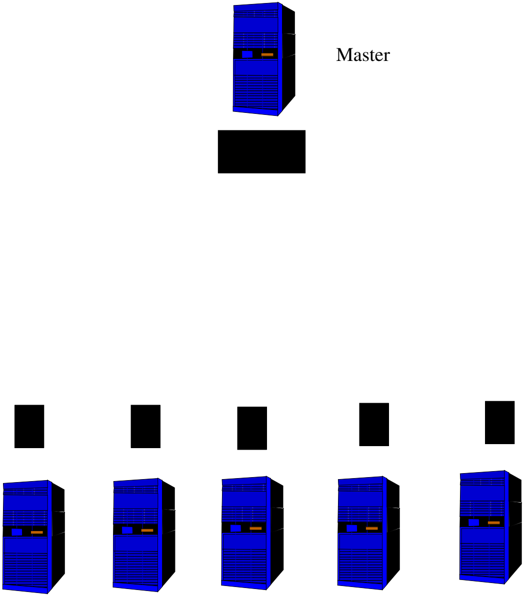


A realistic model (2/2)

Platform

- Master-worker execution
- p heterogeneous resources/workers
  $\Rightarrow$ W_u cost of a task on processor P_u
- Limited bandwidth
  $\Rightarrow$ BW for master and bw for workers
  $\Rightarrow$ $n_{com} = \lfloor \frac{\text{BW}}{\text{bw}} \rfloor$ max number of simultaneous comms
- Overlap between computation and communication

Communication model



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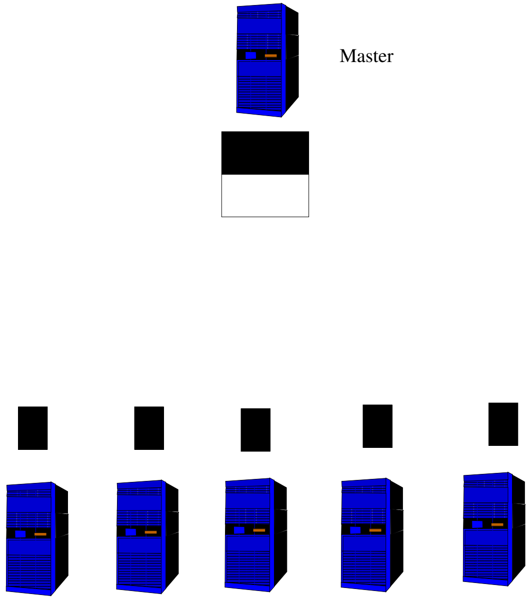

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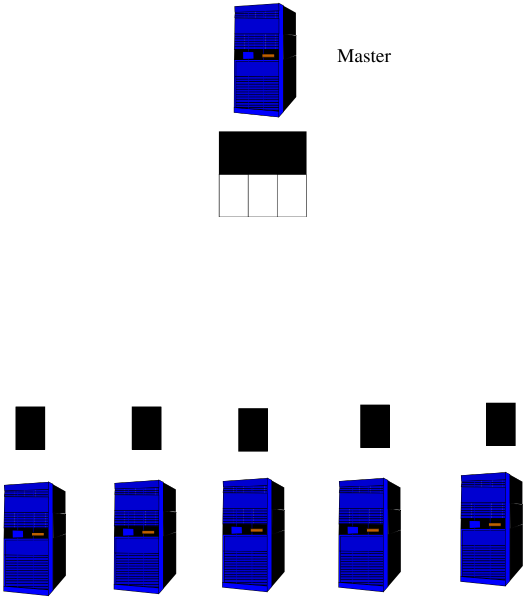

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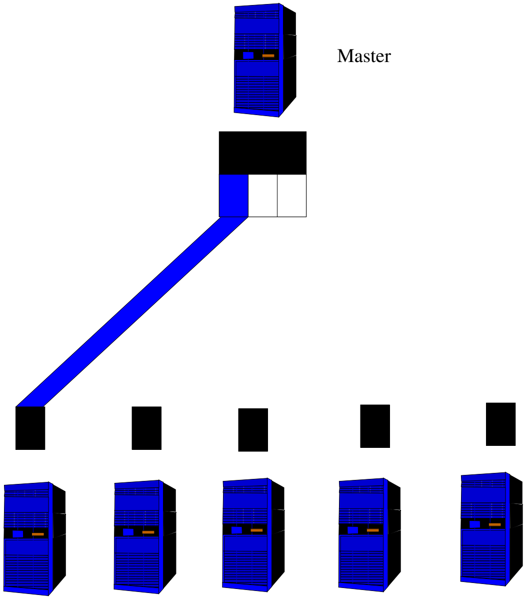

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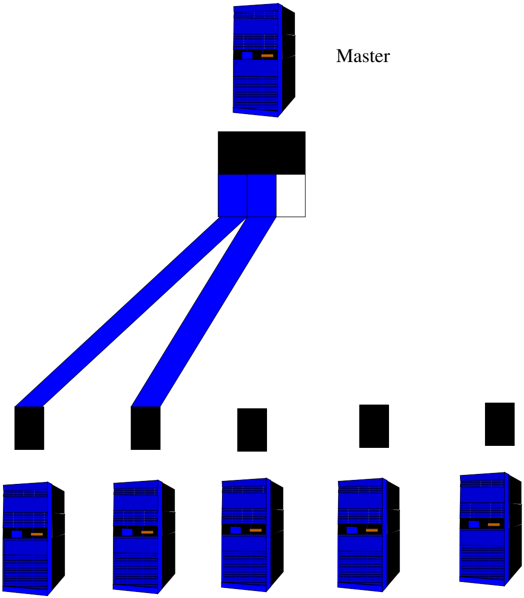

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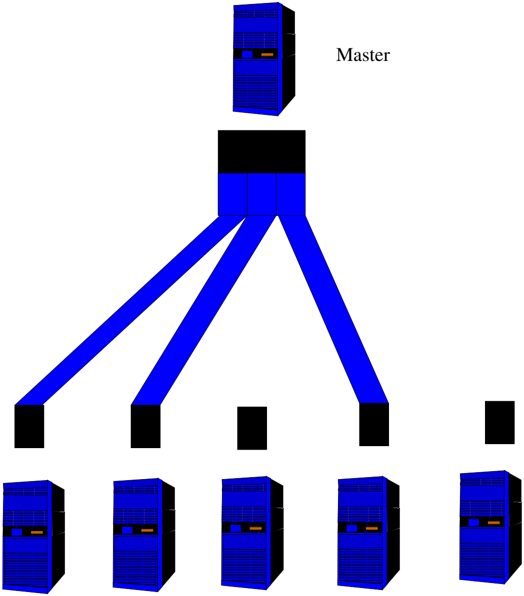

Resource availability

Three possible processor states: *UP*, *RECLAIMED*, *DOWN*

- Preemption delays current operations
- Failure ⇒ current communications and computations are lost
- Program and data need be received again

On-line study

Availability modeled by (independent) 3-state Markov chains

Off-line study

For each processor P_u, state array T_u:

- $T_u[t] = -1$: processor *DOWN* at time slot t
- $T_u[t] = 0$: processor *RECLAIMED* at time slot t
- $T_u[t] = 1$: processor *UP* and available for comms and/or computing

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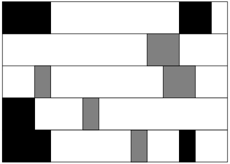
Example: INDEPENDENT scenario

Instance with $m = 5$ tasks

- $p = 5$ processors, $w_i = i$
- $n_{com} = 2$
- $T_{prog} = 2,\ T_{data} = 1$



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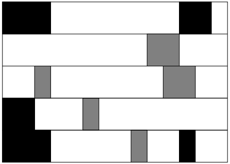

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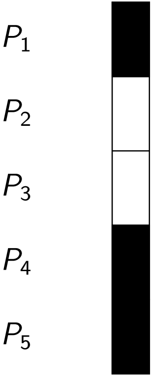

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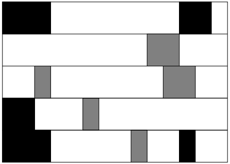

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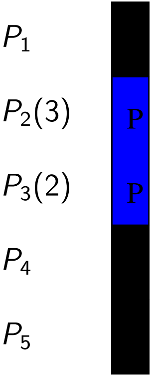

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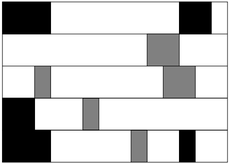

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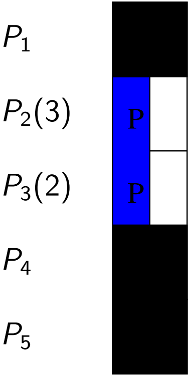

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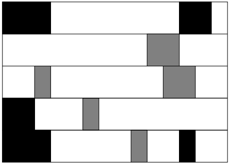

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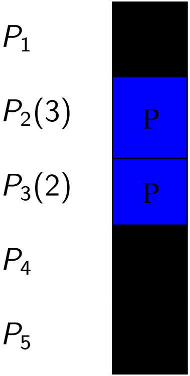

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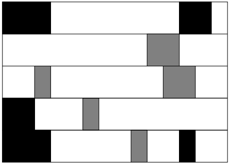

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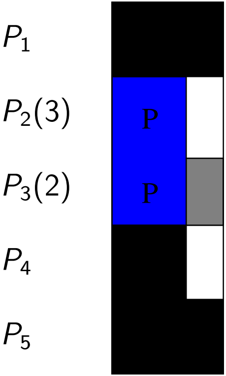

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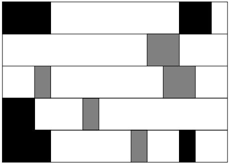

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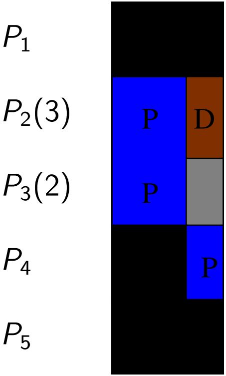

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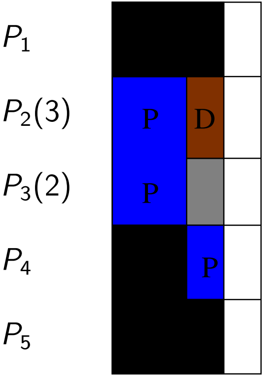

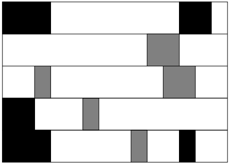

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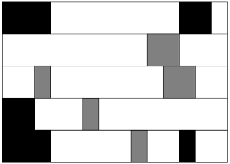
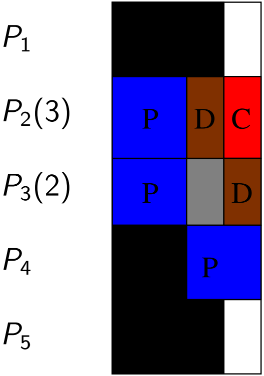


State array



Example: INDEPENDENT scenario

Instance with $m = 5$ tasks

- $p = 5$ processors, $w_i = i$
- $n_{com} = 2$
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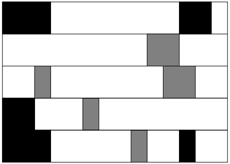

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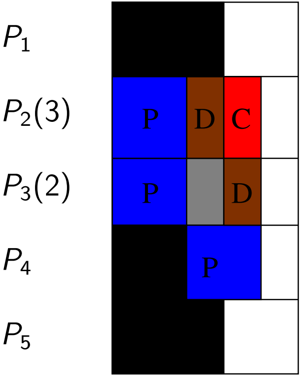

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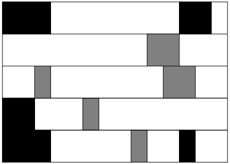

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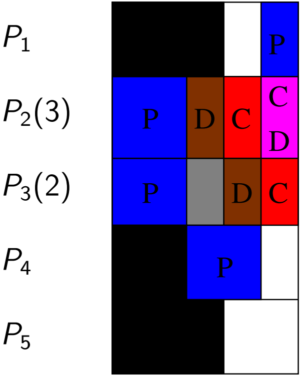

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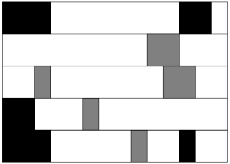

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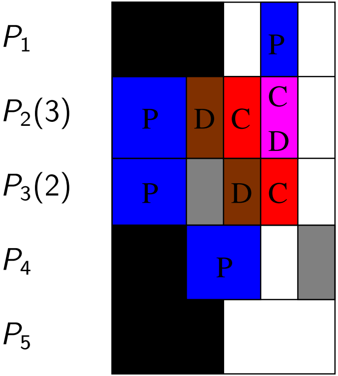

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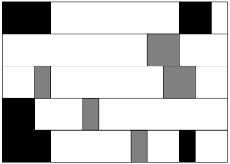

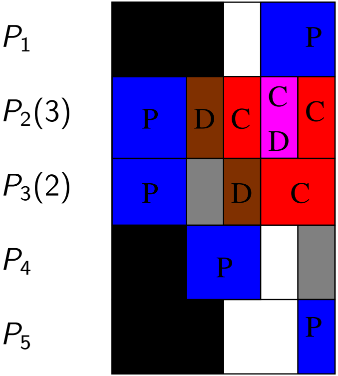

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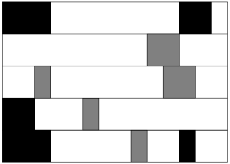
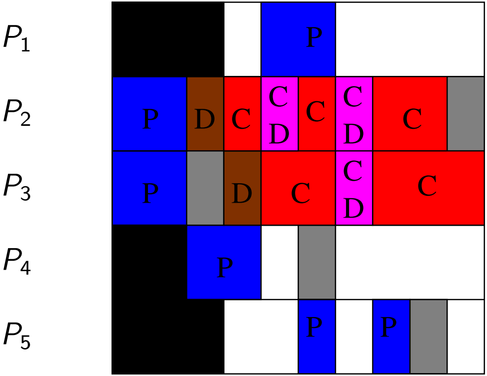


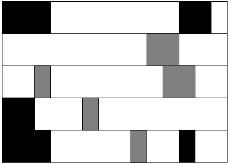
State array



Example: TIGHTLY-COUPLED scenario

Instance with $m = 5$ tasks

- $p = 5$ processors, $w_i = i$
- $n_{com} = 2$
- $T_{prog} = 2$, $T_{data} = 1$



State array

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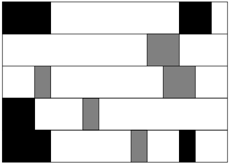

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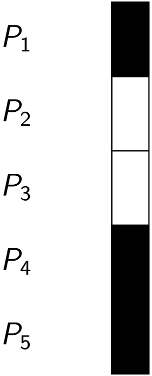

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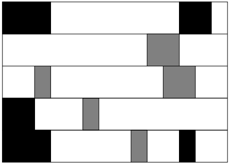

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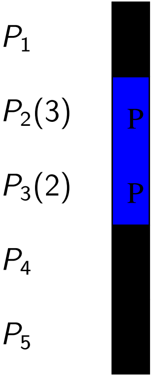

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- $p = 5$ processors, $w_i = i$
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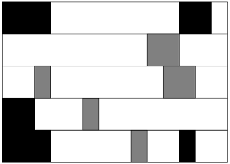

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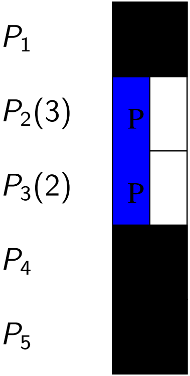

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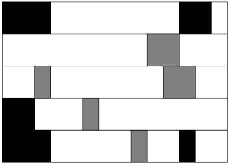

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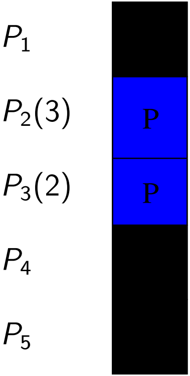

Example: TIGHTLY-COUPLED scenario

Instance with $m = 5$ tasks

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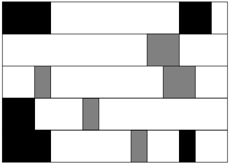

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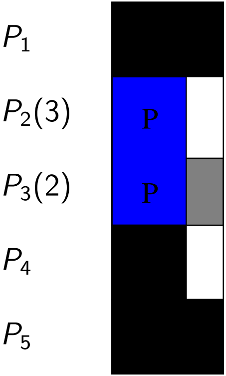

Example: Tightly-Coupled scenario

Instance with $m = 5$ tasks

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- $n_{com} = 2$
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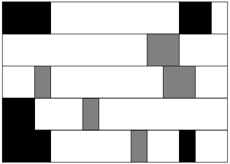

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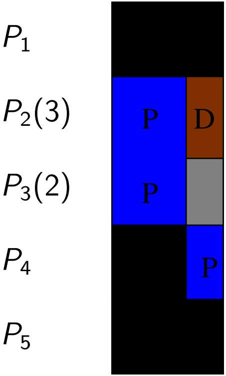

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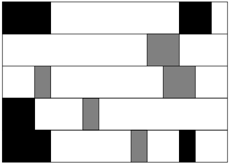

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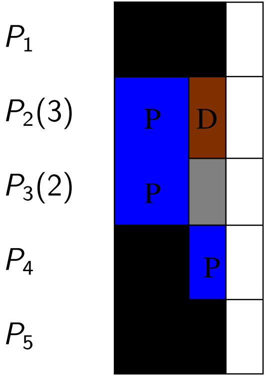

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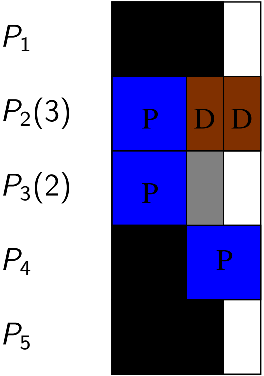

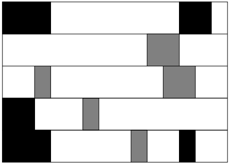

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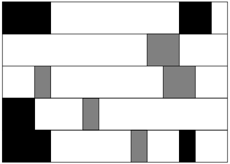

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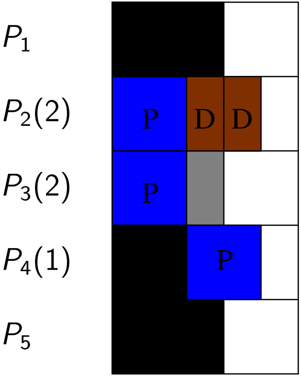

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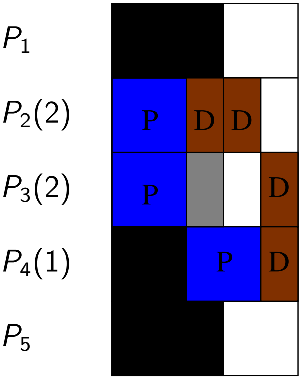

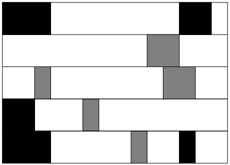

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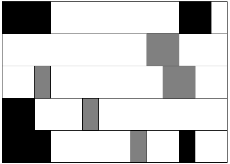

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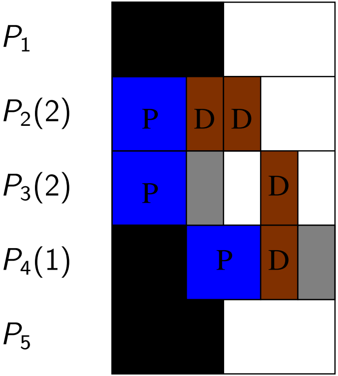

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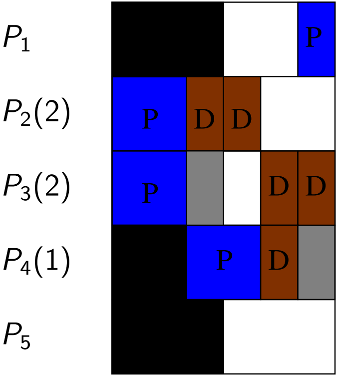

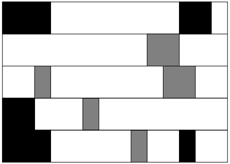

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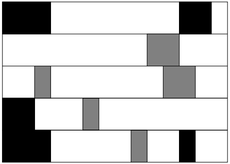

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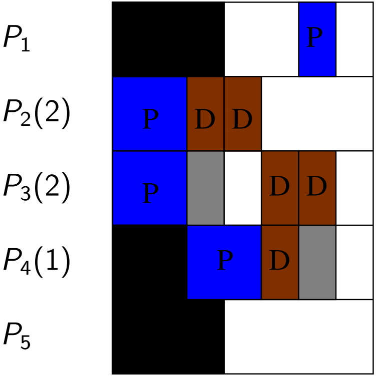

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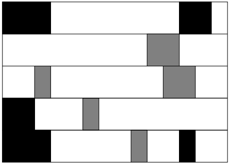

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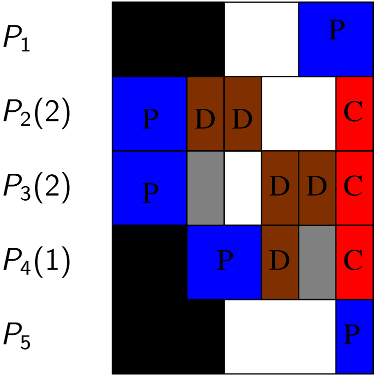

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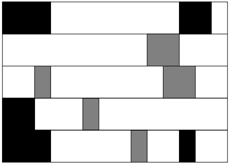

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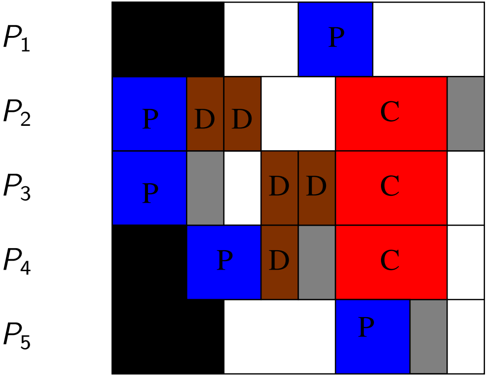

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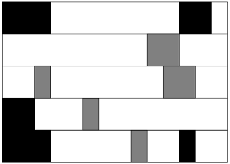

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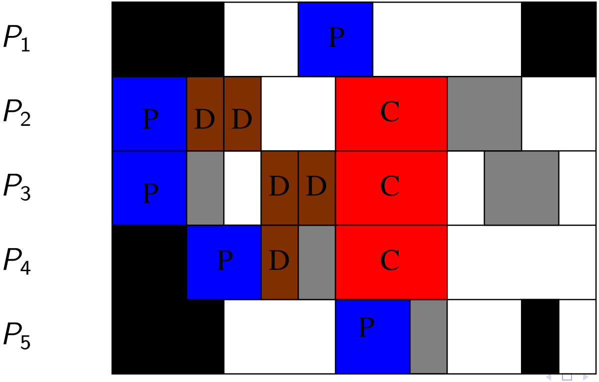

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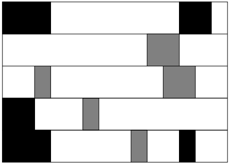
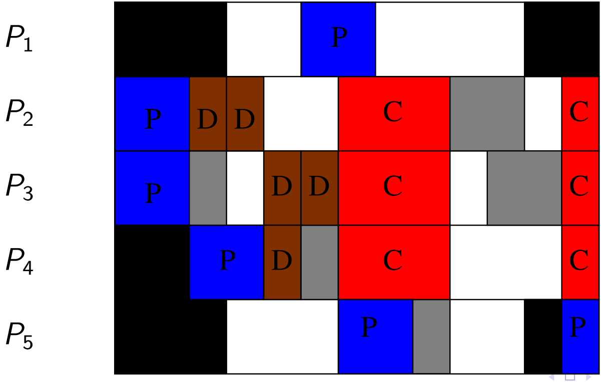


State array



Example: Tightly-Coupled scenario

Instance with $m = 5$ tasks

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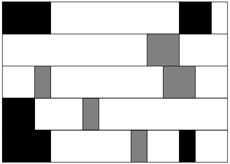
- $n_{com} = 2$
- $T_{prog} = 2$, $T_{data} = 1$



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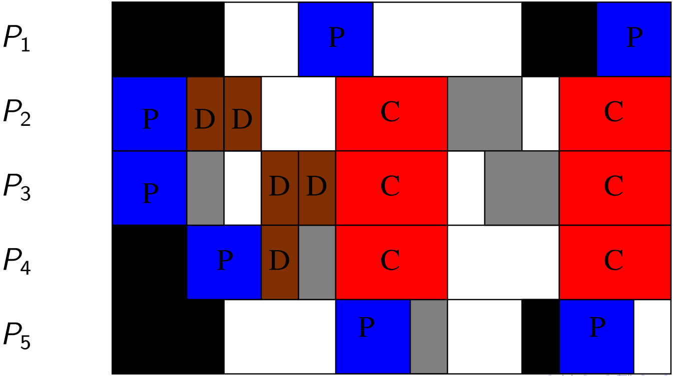


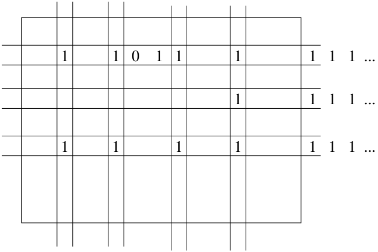
NP-completeness of TIGHTLY-COUPLED (1/2)

Theorem

TIGHTLY-COUPLED *is NP-complete even with uniform resources and no communications* ($T_{prog} = Tdata = 0$)

Reduction from ENCD (Exact Node Cardinality Decision problem): Given a bipartite graph $G = (U \cup V, E)$ and two integers a and b, does there exist a **bi-clique** with exactly a nodes in U and b nodes in V?

NP-completeness of TIGHTLY-COUPLED (2/2)



$p = |U|$ processors, $N = 2|V| + 1$ time slots
$T_i[j] = 1 \iff (u_i, v_j) \in E$ or $j \geq |V| + 1$, otherwise $T_i[j] = 0$
$m = a$ tasks of cost $W = b + |V| + 1$

Not enough time to compute two tasks on same processor
Need a processors available during **the same** W time slots
$\Rightarrow$ need b time slots of computation before time slot $|V|$

NP-completeness of INDEPENDENT (1/2)

Theorem

INDEPENDENT *is NP-complete even with uniform resources*

Reduction from 3SAT:
Given a set $U = \{x_1, ..., x_n\}$ of variables and a collection $\{C_1, ..., C_m\}$ of clauses, does there exist a truth assignment for U?

NP-completeness of INDEPENDENT (2/2)

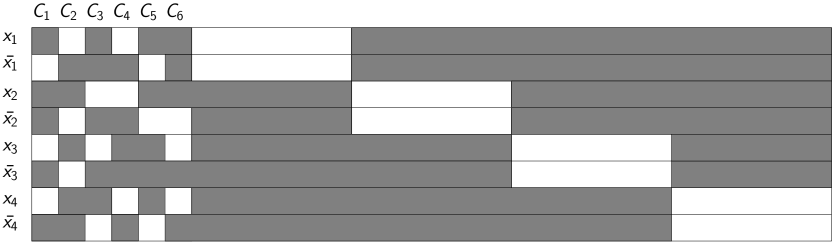
$(\bar{x_1} \vee x_3 \vee x_4) \wedge (x_1 \vee \bar{x_2} \vee \bar{x_3}) \wedge (x_2 \vee x_3 \vee \bar{x_4}) \wedge (x_1 \vee x_2 \vee x_4) \wedge (\bar{x_1} \vee \bar{x_2} \vee \bar{x_4}) \wedge (\bar{x_2} \vee x_3 \vee x_4)$

m tasks, $2n$ procs, $n_{com} = 1$, $T_{prog} = m$, $T_{data} = 0$, $W = 1$
Two processors x and $\bar{x}$ per variable x

Processors x and $\bar{x}$ cannot both compute tasks
$\#$ executed tasks $= \#$ computation time slots $t \leq m$
m tasks executed $\Rightarrow$ enrolled processors validate 3SAT instance

NP-completeness of INDEPENDENT (2/2)

$(\bar{x}_1 \vee x_3 \vee x_4) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_2 \vee x_3 \vee \bar{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge$
$(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee x_3 \vee x_4)$



m tasks, $2n$ procs, $n_{com} = 1$, $T_{prog} = m$, $T_{data} = 0$, $W = 1$
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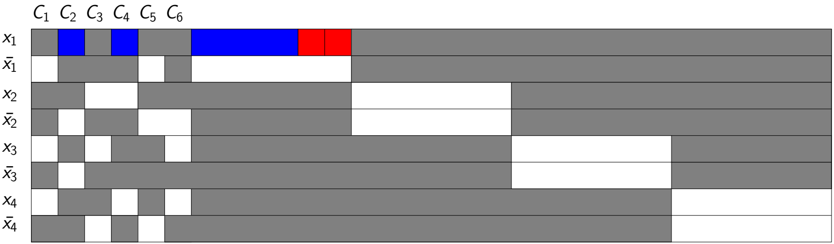
NP-completeness of INDEPENDENT (2/2)

$(\bar{x_1} \vee x_3 \vee x_4) \wedge (x_1 \vee \bar{x_2} \vee \bar{x_3}) \wedge (x_2 \vee x_3 \vee \bar{x_4}) \wedge (x_1 \vee x_2 \vee x_4) \wedge (\bar{x_1} \vee \bar{x_2} \vee \bar{x_4}) \wedge (\bar{x_2} \vee x_3 \vee x_4)$



m tasks, $2n$ procs, $n_{com} = 1$, $T_{prog} = m$, $T_{data} = 0$, $W = 1$
Two processors x and $\bar{x}$ per variable x

Processors x and $\bar{x}$ cannot both compute tasks
\# executed tasks = \# computation time slots $t \leq m$
m tasks executed $\Rightarrow$ enrolled processors validate 3SAT instance

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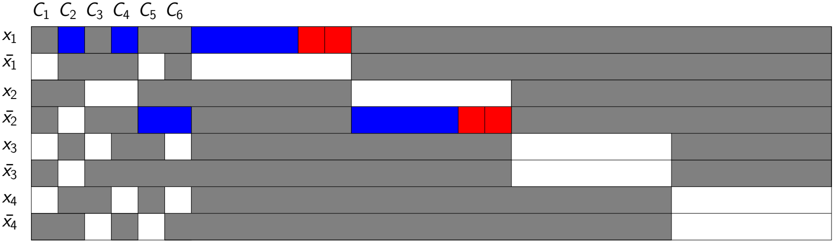

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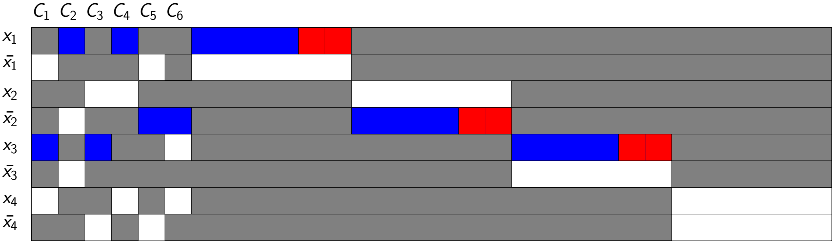

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Markov chain

Transition matrix for processor P_i:

$$\begin{vmatrix} P_{u,u}(i) & P_{r,u}(i) & P_{d,u}(i) \\ P_{u,r}(i) & P_{r,r}(i) & P_{d,r}(i) \\ P_{u,d}(i) & P_{r,d}(i) & P_{d,d}(i) \end{vmatrix}$$

Probabilities:

- $\pi_u^{(i)}$ for P_i being *UP*
- $\pi_r^{(i)}$ for P_i being *RECLAIMED*
- $\pi_d^{(i)}$ for P_i being *DOWN*

Probability of success of a computation in INDEPENDENT

Probability of another time slot of computation:

$$
\begin{aligned}
P_{+}^{(q)} &= P_{u,u}^{(q)} + \sum_t P_{u,r} P_{r,r}^{t} P_{r,u} \\
&= P_{u,u}^{(q)} + \frac{P_{u,r}^{(q)} P_{r,u}^{(q)}}{1 - P_{r,r}^{(q)}}
\end{aligned}
$$

Probability of success of a computation of W time-slots:

$$P_{+}^{(q)}(W) = (P_{+}^{(q)})^{W-1}$$

Probability of success of a computation in INDEPENDENT

Probability of another time slot of computation:

$$P_+^{(q)} = P_{u,u}^{(q)} + \sum_t P_{u,r} P_{r,r}^t P_{r,u}$$
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Probability of success of a computation of W time-slots:

$$P_+^{(q)}(W) = (P_+^{(q)})^{W-1}$$

Expected completion time in INDEPENDENT

Expected time of the next time slot of computation:

$$E^{(q)}(up) = 1 + \frac{P_{u,r}^{(q)} P_{r,u}^{(q)}}{1 - P_{r,r}^{(q)}} \times \frac{1}{P_{u,u}^{(q)}(1 - P_{r,r}^{(q)}) + P_{u,r}^{(q)} P_{r,u}^{(q)}}$$

Expected time of the completion of a computation of W time-slots:

$$E^{(q)}(W) = 1 + (W - 1)E^{(q)}(up)$$

Probability of success in TIGHTLY-COUPLED

A set S of processors with W time slots of workload
All processors of S are UP at time 0

Probability that all processors of S will be UP at time t:

$$P^{(S)}(t)$$

Let $E_u(S) = \sum_{t>0} P^{(S)}(t)$

Probability of another time slot of computation:

$$P_+^{(S)} = \frac{E_u(S)}{1 + E_u(S)}$$

Probability of success in TIGHTLY-COUPLED

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Expected computation time in TIGHTLY-COUPLED

Let $E_u(S) = \sum_{t>0} P^{(S)}(t)$
Let $A(S) = \sum_{t>0} t \times P^{(S)}(t)$

Expected time of the next time slot of computation:

$$E_c^{(S)} = \frac{A(S)\left(1 - P_+^{(S)}\right)}{1 + E_u(S)}$$

General description of heuristics

Three classes of heuristics:

- Passive: the configuration may change only when one of the hosts in it goes to the DOWN state
- Dynamic: the configuration may change if a "better" processor becomes *UP*, but no ongoing communication/computation is terminated
- Proactive: like dynamic, but aggressive termination of ongoing communication/computation

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Proactive criteria for TIGHTLY-COUPLED

After t time slots on a same iteration:

- Success probability: P
- Expected completion time: E
- Expected yield: $\frac{P}{E+t}$
- Apparent yield: $\frac{P}{E}$

Random heuristics

RANDOM: randomly select processor for next task

Weighted random:

- RANDOM1: weight $P_{u,u}^{(q)}$ for P_q
- RANDOM2: weight $P_{+}^{(q)}$ for P_q
- RANDOM3: weight $\pi_u^{(q)}$ for P_q
- RANDOM4: weight $1 - \pi_d^{(q)}$ for P_q

variants RANDOMXW: weight divided by w_q

Greedy heuristics for INDEPENDENT

- MCT: Minimum completion time

$$CT(P_q, n_q) = \text{Delay}(q) + T_{\text{data}} + \max(n_q - 1, 0) \max(T_{\text{data}}, w_q) + w_q$$

- EMCT: Expected MCT
- LW: Likely to work

$$\left(P_+^{(q)}\right)^{CT(P_q, n_q+1)}$$

- UD: Unlikely Down
$P_{UD}^{(q)}(k)$ probability to not fail *DOWN* during k time slots

$$\left(P_{UD}^{(q)}\right)^{(E^{(q)}(CT(P_q, n_q+1)))}$$

variant *: $T_{\text{data}} \rightarrow \left\lceil \frac{n_{active}}{n_{com}} \right\rceil T_{\text{data}}$

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Greedy heuristics for Tightly-Coupled

First possibility: Compute workload independently on each processor

- MCT: Minimum completion time

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- EMCT: Expected MCT
- LW: Likely to work

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- UD: Unlikely Down
  $P_{UD}^{(q)}(k)$ probability to not fail down during k time slots

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Greedy heuristics designed for TIGHTLY-COUPLED

- IP: Incremental: Probability of success

$$q_0 = \text{ArgMax}\left\{P^S(q)\right\}$$

- IE: Incremental: Expected completion time

$$q_0 = \text{ArgMin}\left\{T^q_{comm} + T^q_{comp}\right\}$$

- IY: Incremental: Expected yield

$$q_0 = \text{ArgMax}\left\{\frac{P^S(q)}{t + T^S(q)}\right\}$$

- IAY: Incremental: Expected apparent yield

$$q_0 = \text{ArgMax}\left\{\frac{P^S(q)}{T^S(q)}\right\}$$

Instances for INDEPENDENT

Parameter values for Markov simulations

parameter	values
p	20
m	5, 10, 20, 40
n_{com}	5, 10, 20
w_{min}	1, 2, 3, 4, 5, 6, 7, 8, 9, 10

- $0.9 \leq P_{x,x} \leq 0.99$
- $P_{x,y} = \frac{1}{2}(1 - P_{x,x})$
- $w_{min} \leq w_q \leq 10 * w_{min}$
- $T_{data} = w_{min}$ and $T_{prog} = 5 * w_{min}$

Comparison of average dfb (degradation from best, percentage)

Instances for Tightly-Coupled

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Results over all problem instances for INDEPENDENT

Algorithm	Average *dfb*	#wins
EMCT	4.77	80320
EMCT*	4.81	78947
MCT	5.35	73946
MCT*	5.46	70952
UD*	7.06	42578
UD	8.09	31120
LW*	11.15	28802
LW	12.74	19529
RANDOM1W	28.42	259
RANDOM2W	28.43	301
RANDOM4W	28.51	278
RANDOM3W	31.49	188
RANDOM3	44.01	87
RANDOM4	47.33	88
RANDOM1	47.44	36
RANDOM2	47.53	73
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Results with higher communication costs

Table: Results for contention-prone simulations

Communication times ×5

Algorithm	Average *dfb*
EMCT*	3.87
MCT*	4.10
UD*	5.23
EMCT	6.13
UD	6.42
MCT	7.70
LW*	8.76
LW	10.11

Communication times ×10

Algorithm	Average *dfb*
UD*	2.76
UD	3.20
EMCT*	3.66
LW*	4.02
MCT*	4.22
LW	4.46
EMCT	8.02
MCT	15.50

Results with higher communication costs

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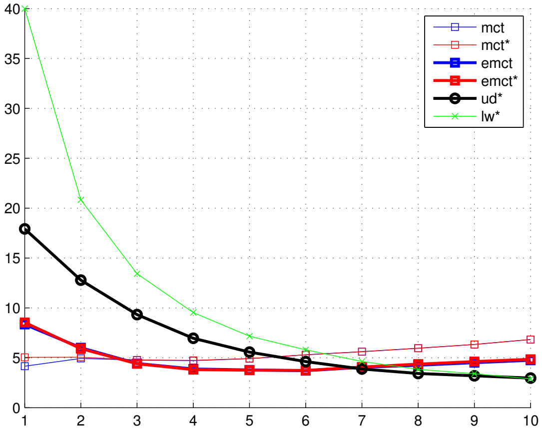
Influence of w_{min} for INDEPENDENT



Averaged dfb results vs. w_{min}

Results for best 10 heuristics for TIGHTLY-COUPLED

Algorithm	Average *dfb*	#wins	#good rate	*stdv*
Y-IE	33.06	17.76	69.71	40.33
P-IE	34.48	16.66	68.02	41.34
E-IAY	35.26	24.83	71.37	55.80
E-IY	45.44	20.38	64.22	69.38
IE	51.40	10.81	69.71	59.83
IAY	59.32	8.46	70.12	93.12
IY	74.69	6.02	63.08	106.61
E-IP	77.08	15.41	49.51	103.19
E-EMCT*	92.23	8.63	43.14	164.71
E-LW*	92.80	12.65	44.65	123.30

General results for the best 10 heuristics

Results for best 10 heuristics for TIGHTLY-COUPLED

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General results for the best 10 heuristics

Results for best 10 heuristics with 5 tasks

Algorithm	Average *dfb*	#wins	#good rate	*stdv*
Y-IE	32.30	17.69	70.44	40.21
P-IE	33.83	16.36	68.67	41.19
E-IAY	35.34	23.74	70.99	56.98
E-IY	44.10	20.48	64.86	67.89
IE	47.59	11.33	70.44	50.82
IAY	57.57	9.18	69.66	96.75
IY	71.02	6.63	63.67	108.21
E-IP	73.82	15.68	50.17	98.56
E-EMCT*	87.23	9.32	44.80	162.71
E-LW	90.68	12.82	45.07	119.19

Results with 5 tasks for the best 10 heuristics

Results for best 10 heuristics with 5 tasks

Algorithm	Average *dfb*	#wins	#good rate	*stdv*
Y-IE	32.30	17.69	70.44	40.21
P-IE	33.83	16.36	68.67	41.19
E-IAY	35.34	23.74	70.99	56.98
E-IY	44.10	20.48	64.86	67.89
IE	47.59	11.33	70.44	50.82
IAY	57.57	9.18	69.66	96.75
IY	71.02	6.63	63.67	108.21
E-IP	73.82	15.68	50.17	98.56
E-EMCT*	87.23	9.32	44.80	162.71
E-LW	90.68	12.82	45.07	119.19

Results with 5 tasks for the best 10 heuristics

Results for best 10 heuristics with 10 tasks

Algorithm	Average *dfb*	#wins	#good rate	*stdv*
E-IAY	34.83	31.20	73.60	48.23
Y-IE	37.48	18.20	65.40	40.98
P-IE	38.31	18.40	64.20	42.21
E-IY	53.32	19.80	60.40	77.59
IAY	69.62	4.20	72.80	67.90
IE	73.74	7.80	65.40	97.15
E-IP	96.20	13.80	45.60	127.04
IY	96.29	2.40	59.60	96.62
E-LW	105.30	11.60	42.20	145.12
E-EMCT*	121.64	4.60	33.40	175.99

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Results with 10 tasks for the best 10 heuristics

Conclusion for this study

Theoretical results

- Realistic models
- Complexity results for off-line problems
- Probability computations

Simulation results

- Large set of efficient heuristics
- Simulations

Perspectives

- Real life traces are not Markovian (Weibull or Pareto distributions)
- The yield is not used in INDEPENDENT heuristics

Outline

1. Introduction
2. Scheduling filtering applications (overview)
3. Reliability of pipelined real-time systems (overview)
4. Scheduling on volatile ressources
5. Conclusion and perspectives

Conclusion

Application models:

- Filtering tasks
- Pipelined real-time systems
- Iterative applications

Failure models:

- Transient failures
- Desktop grids

Results:

- Complete sets of complexity results
- Some approximation results and ILP formulations
- Heuristics and simulations

Perspectives

- More approximation results
- Design more heuristics for some models
- Consider additional criteria (power consumption,...)
- Study other methods to increase reliability (checkpointing, migration)
- More realistic probability distribution for failures

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