Computing the throughput of probabilistic and replicated streaming applications

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Roma Working Group

Outline

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Problem description

- We are given
  - (i) a streaming application, dependence graph = linear chain;
  - (ii) a one-to-many mapping of appliction onto heterogeneous platform;
  - (iii) a set of I.I.D. (Independent and Identically-Distributed) variables to model computation/communication time in the mapping.
- How can we compute the throughput of the application, i.e., the rate at which data sets can be processed?
- Two execution models: **Strict** and **Overlap**

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- No replication, i.e., one-to-one mapping: throughput dictated by critical hardware resource
- With replication, deterministic case: surprisingly difficult! (remember previous work, cases with no critical resources)
- Contributions:
  - (i) general method (exponential cost) to compute throughput with I.I.E. exponential laws;
  - (ii) bounds for arbitrary I.I.E. and N.B.U.E. (New Better than Used in Expectation) variables: between exponential and deterministic values;
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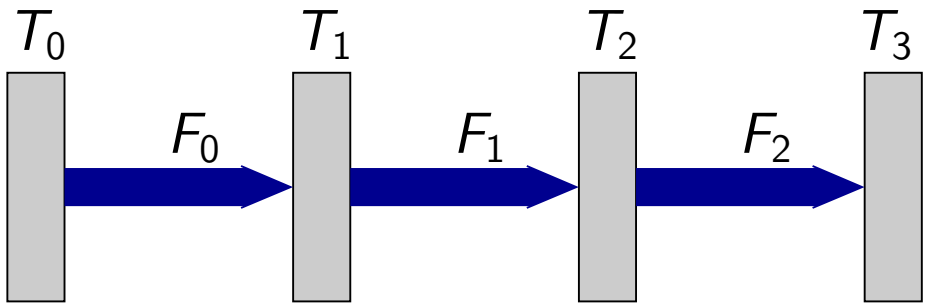

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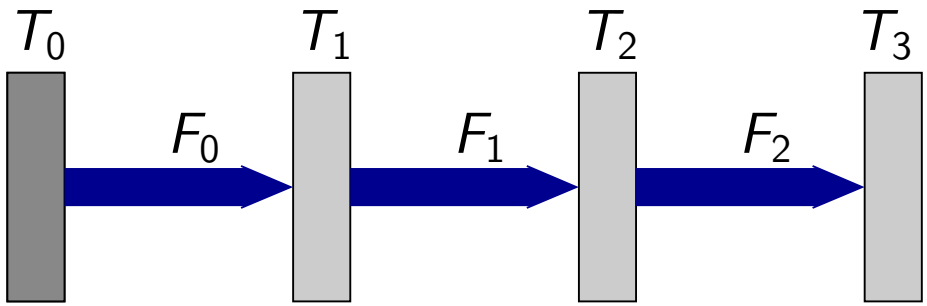

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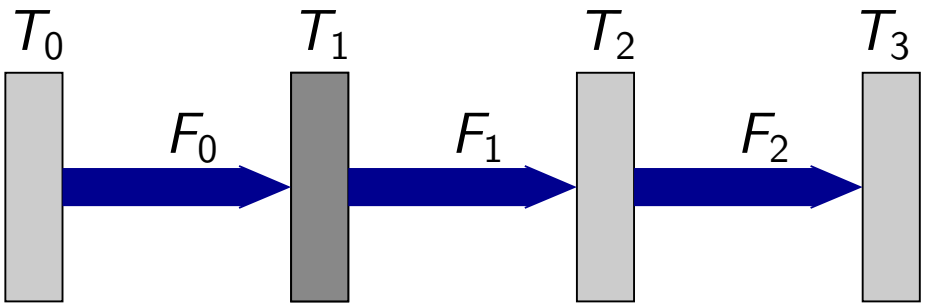

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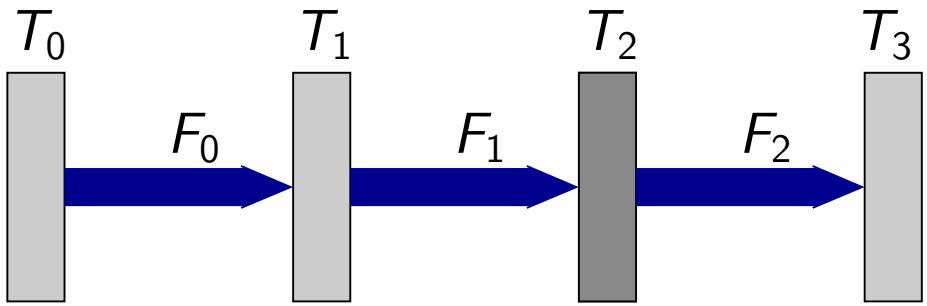

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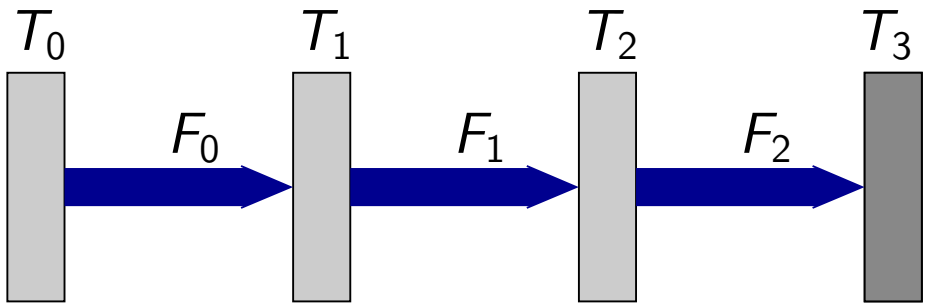

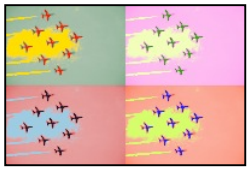

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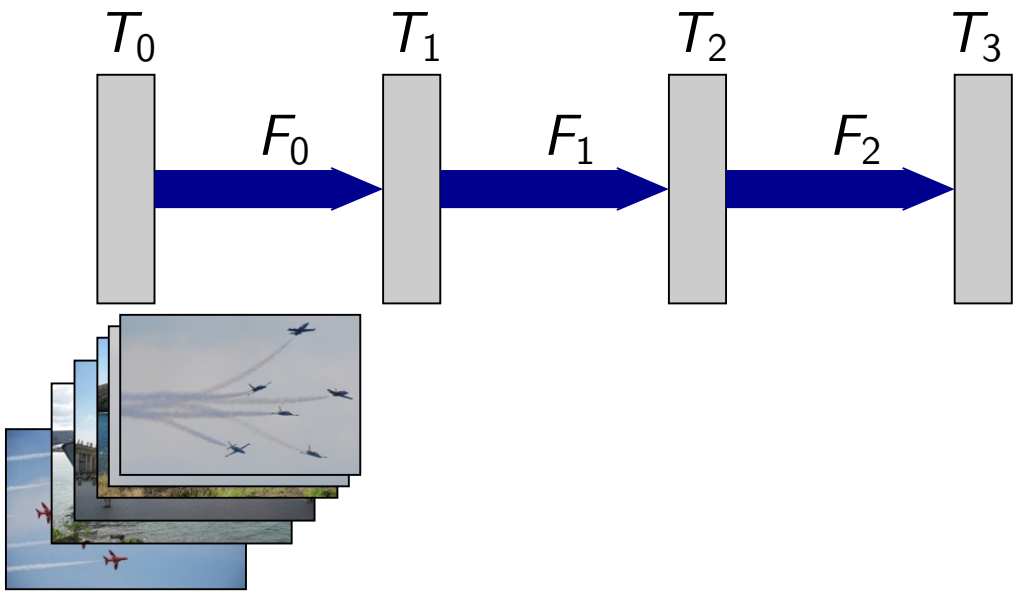

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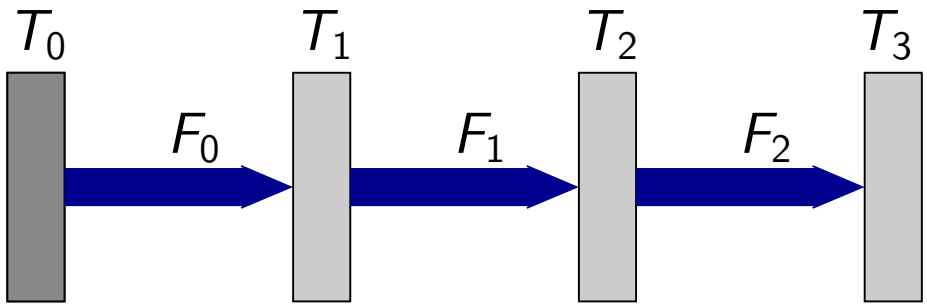

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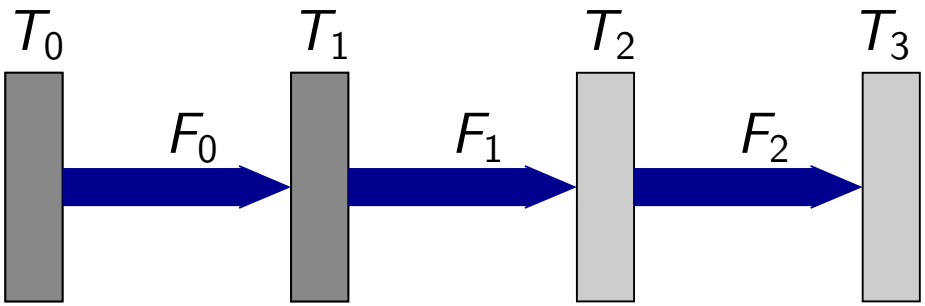

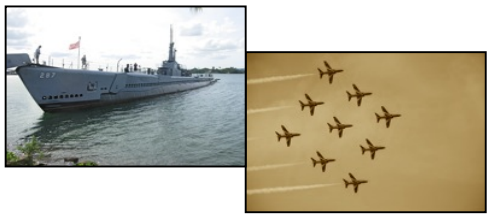

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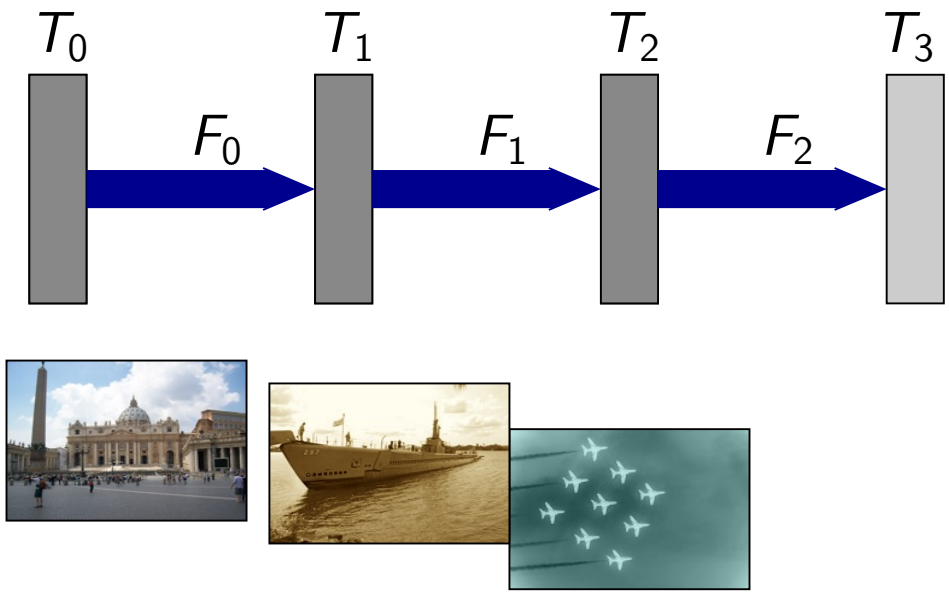

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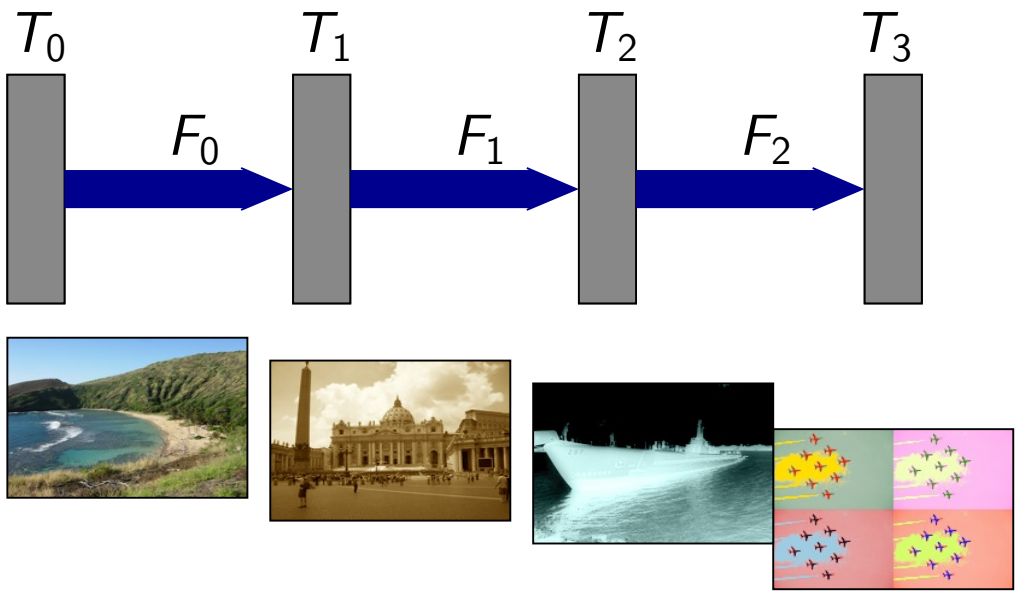

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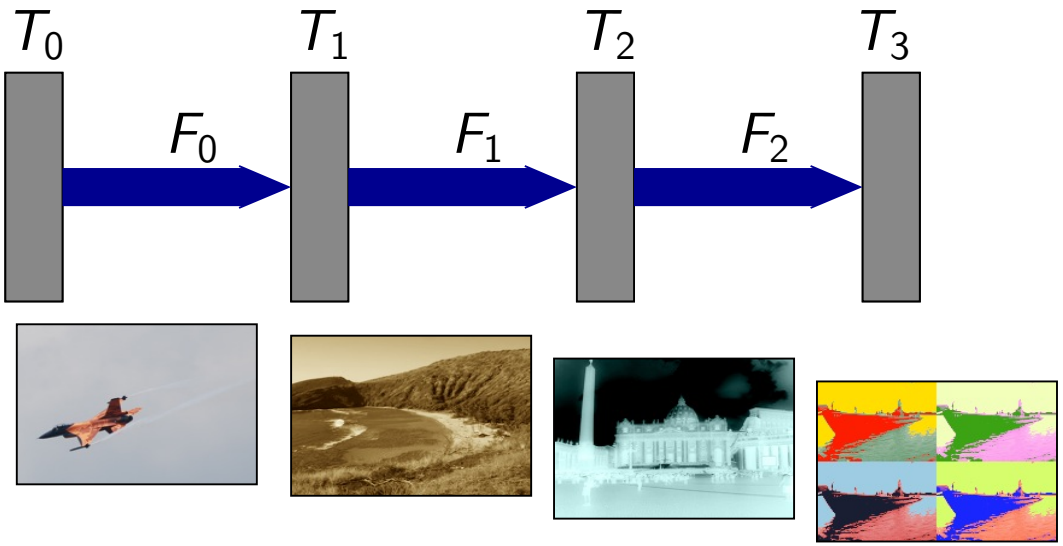

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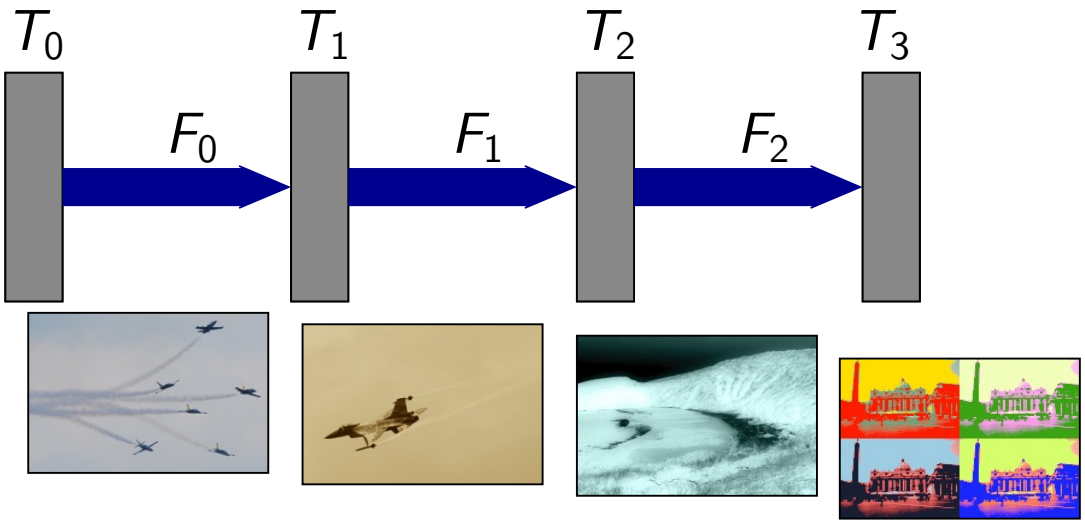

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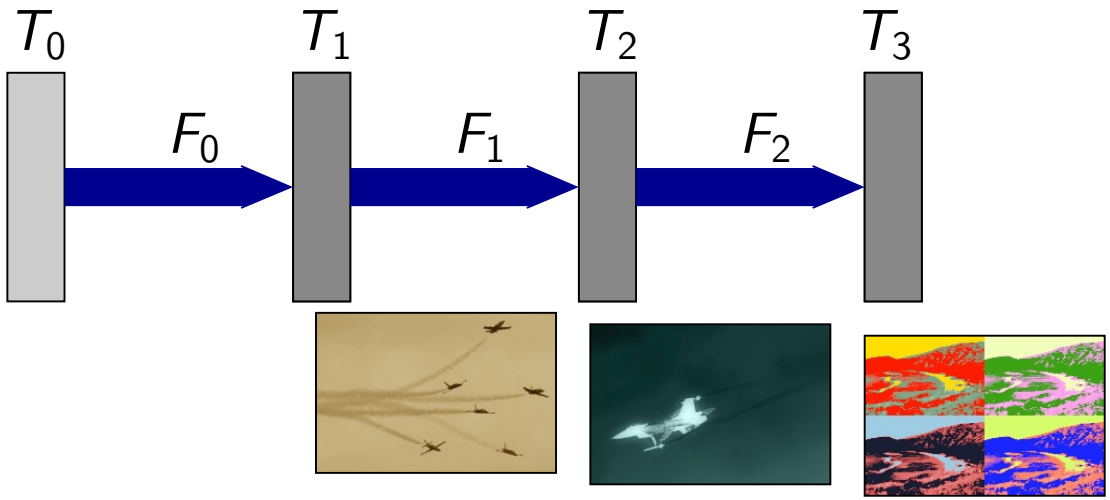

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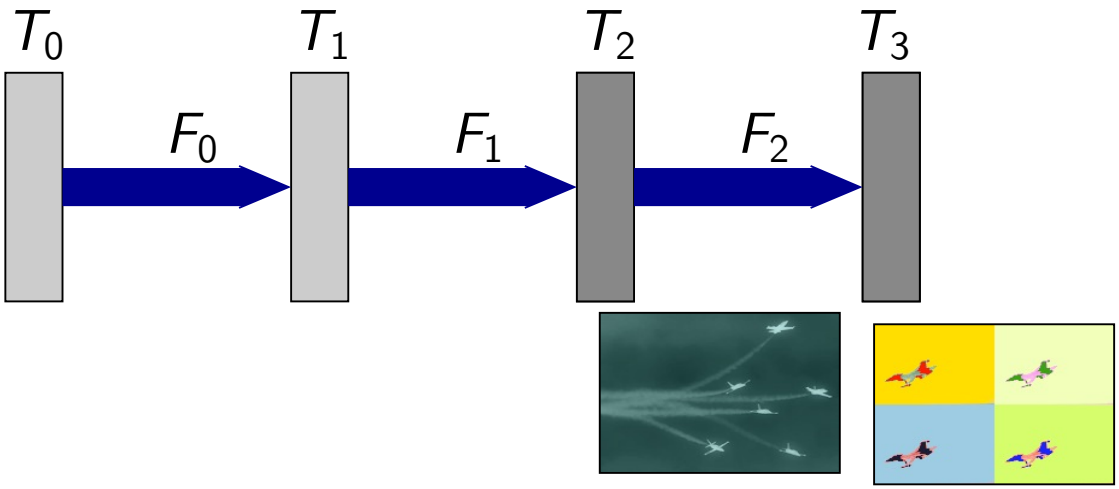

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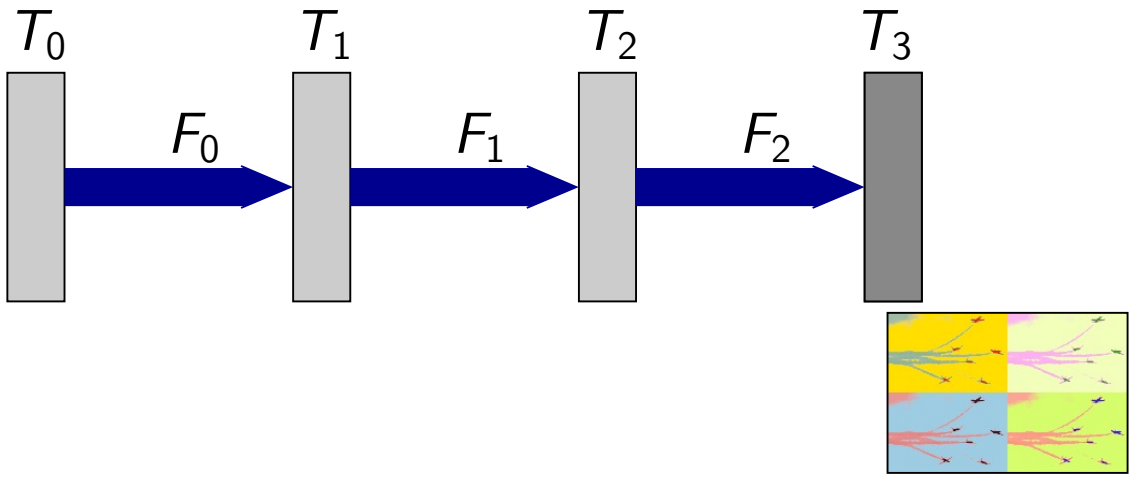

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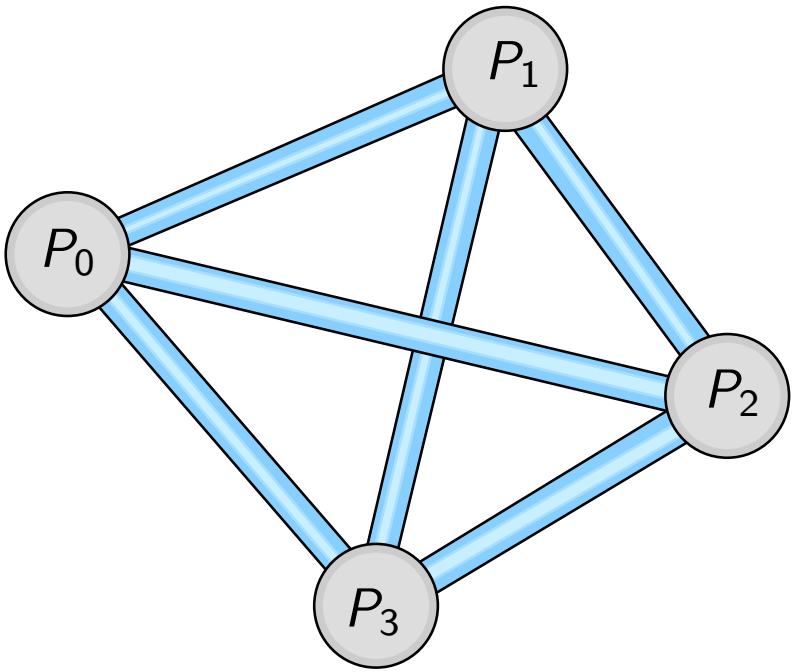
- Heterogeneous processors and communication links



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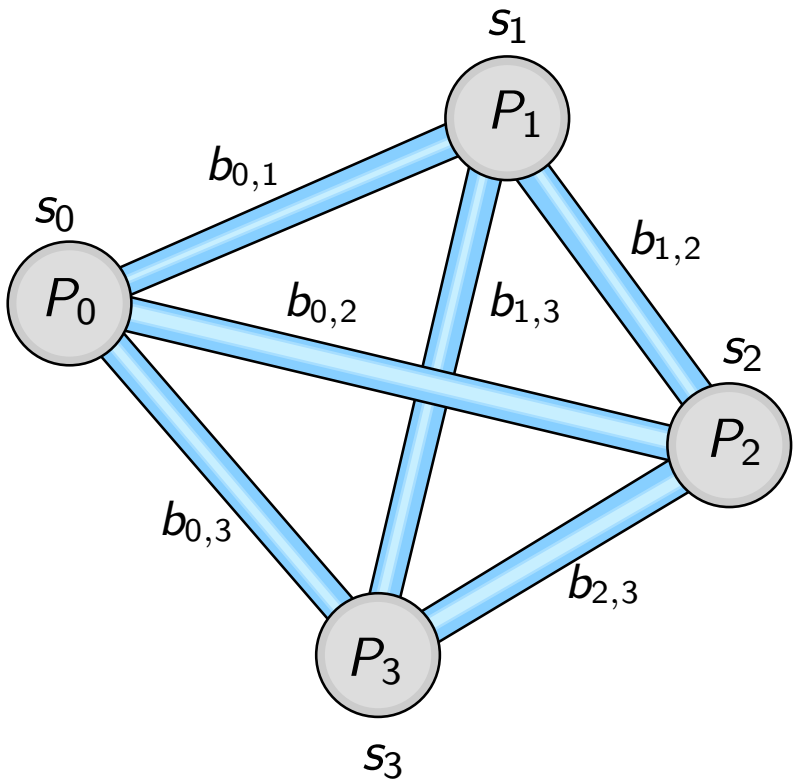
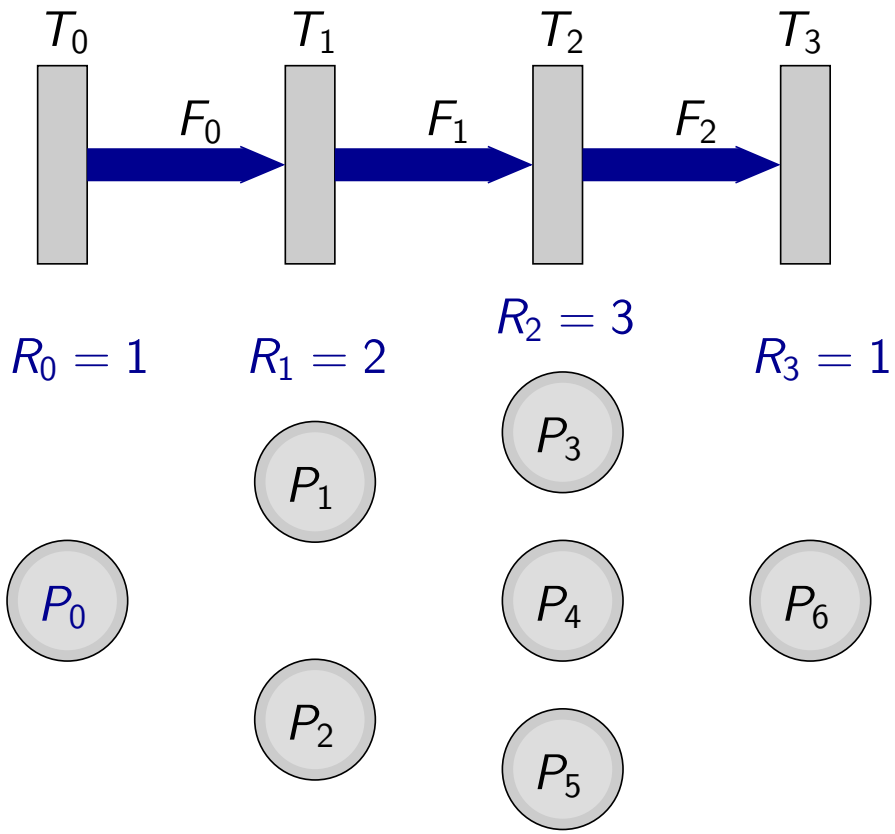


Mapping

- A processor processes at most 1 task
- A task is mapped on possibly many processors
- Replication count of T_i: R_i
- Round-Robin distribution of each task



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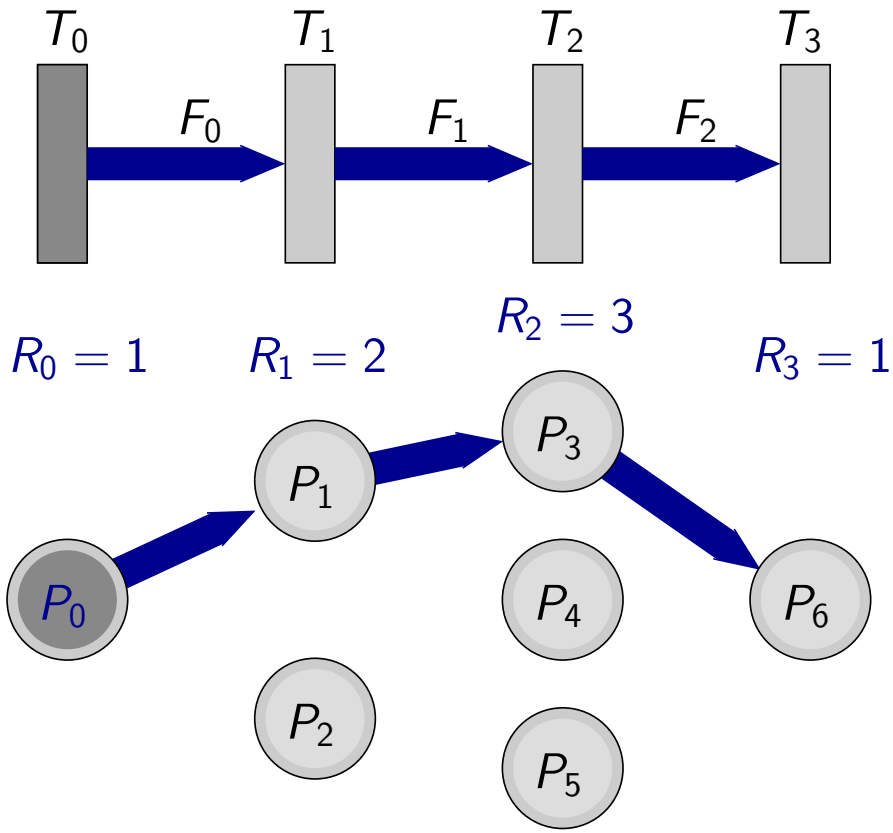

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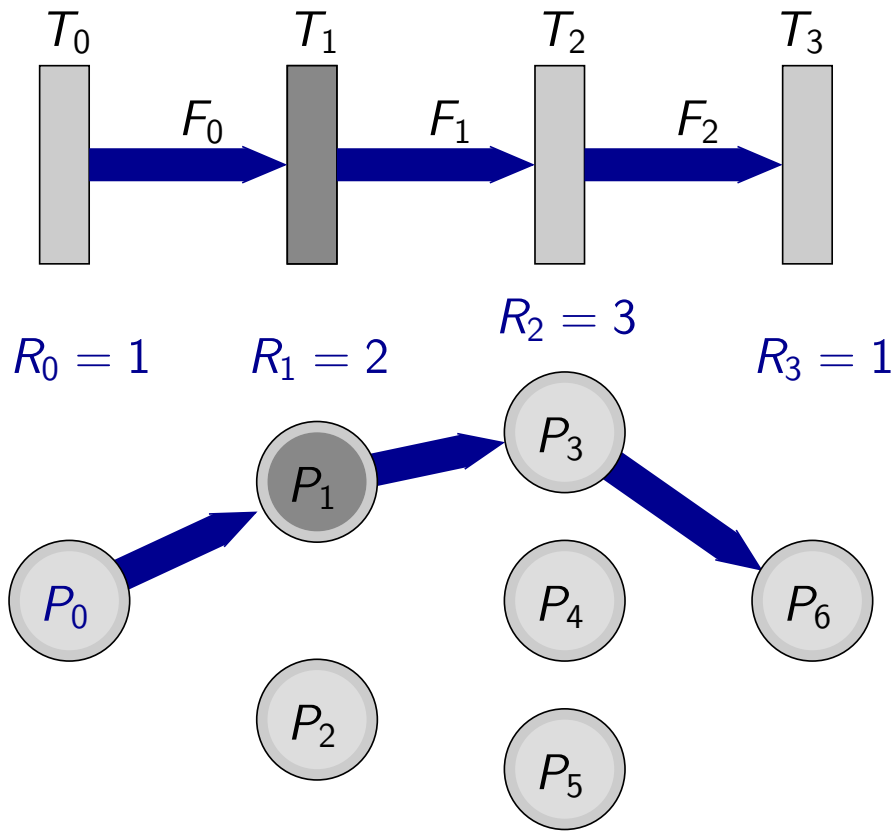

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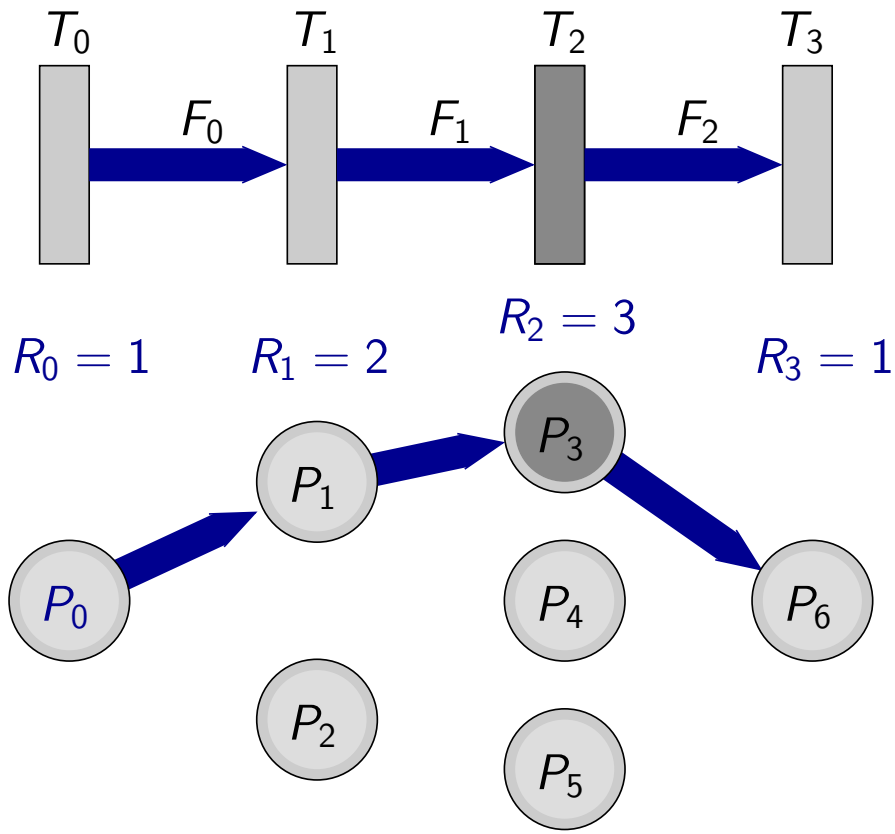

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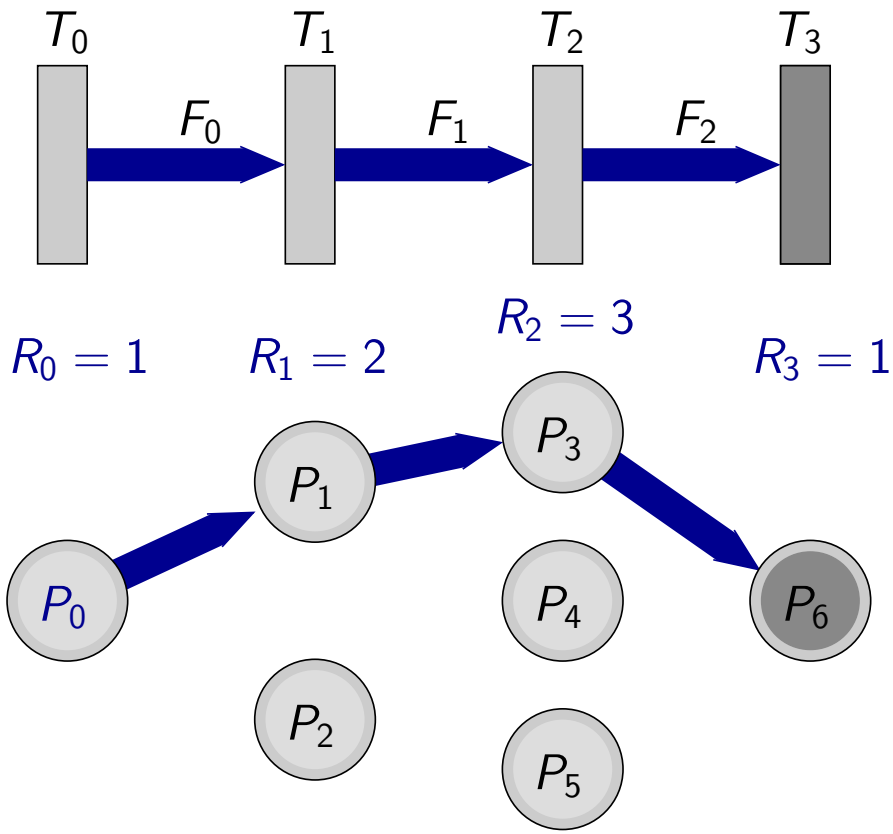

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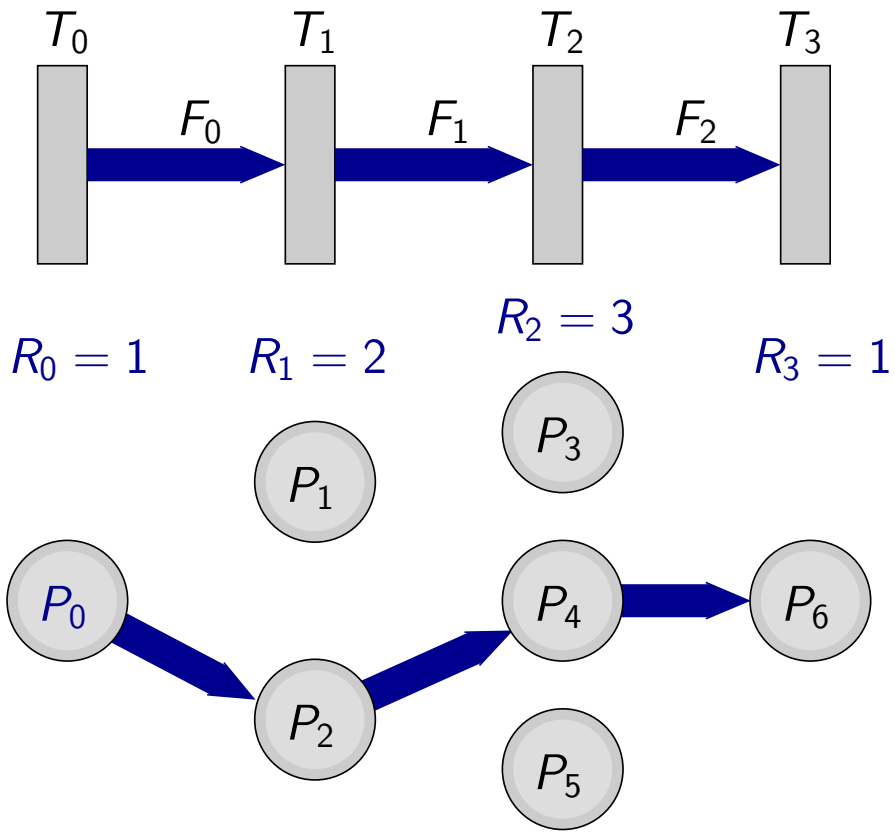

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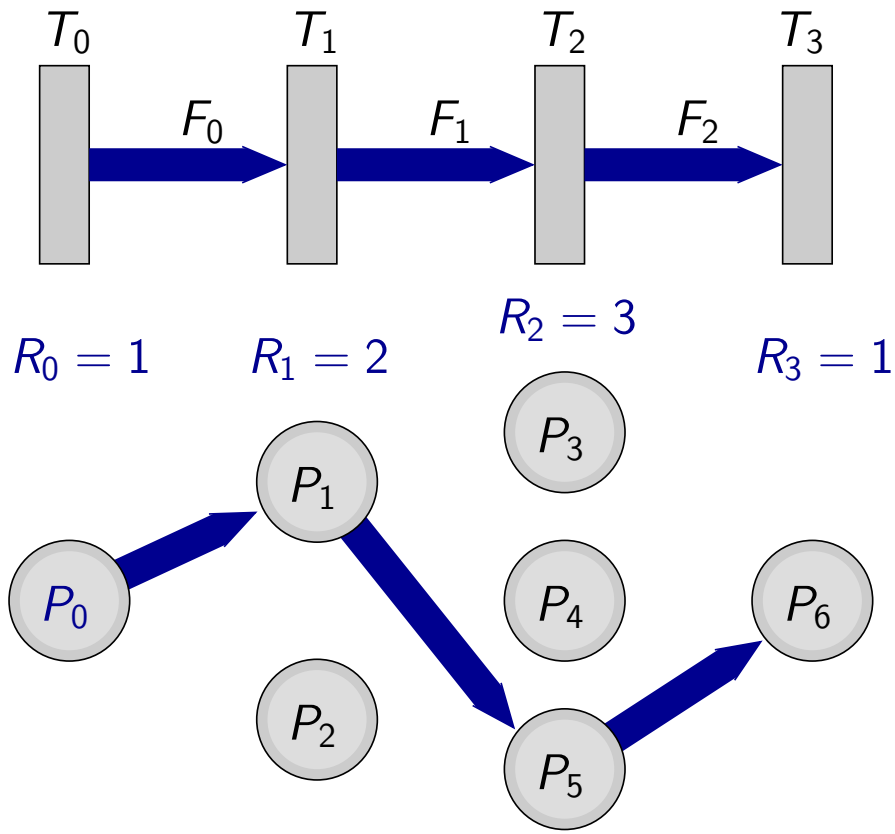

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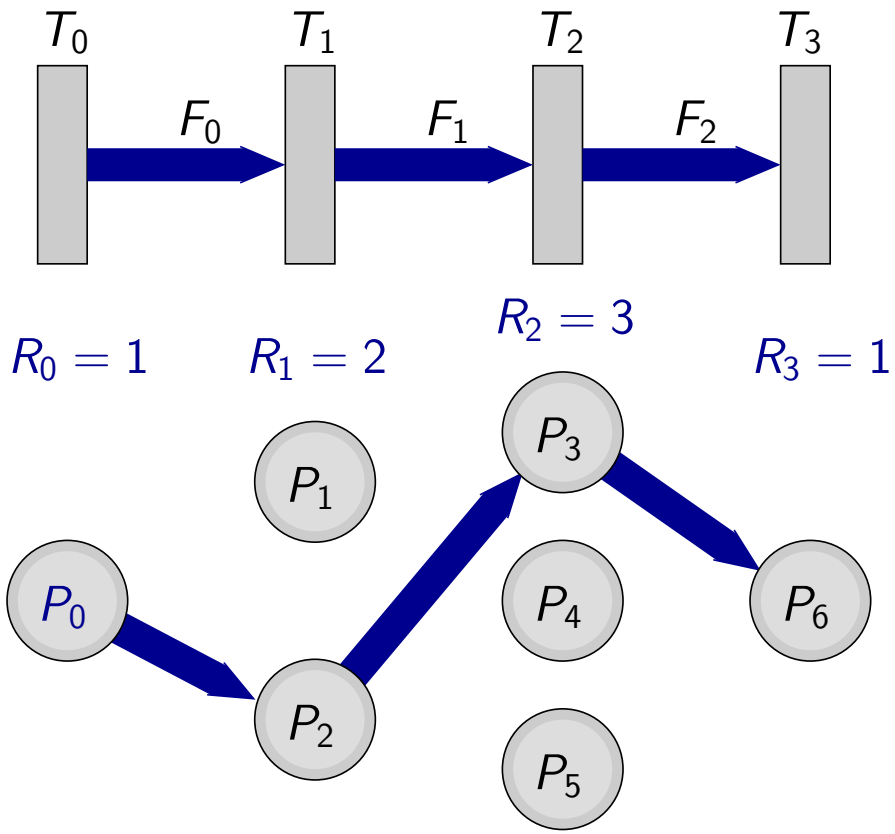

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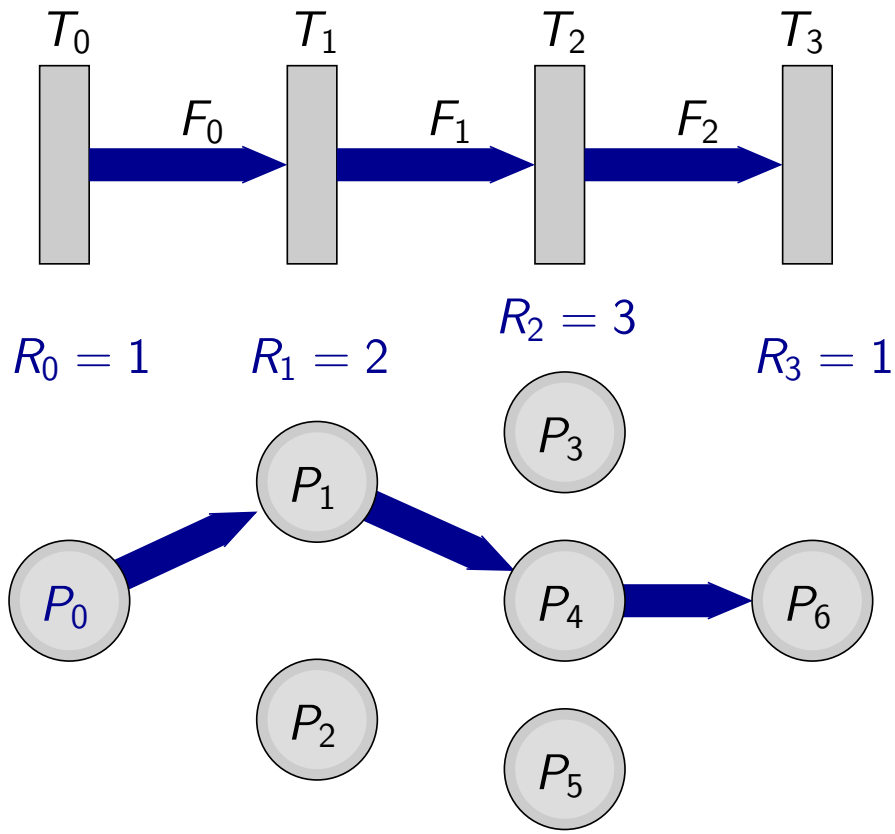

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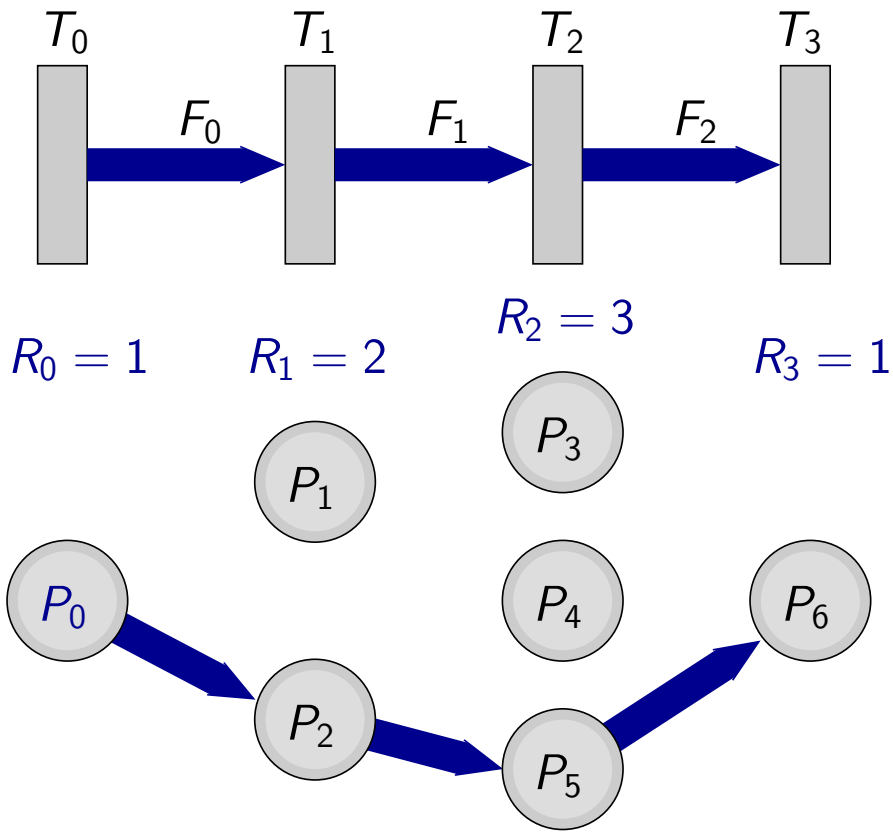

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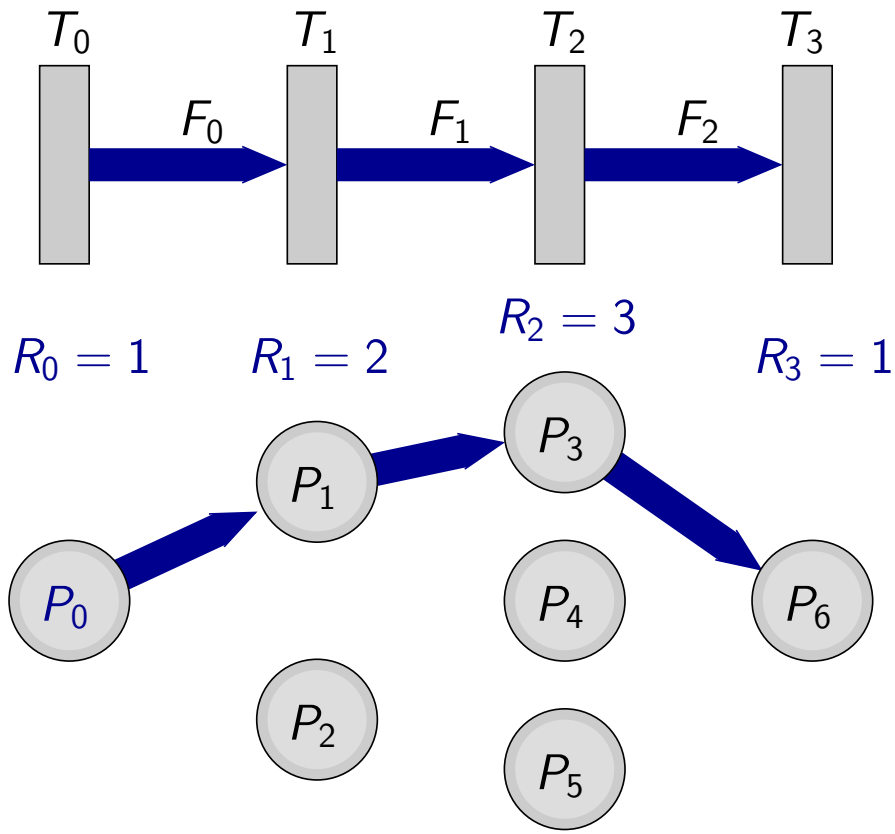

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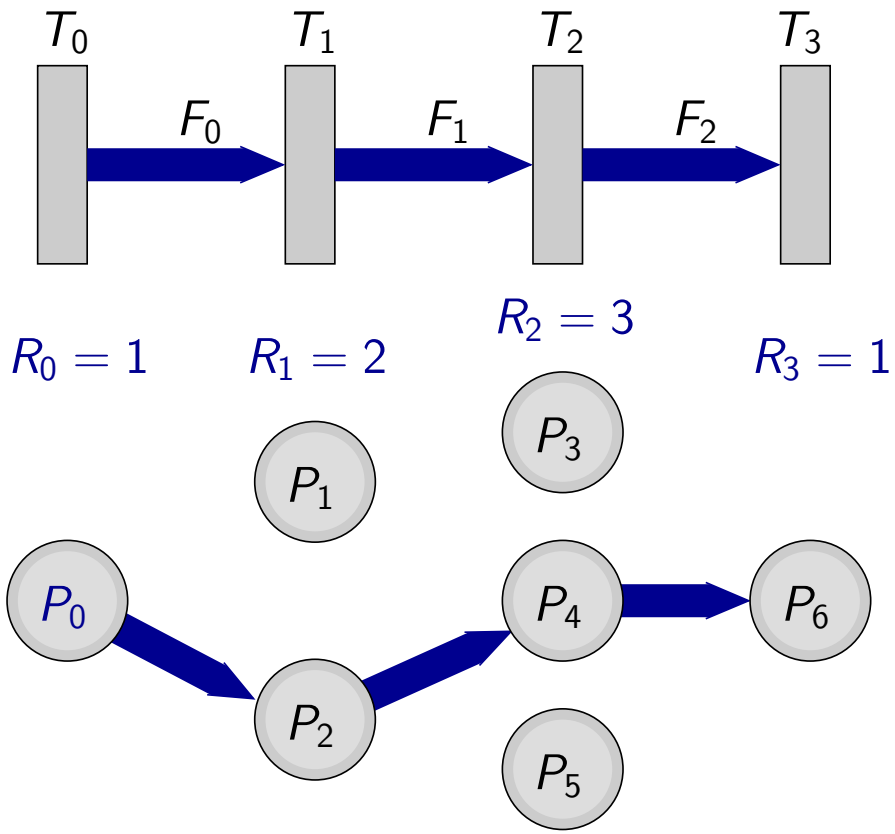

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Input data	Path in the system
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1	$P_0 \rightarrow P_2 \rightarrow P_4 \rightarrow P_6$
2	$P_0 \rightarrow P_1 \rightarrow P_5 \rightarrow P_6$
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Theorem

Assume that stage T_i is mapped onto R_i distinct processors. Then the number of paths is equal to $R = \mathrm{lcm}(R_0, \ldots, R_{n-1})$.

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Random variables

- $X_i(n)$: time required by P_i to process its n-th data set
- $Y_{i,j}(n)$: time required by P_i to send its n-th file to P_j
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- Exponential variables
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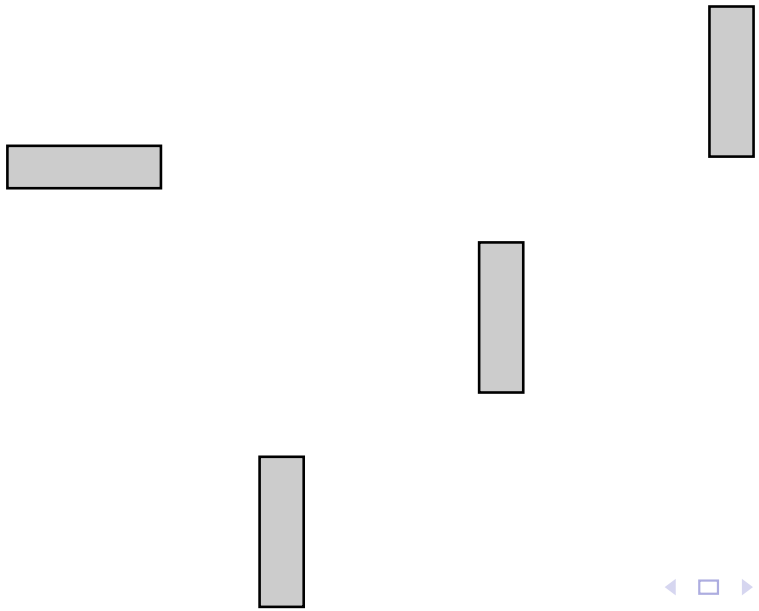
- Some places
- Connections between transitions and places. . . and between places and transitions
- Some tokens allowing transitions to be fired
- Time between the consumption of the input tokens and the creation of the output tokens



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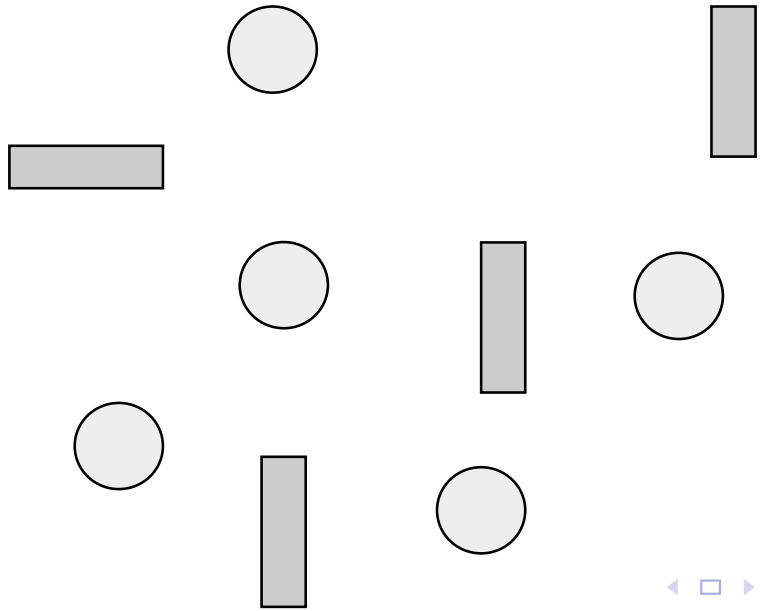

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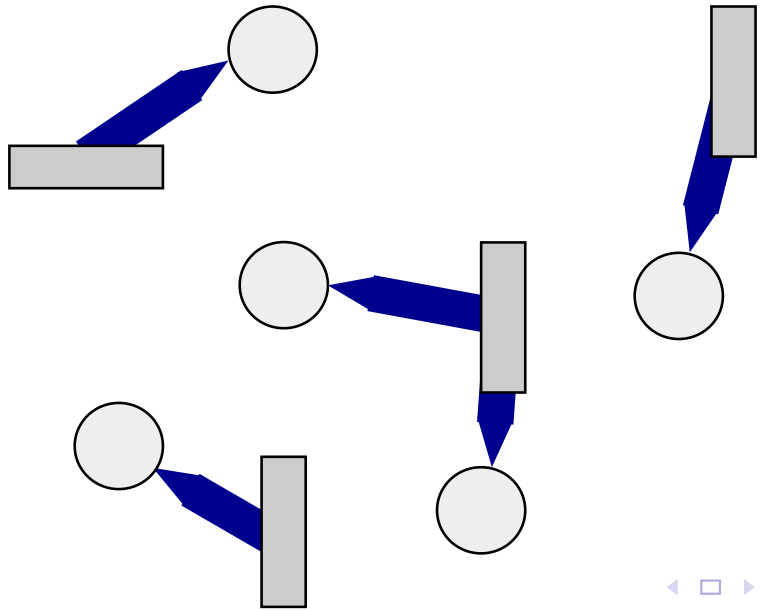

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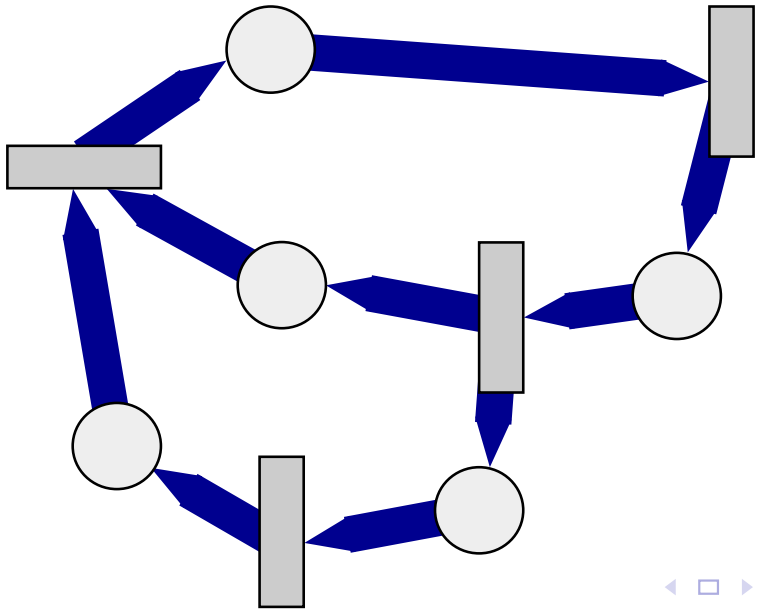

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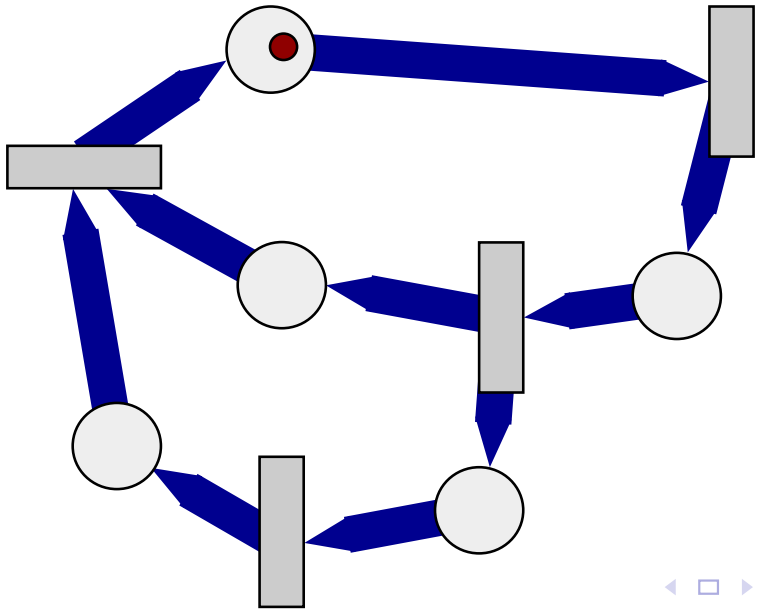

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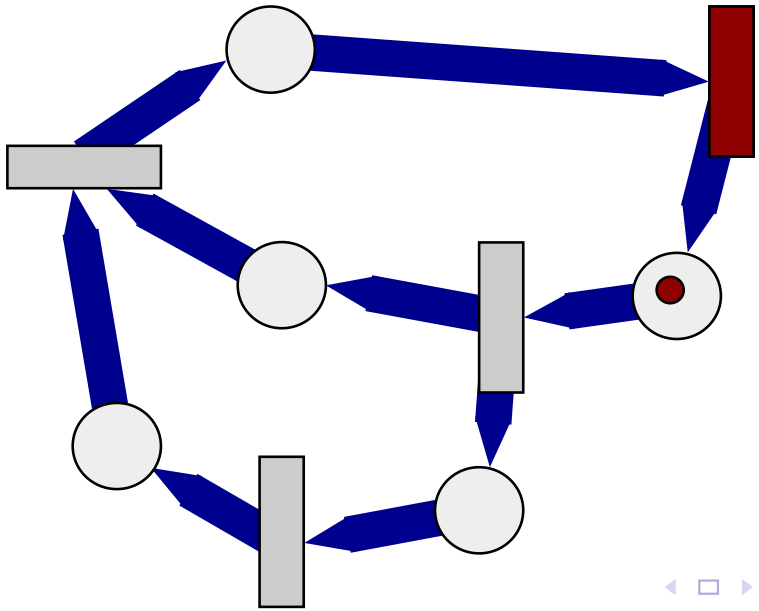

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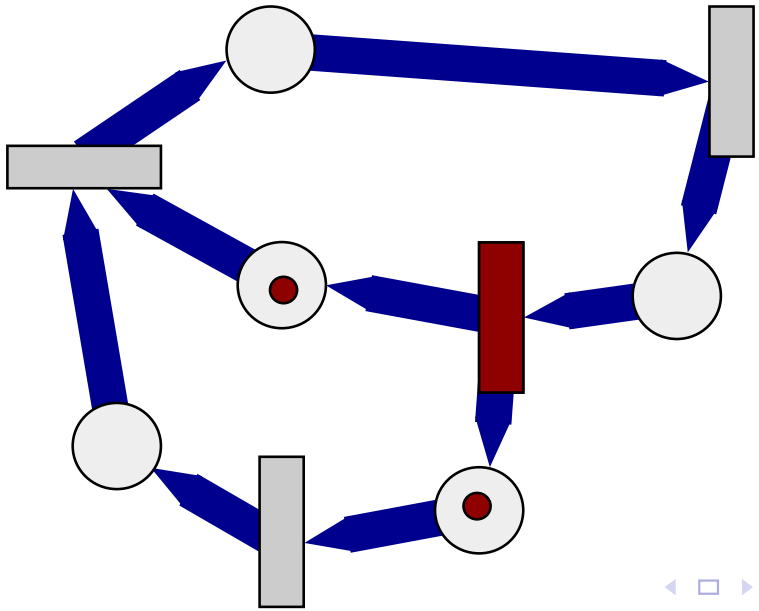

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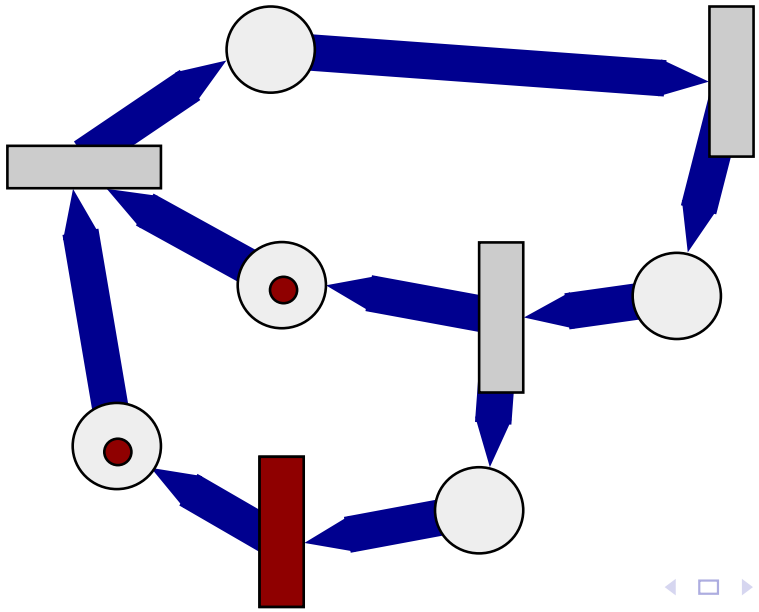

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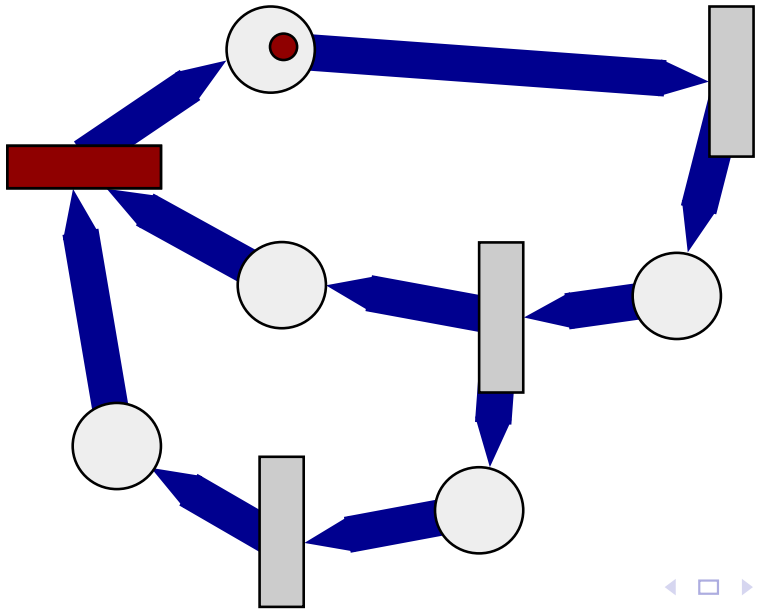

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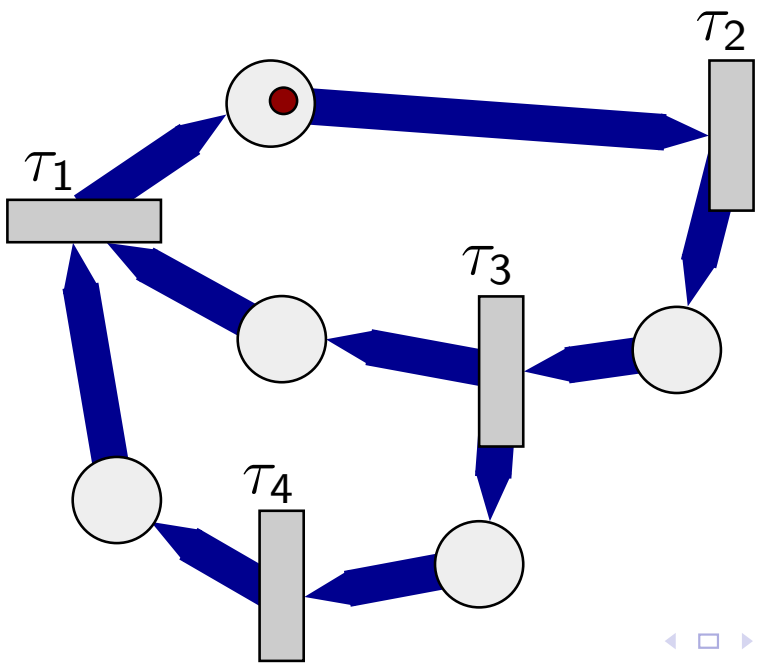
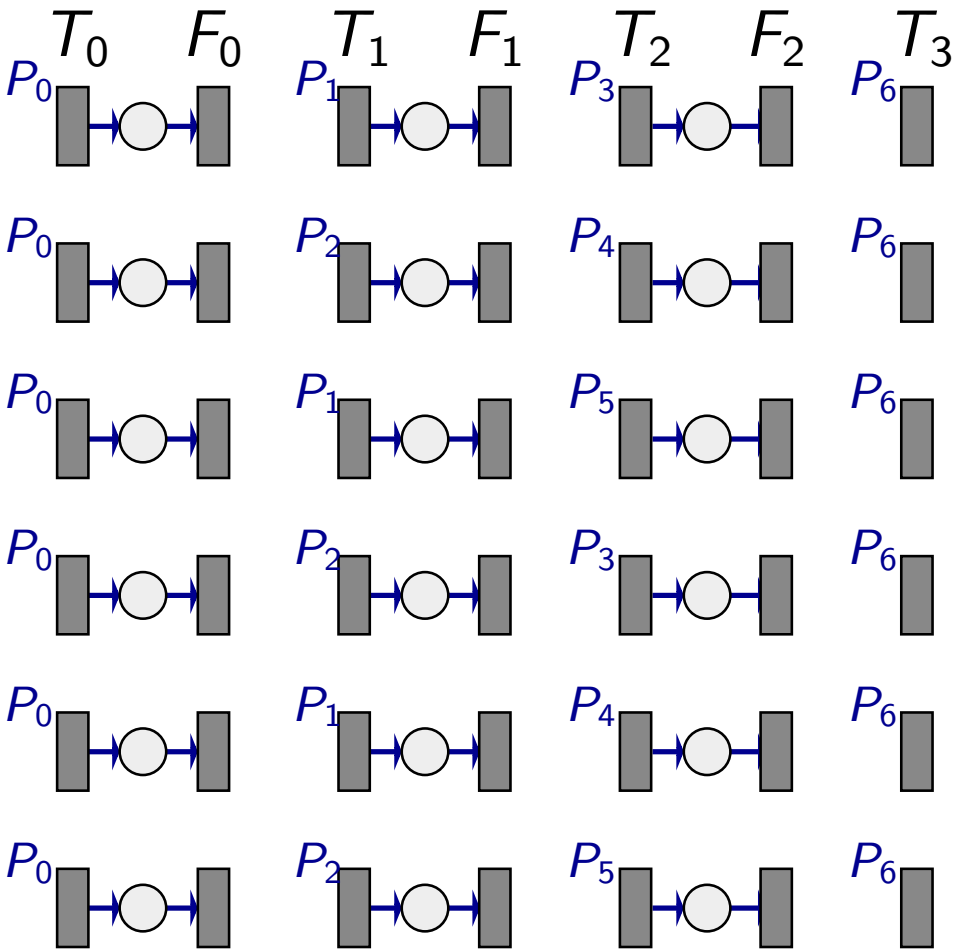


Timed Event Graph model

- Transitions: communications and computations
- Places: dependences between two successive operations
- Each path followed by the input data must be fully developed in the TEG
- Exponential size of the TEG

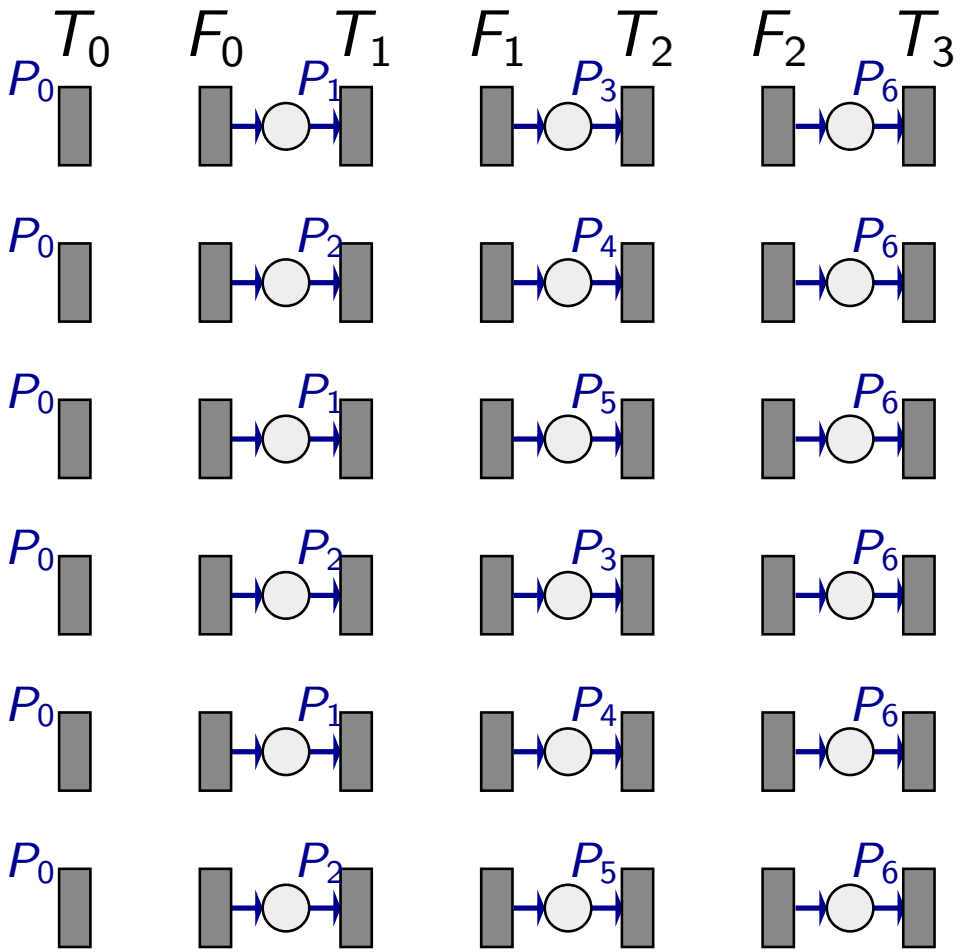
Overlap model

A communication cannot begin before the end of the computation



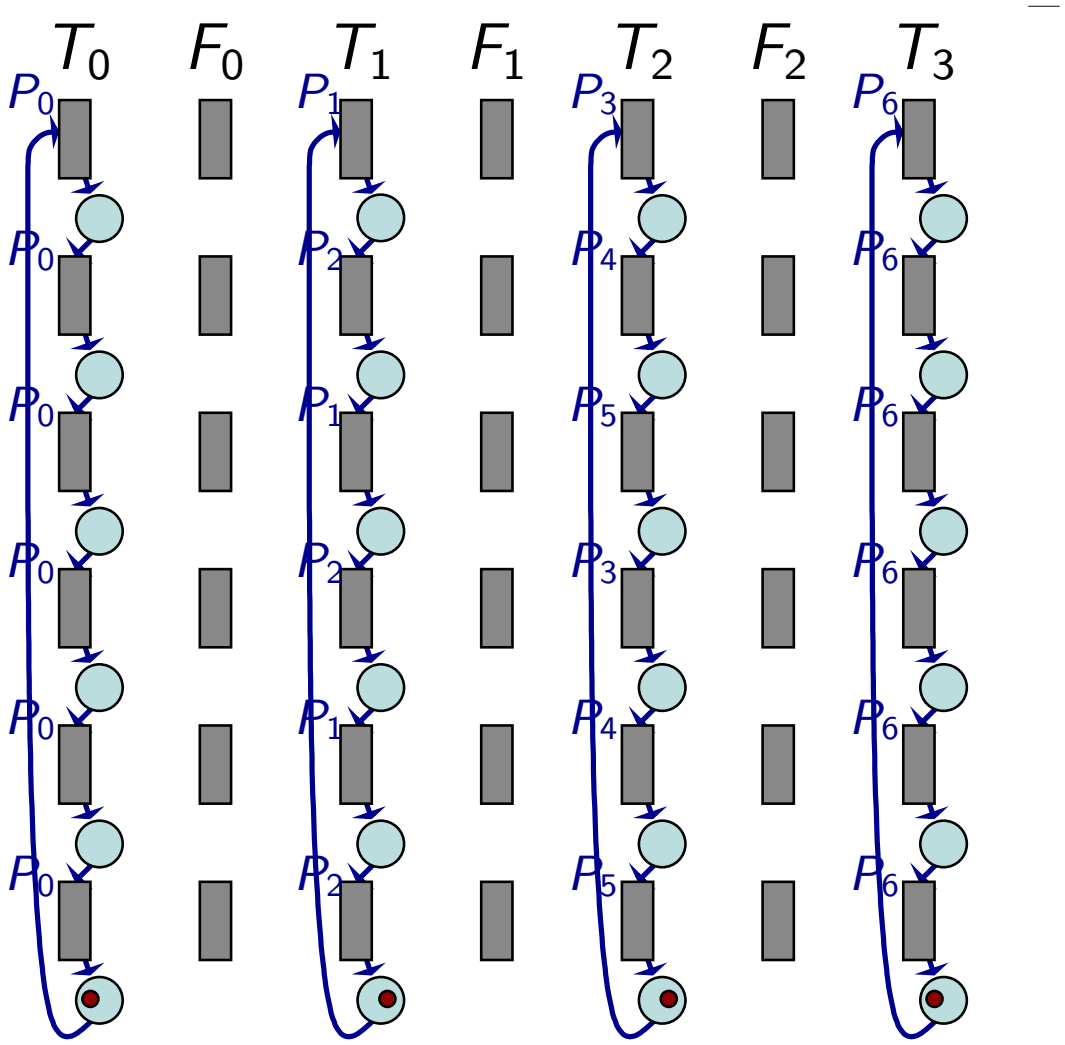
Overlap model

A computation cannot begin before the end of the communication



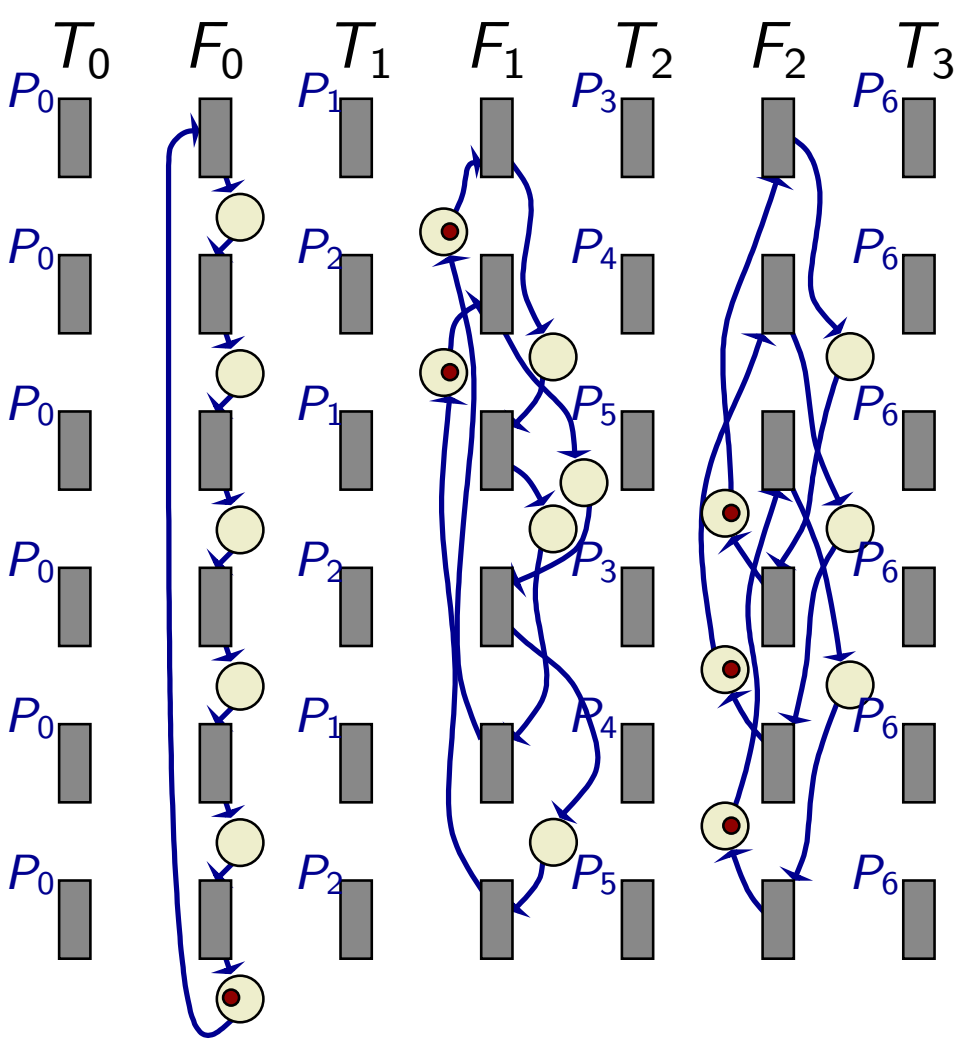
Overlap model

Dependences due to the round-robin distribution of computations



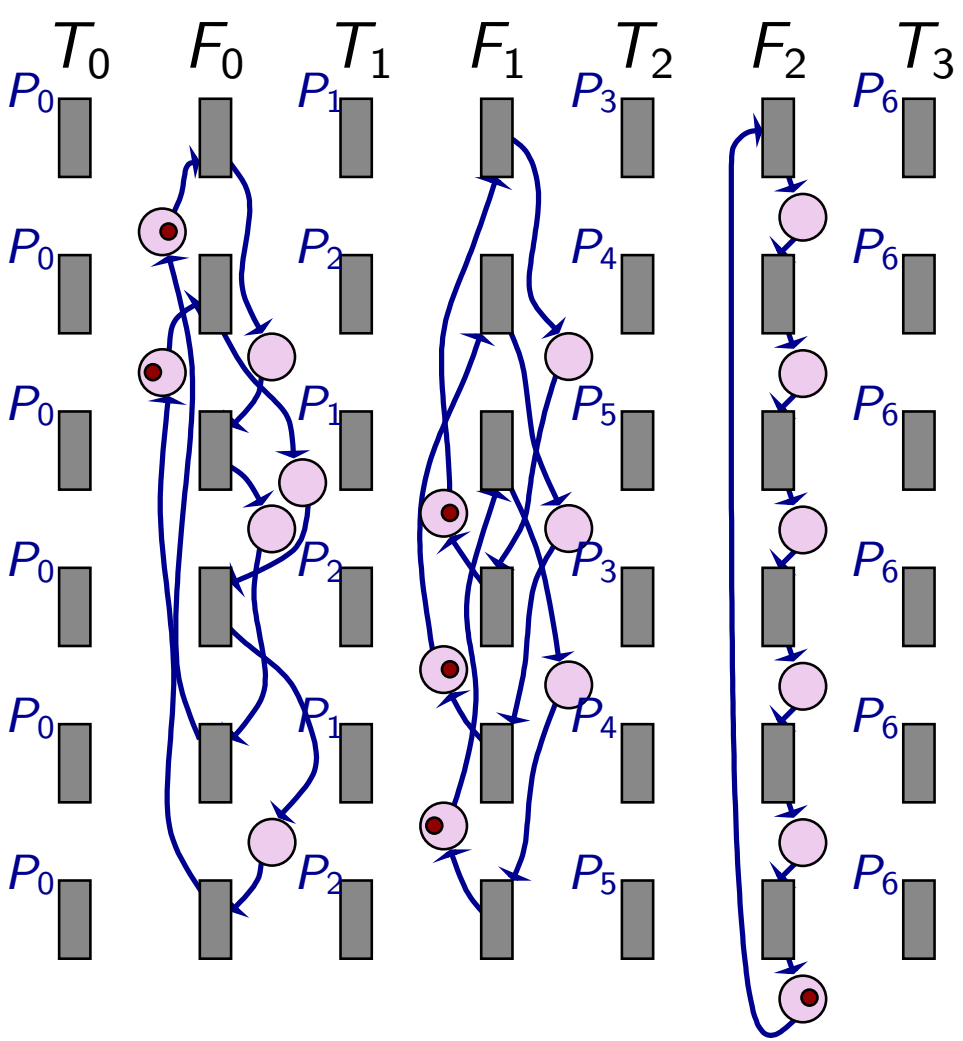
Overlap model

Dependences due to the round-robin distribution of outgoing communications



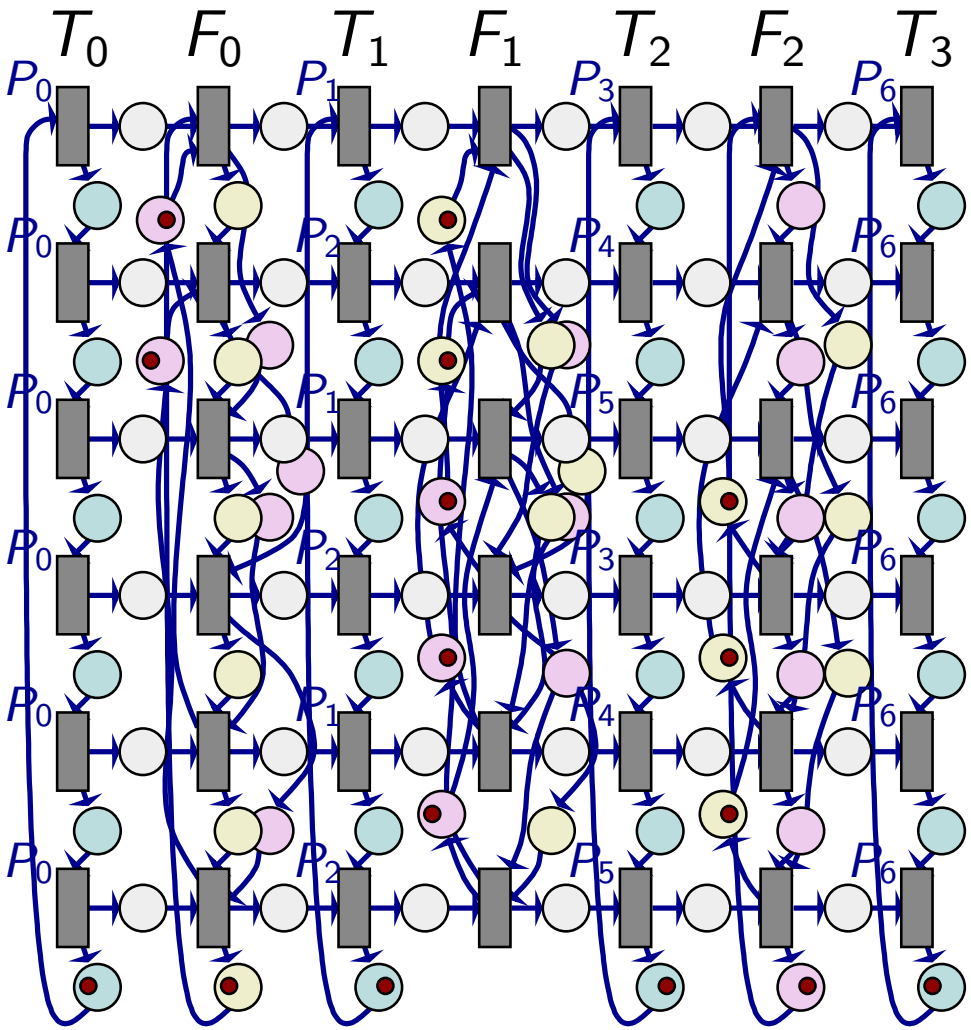
Overlap model

Dependences due to the round-robin distribution of incoming communications



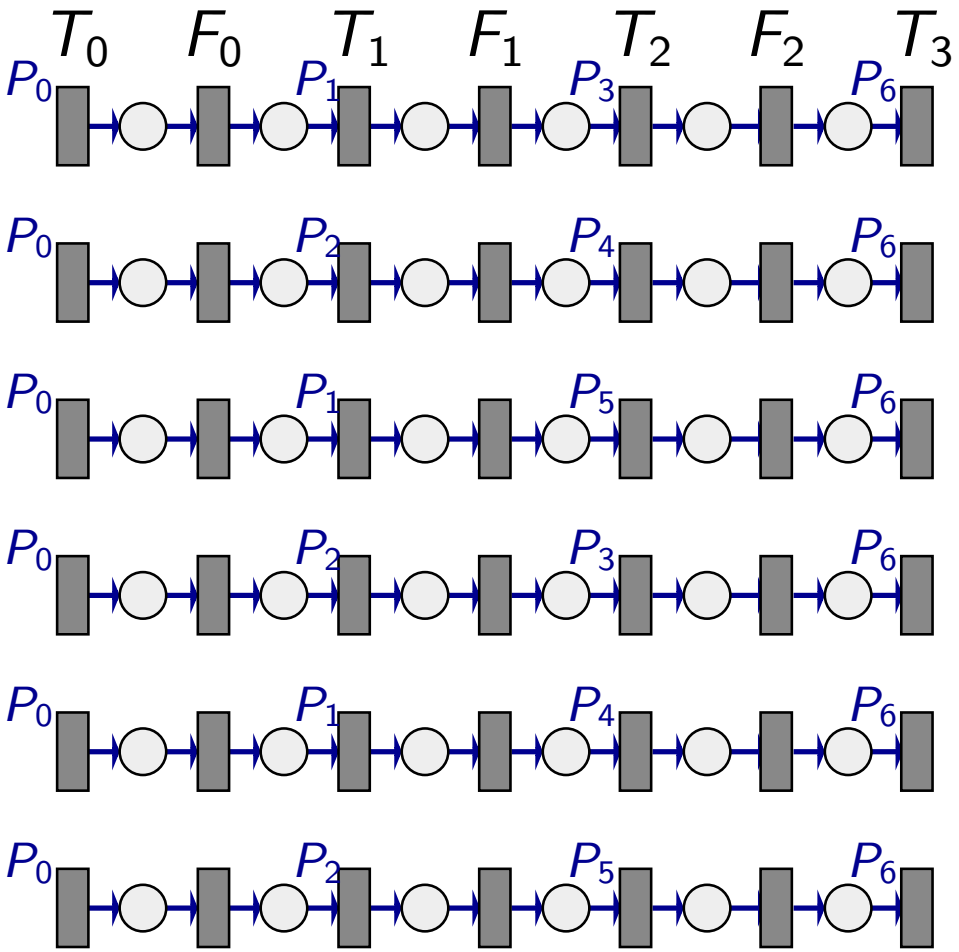
Overlap model

All dependences!



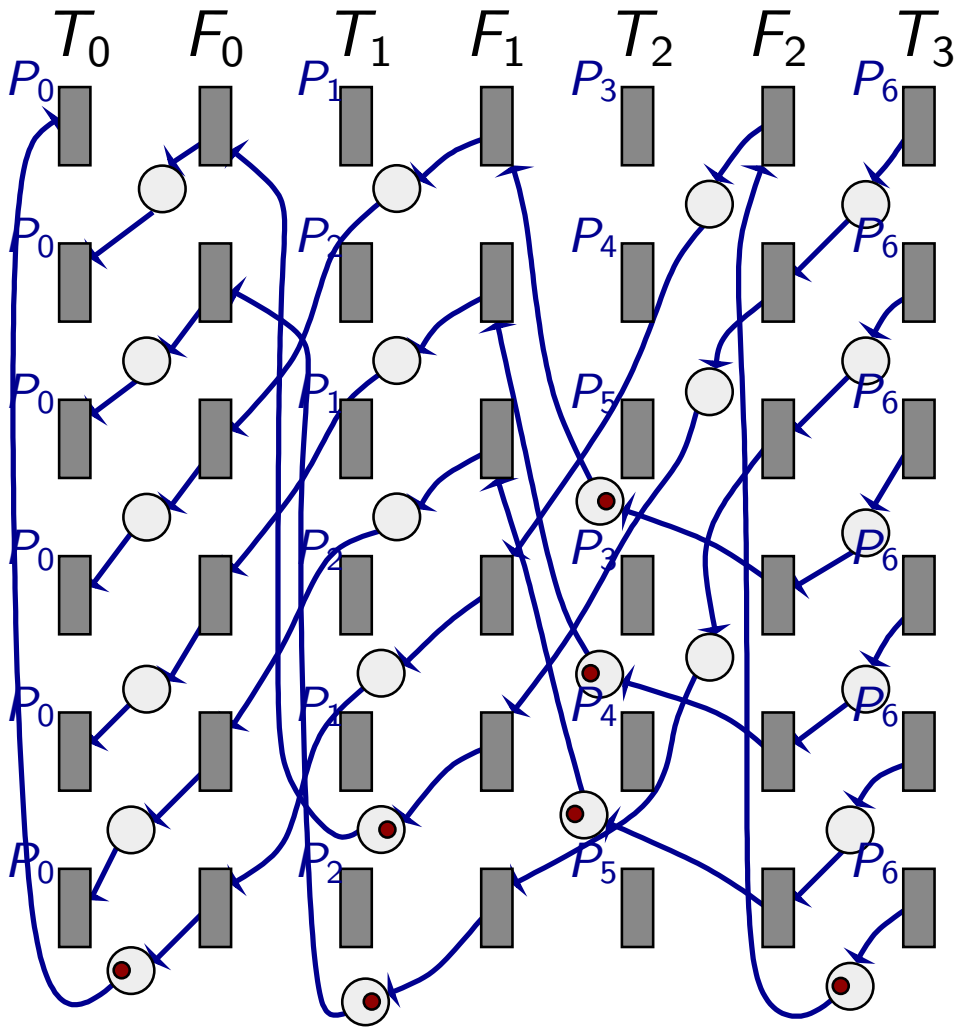
Strict model

Dependences between communications and computations



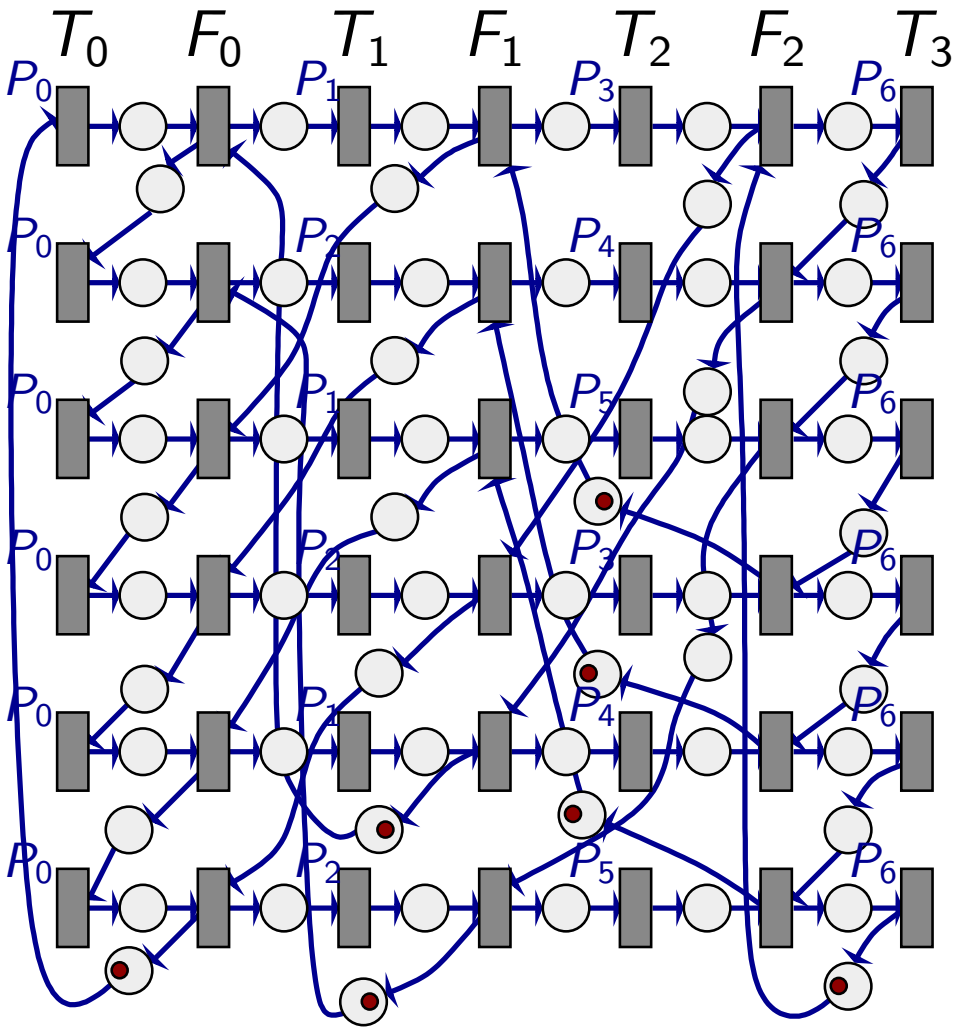
Strict model

Dependences due to the **Strict** model



Strict model

All dependences!



Outline

Computing the throughput – deterministic case

- Equivalent to find critical cycles
- $\mathcal{C}$ is a cycle of the TEG
- $\mathcal{L}(\mathcal{C})$ is its length (total time of transitions)
- $t(\mathcal{C})$ is the total number of tokens in places traversed by $\mathcal{C}$
- A critical cycle achieves the largest ratio $\max_{\mathcal{C}\text{cycle}} \frac{\mathcal{L}(\mathcal{C})}{t(\mathcal{C})}$
- This ratio gives the period $\mathcal{P}$ of the system
- Can be computed in time $\mathcal{O}(M^3R^3)$
  $(R = \operatorname{lcm}(R_0, \ldots, R_{M-1}))$

Computing the throughput – deterministic case

(previous result)

- **Strict** model: the TEG has an exponential size!
- **Overlap** model:

Theorem

Consider a pipeline of M stages $T_0, \ldots, T_{M-1}$, such that stage T_i is mapped onto R_i distinct processors. Then the average throughput of this system can be computed in time $\mathcal{O}\left(\sum_{i=0}^{M-2}\left((R_i R_{i+1})^3\right)\right)$.

Computing the throughput – exponential laws

General case:

Theorem

Let us consider a system formed by the mapping of an application onto a platform. Then the throughput can be computed in time $O\left(\exp(R)^3\right)$.

Computing the throughput – exponential laws

General case:

- model the system by a timed event graph
  Exponential in the size of the system
- transform this timed event graph into a Markov chain
  Exponential in the size of the TEG
- compute the stationary measure of this Markov chain
- derive the throughput from the marginals of the stationary measure

Computing the throughput – exponential laws

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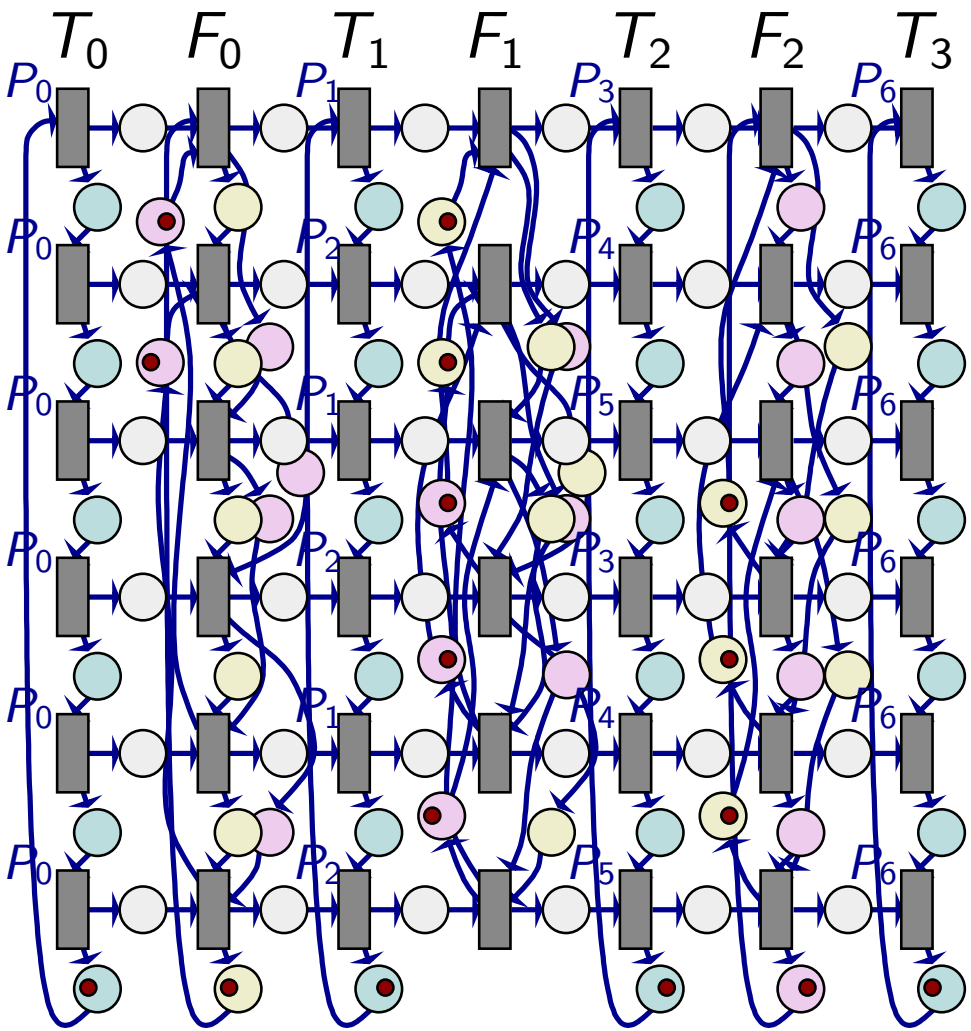
Computing the throughput – exponential laws

Transformation into a Markov chain: each marking of the TEG becomes a state



Computing the throughput – exponential laws

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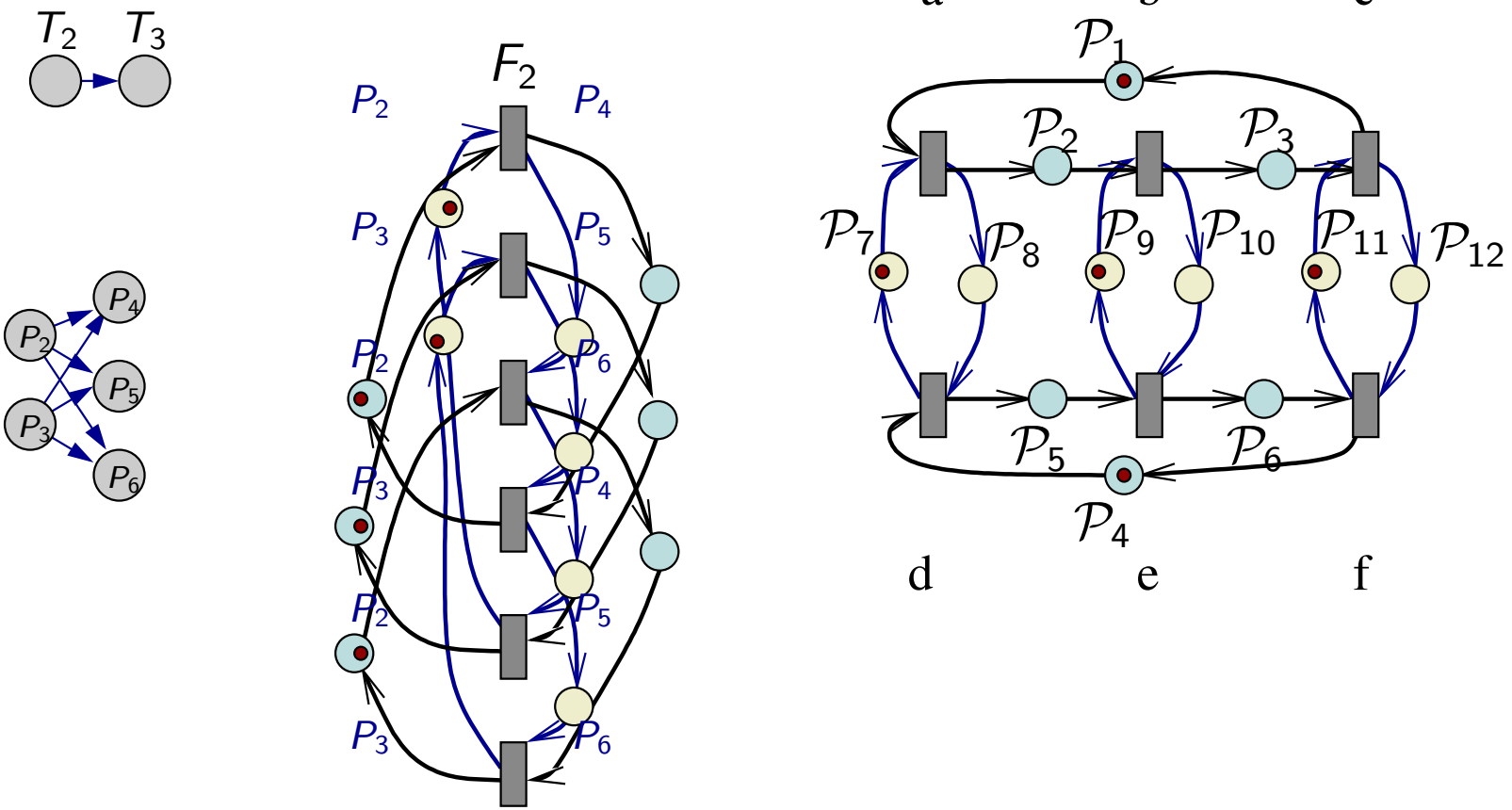
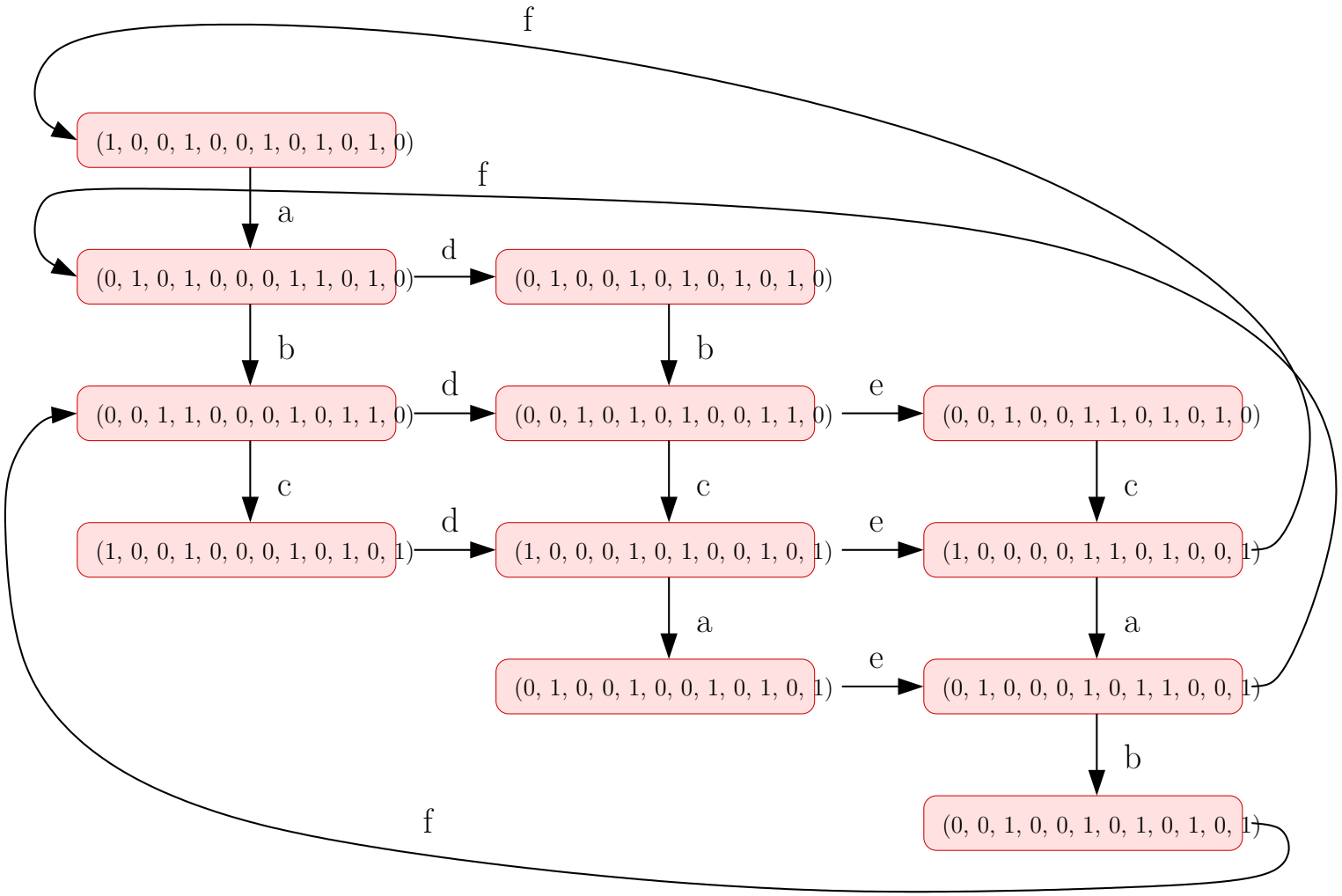


Computing the throughput – exponential laws

Transformation into a Markov chain: list of all possible states



Computing the throughput – exponential laws

Overlap model:

Theorem

Let us consider a system formed by the mapping of an application onto a platform. Then the throughput can be computed in time

$$O\left(N \exp(\max_{1 \leq i \leq N}(R_i))^3\right).$$

Computing the throughput – exponential laws

Overlap model:

- split the timed event graph into columns C_i, with $1 \leq i \leq 2N-1$
- separately consider each column C_i
- separately consider each connected component D_j of C_i
- single component D_j: many copies of the same pattern $\mathcal{P}_j$, of size $u_j \times v_j$
- transform $\mathcal{P}_j$ into a Markov chain $\mathcal{M}_j$
- determine a stationary measure of $\mathcal{M}_j$
- compute the throughput of $\mathcal{P}_j$ in isolation
- combine the inner throughputs of all components to get the global throughput of the system

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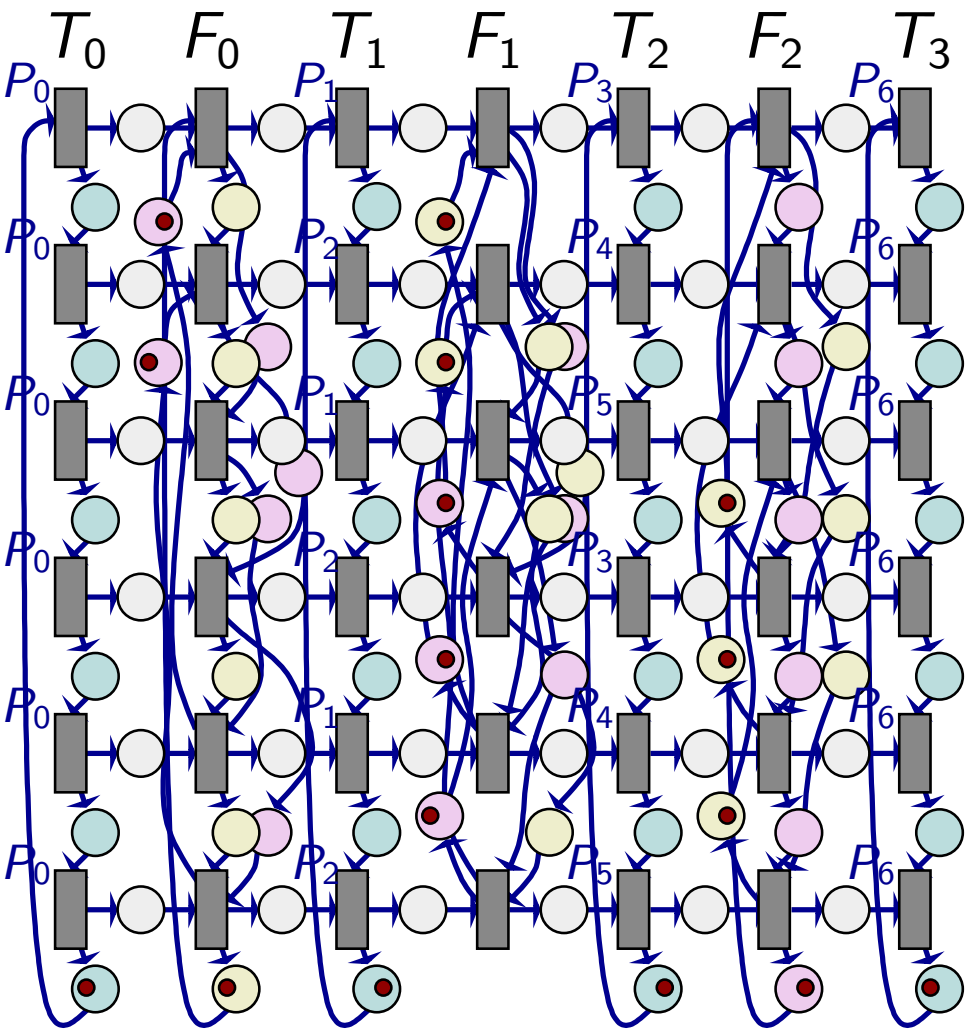
Computing the throughput – exponential laws

Overlap model:

Communication column:



Computing the throughput – exponential laws

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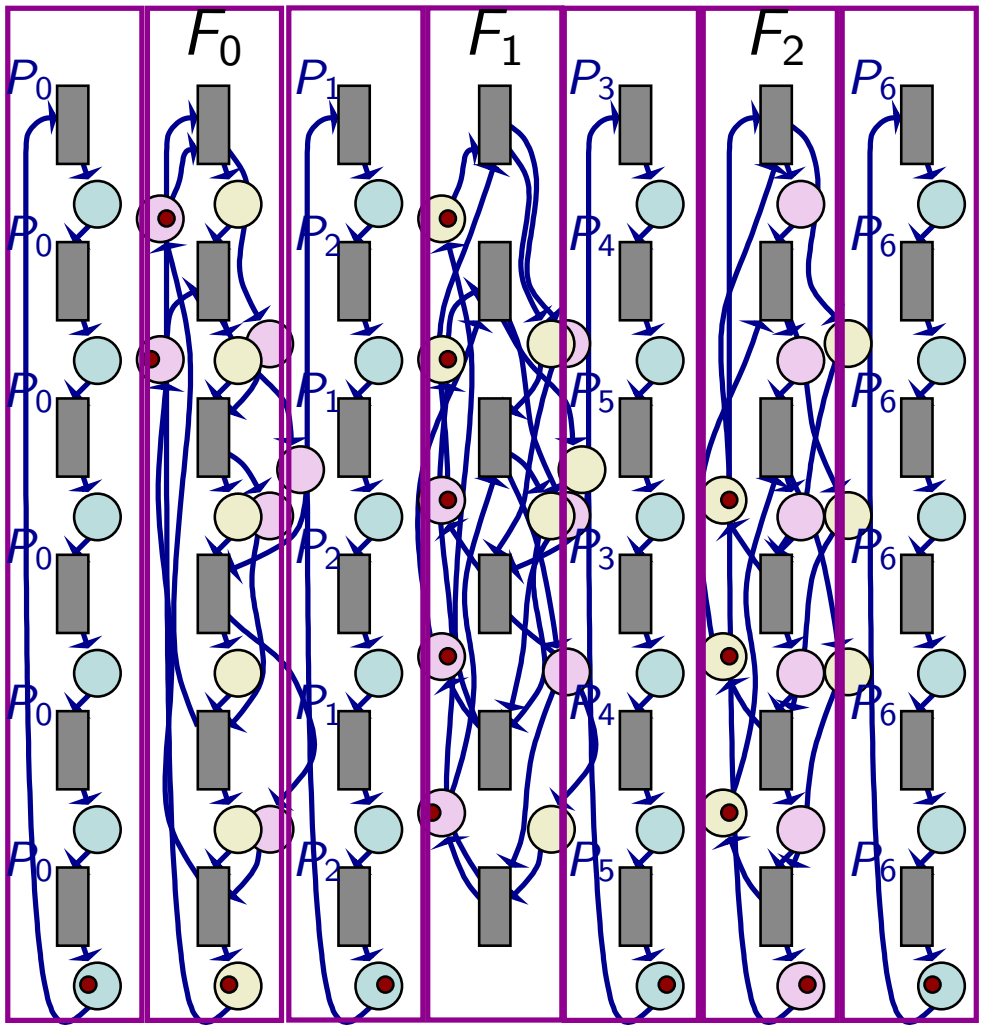

Computing the throughput – exponential laws

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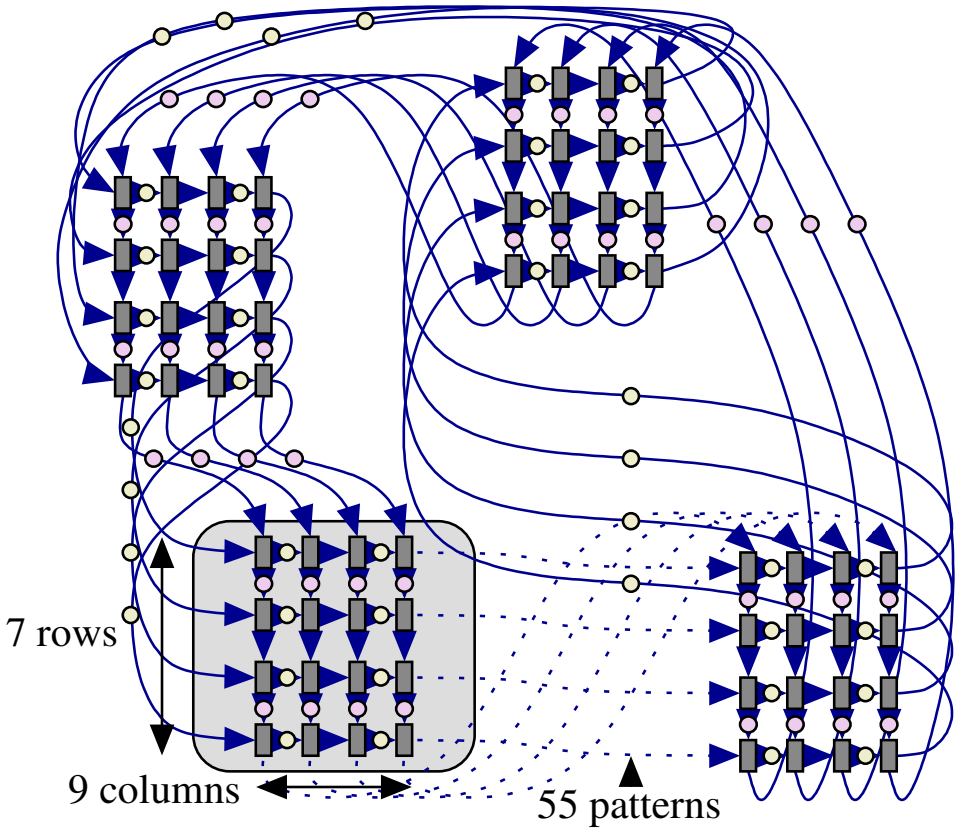
Communication column:

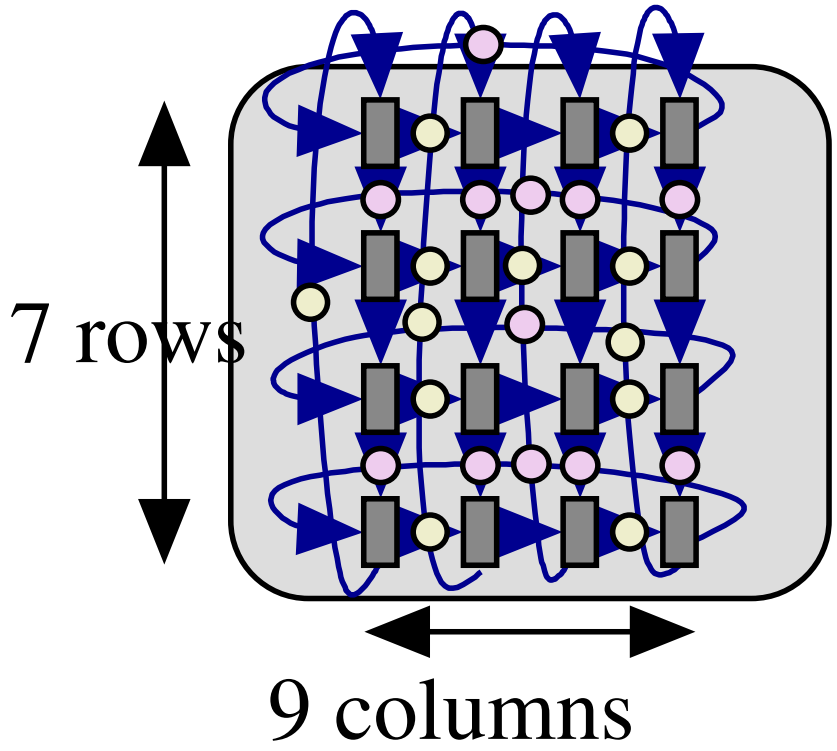


$$R_0 = 5, R_1 = 21, R_2 = 27, R_3 = 11$$

Computing the throughput – exponential laws

Overlap model:

Communication column:



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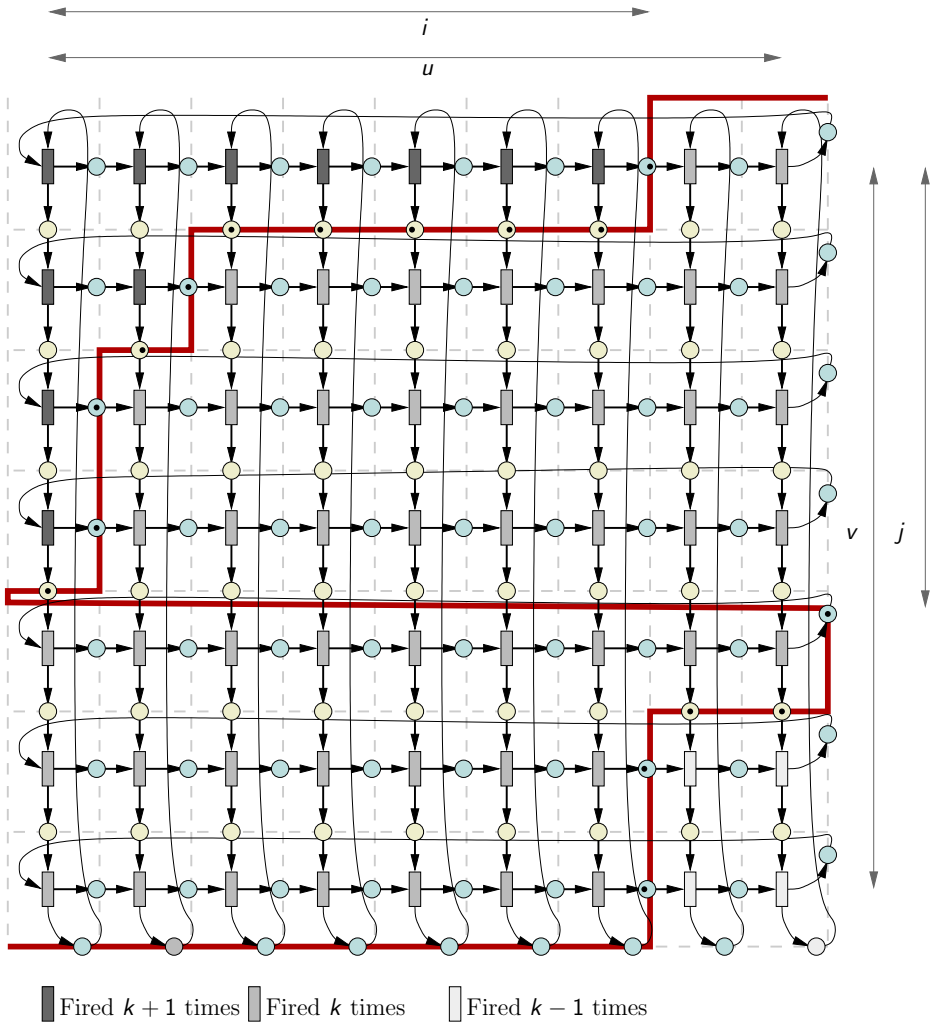
Computing the throughput – exponential laws

Overlap model:

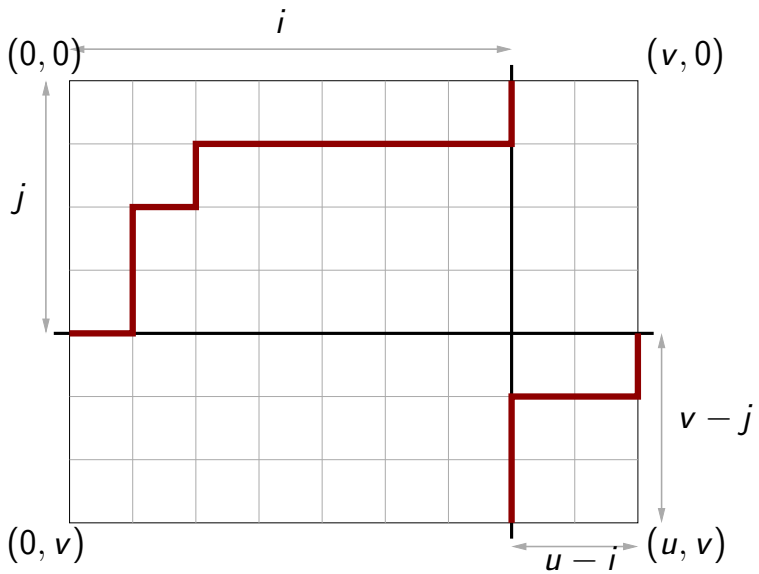
Representation of a valid marking on the TEG



Computing the throughput – exponential laws

Overlap model:

Representation of a valid marking with Young diagrams



$\Rightarrow$ Number of states easily determined

Exponential number of states in each connected component

Computing the throughput – exponential laws

Overlap model, homogeneous communication network:

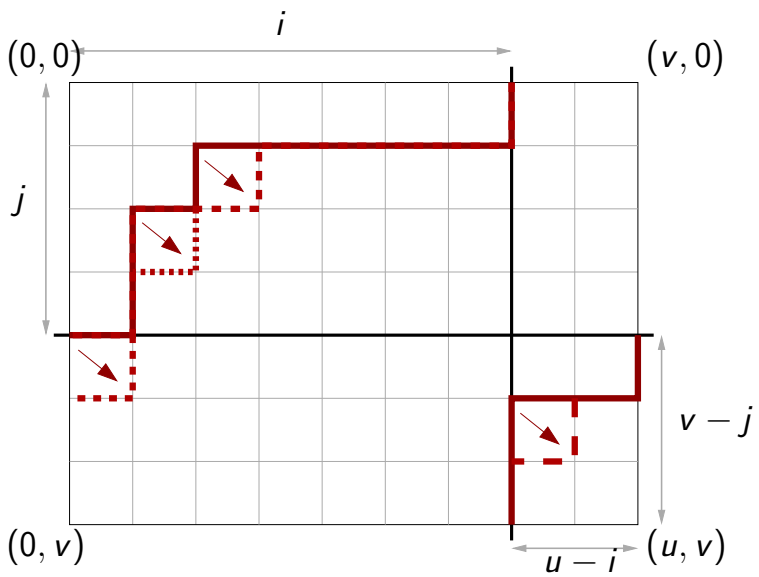
Theorem

Let us consider a system formed by the mapping of an application onto a platform, following the **Overlap** *communication model with a homogeneous communication network. Then the throughput can be computed in polynomial time.*

Computing the throughput – exponential laws

Overlap model:

Reachable states from a given position



$\Rightarrow$ Same number of incoming and outgoing states

$+$ Same firing rate (homogeneous communication network)

$=$ Invariant measure is uniform

Outline

Comparison between two systems

Theorem

Consider two systems $(X^{(1)}, Y^{(1)})$ *and* $(X^{(2)}, Y^{(2)})$. *If we have for all* n,
$\forall 1 \leq p \leq M, X_p^{(1)}(n) \leq_{\mathrm{st}} X_p^{(2)}(n)$ *and*
$\forall 1 \leq p, q \leq M, Y_{p,q}^{(1)}(n) \leq_{\mathrm{st}} Y_{p,q}^{(2)}(n)$, *then* $\rho^{(1)} \geq \rho^{(2)}$.

Comparison between two systems with I.I.D. laws

Theorem

Let us consider two systems with I.I.D. communication and processing times $(X^{(1)}, Y^{(1)})$ *and* $(X^{(2)}, Y^{(2)})$. *If we have for all n,* $\forall 1 \leq p \leq M, X_p^{(1)}(n) \leq_{\text{icx}} X_p^{(2)}(n)$ *and* $\forall 1 \leq p, q \leq M, Y_{p,q}^{(1)}(n) \leq_{\text{icx}} Y_{p,q}^{(2)}(n)$, *then* $\rho^{(1)} \geq \rho^{(2)}$.

Bounds on the expected throughput

Theorem

Let us consider any system $(X^{(1)}, Y^{(1)})$, *such that* $X_p^{(1)}(n)$ *and* $Y_{p,q}^{(1)}(n)$ *are N.B.U.E.. Let us also consider two new systems* $(X^{(2)}, Y^{(2)})$ *and* $(X^{(3)}, Y^{(3)})$ *such that:*

$\forall 1 \leq p \leq M, X_p^{(2)}(n)$ *has an exponential distribution, and* $\mathbf{E}[X_p^{(2)}(n)] = \mathbf{E}[X_p^{(1)}(n)]$,

$\forall 1 \leq p, q \leq M, Y_{p,q}^{(2)}(n)$ *has an exponential distribution, and* $\mathbf{E}[Y_{p,q}^{(2)}(n)] = \mathbf{E}[Y_{p,q}^{(1)}(n)]$,

$\forall 1 \leq p \leq M, X_p^{(3)}(n)$ *is deterministic and for all* n, $X_p^{(3)}(n) = \mathbf{E}[X_p^{(1)}(n)]$,

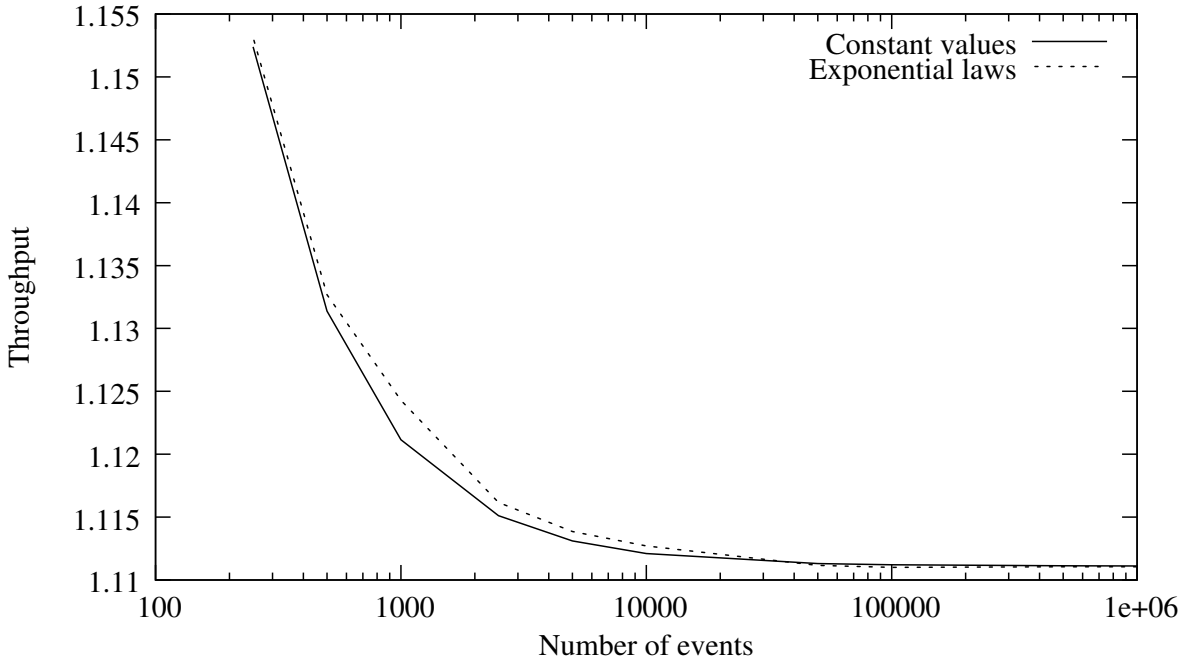
$\forall 1 \leq p, q \leq M, Y_{p,q}^{(3)}(n)$ *is deterministic and for all* n, $Y_{p,q}^{(3)}(n) = \mathbf{E}[Y_{p,q}^{(1)}(n)]$.

Then we have:

$$\rho^{(3)} \geq \rho^{(1)} \geq \rho^{(2)}.$$

Numerical experiments

Evolution of the measured throughput with the number of samples



Distribution	Constant value c	Exponential mean c	Uniform $c/2$ - $3c/2$	Uniform $c/10$ - $19c/10$	Pareto mean c
Throughput	2.0299	2.0314	2.0304	2.0305	2.0300

Table: Throughput obtained with several distributions of same mean.

Outline

Conclusion and future work

- Even if the mapping is given, the throughput is hard to determine
- Expectation of the throughput can be computed in many cases:
  - General case with exponential laws: exponential time
  - **Overlap** model with exponential laws: smaller exponential time
  - **Overlap** model, homogeneous communications: polynomial time
  - General case, N.B.U.E. laws: bounds can be established
- Determining the mapping that maximizes the throughput is an NP-complete problem, even in the simpler deterministic case with no communication costs

Future work:

- Design efficient mapping heuristics

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