A Few Scheduling Problems for Resilience at Scale

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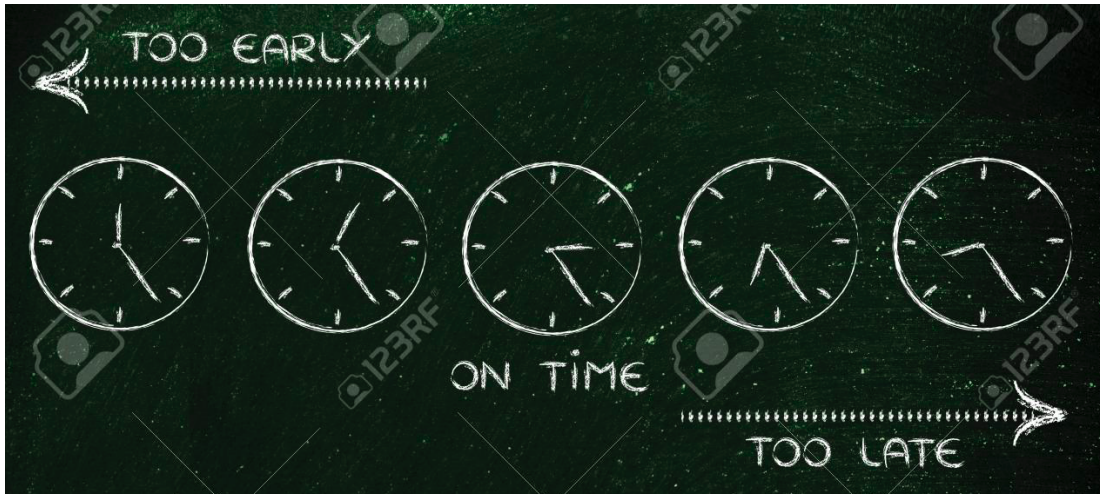
SCALA workshop – November12, 2018

http://graal.ens-lyon.fr/~yrobert/scala18.pdf

Scheduling Matters



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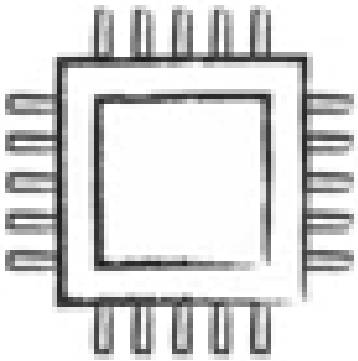
Agenda

Three little scheduling problems

1. Scheduling checkpoints
2. Scheduling against IO interference
3. Scheduling stochastic tasks

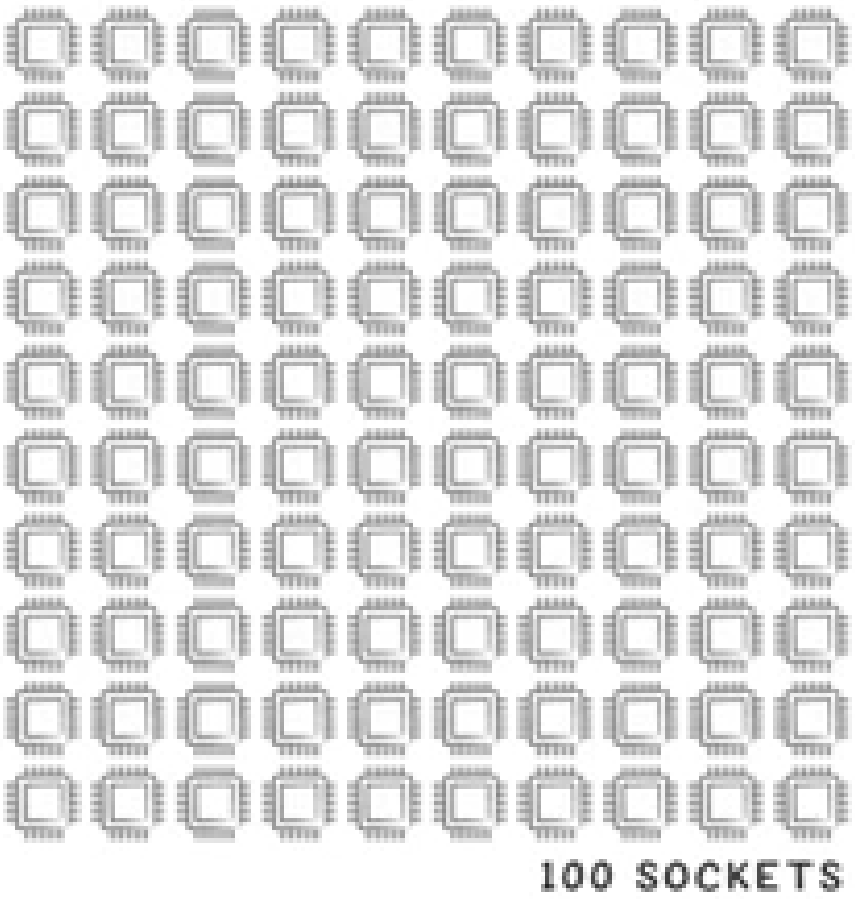
Scale is the enemy



100
YEARS

MEAN TIME
BETWEEN FAILURES

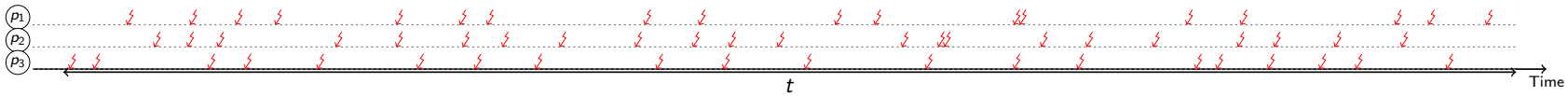
Scale is the enemy



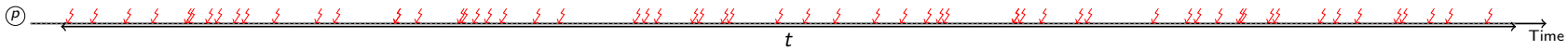
1
YEAR

MEAN TIME
BETWEEN FAILURES

Scale is the enemy

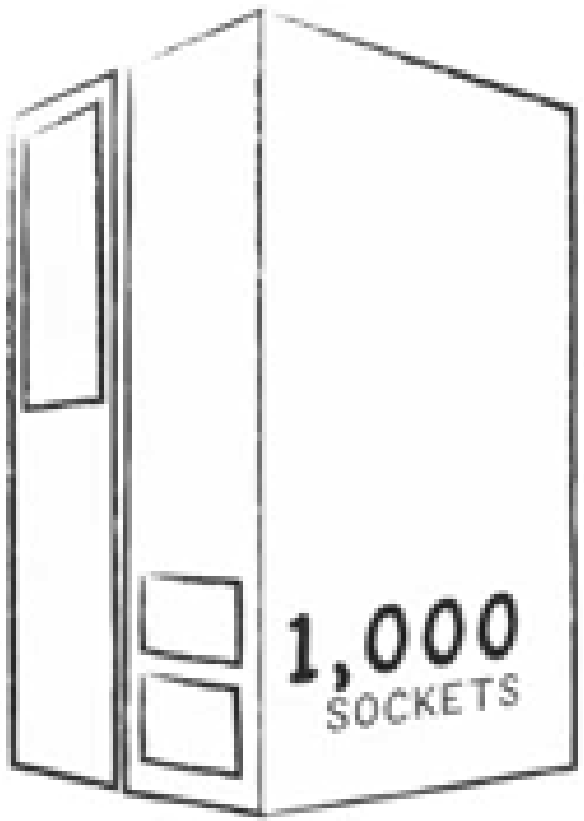


If three processors have around 20 faults during a time t ($\mu = \frac{t}{20}$)...



...during the same time, the platform has around 60 faults ($\mu_p = \frac{t}{60}$)

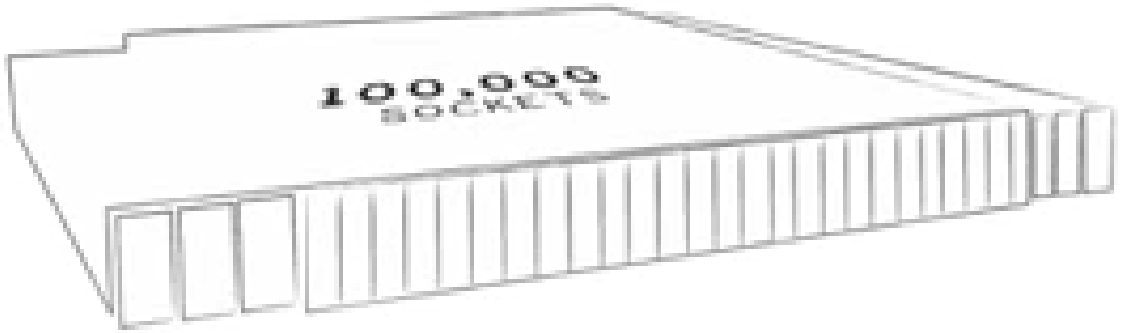
Scale is the enemy



36
DAYS

MEAN TIME
BETWEEN FAILURES

Scale is the enemy



8
HOURS
MEAN TIME
BETWEEN FAILURES

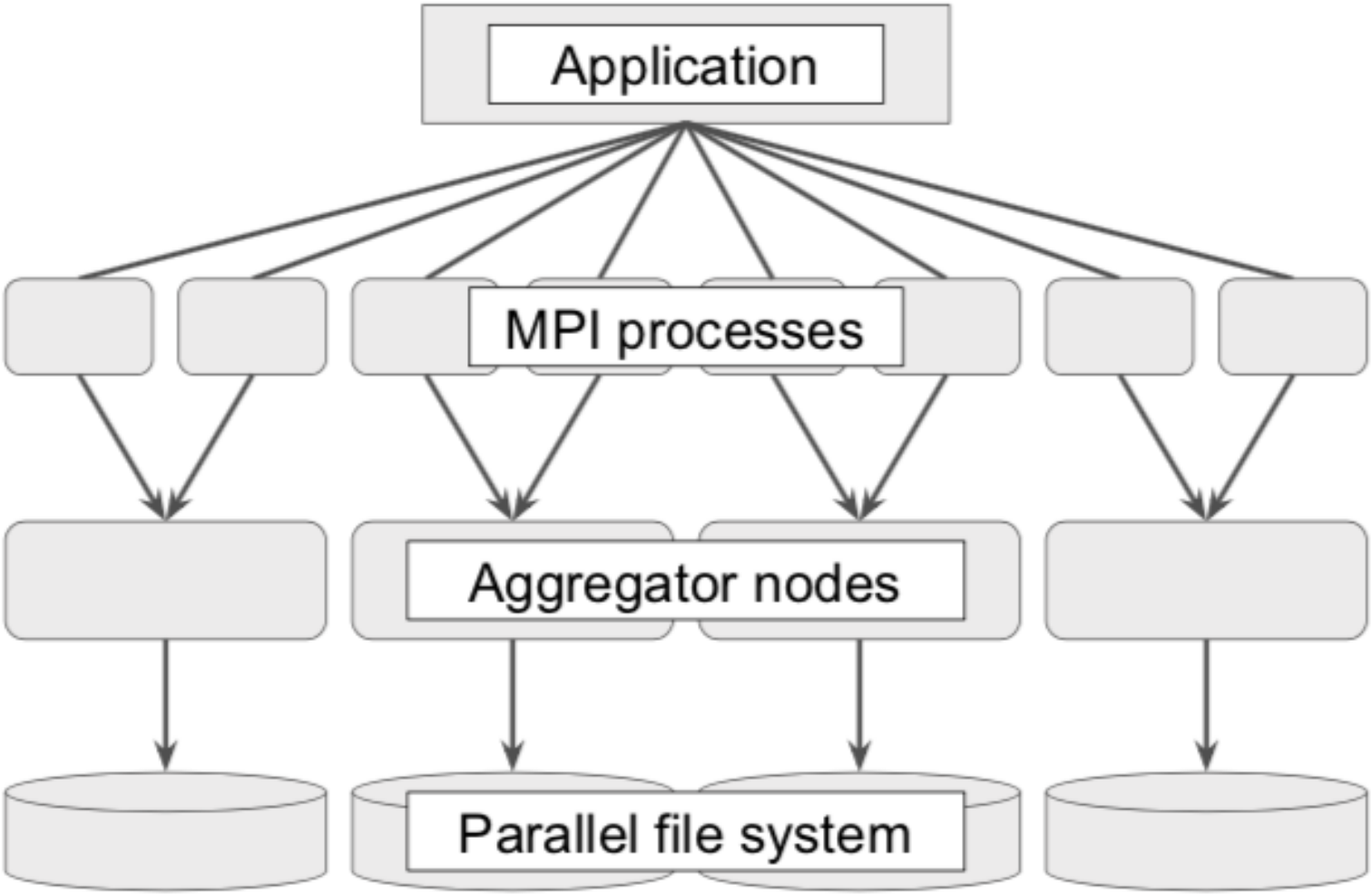
Scale is the enemy

Need to checkpoint!

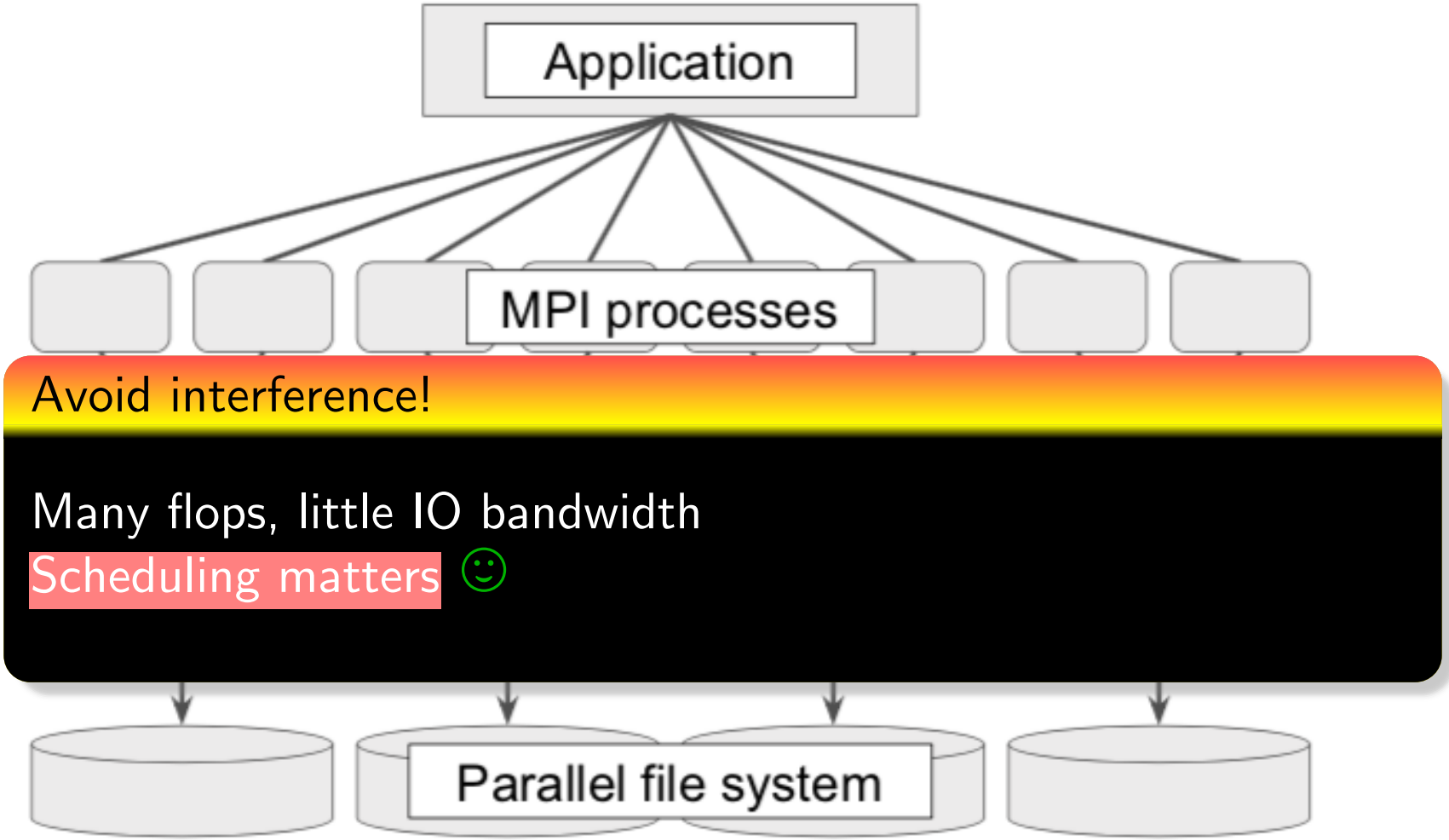
But when?

Scheduling matters ☺

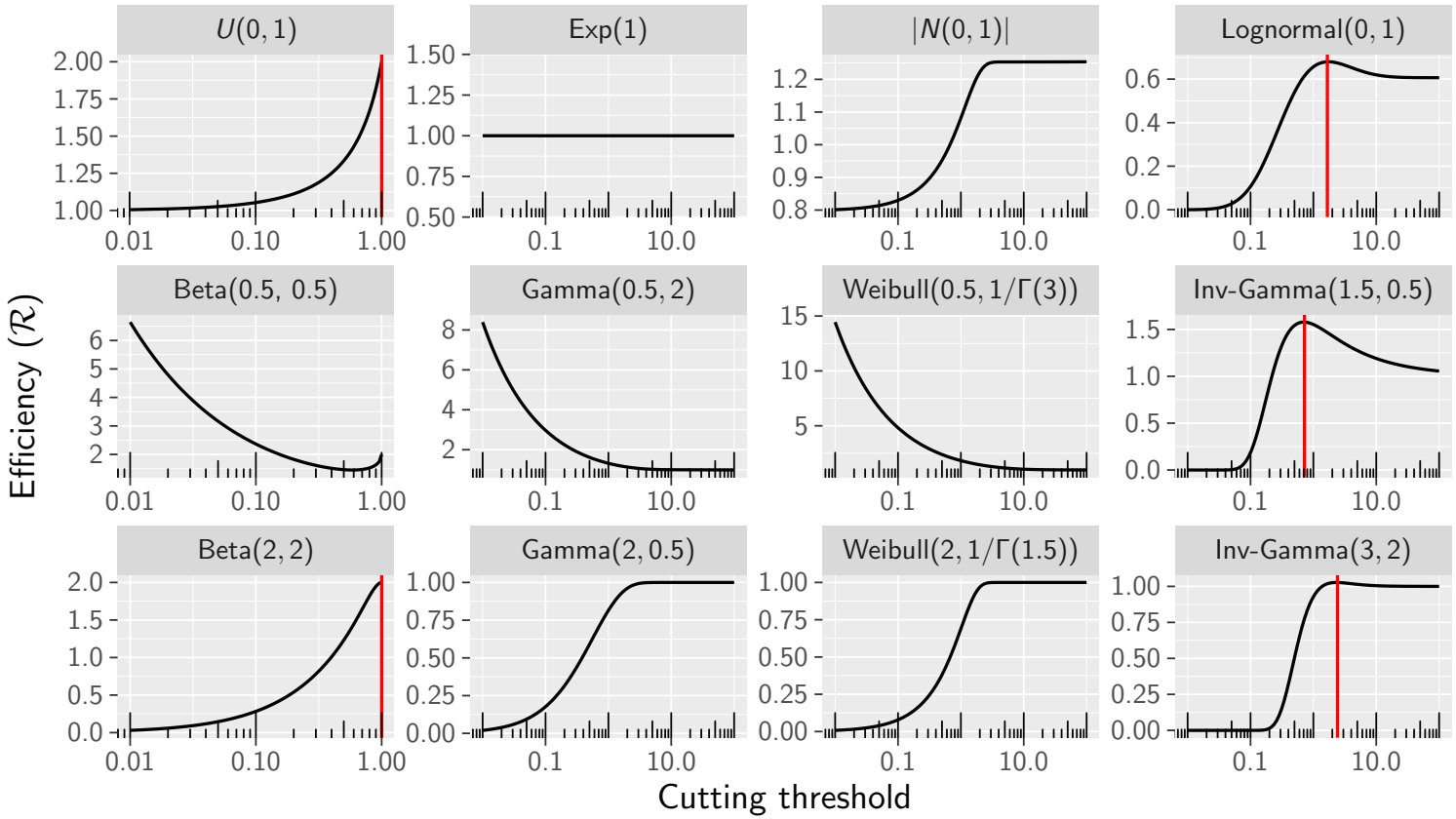
IO gap increases



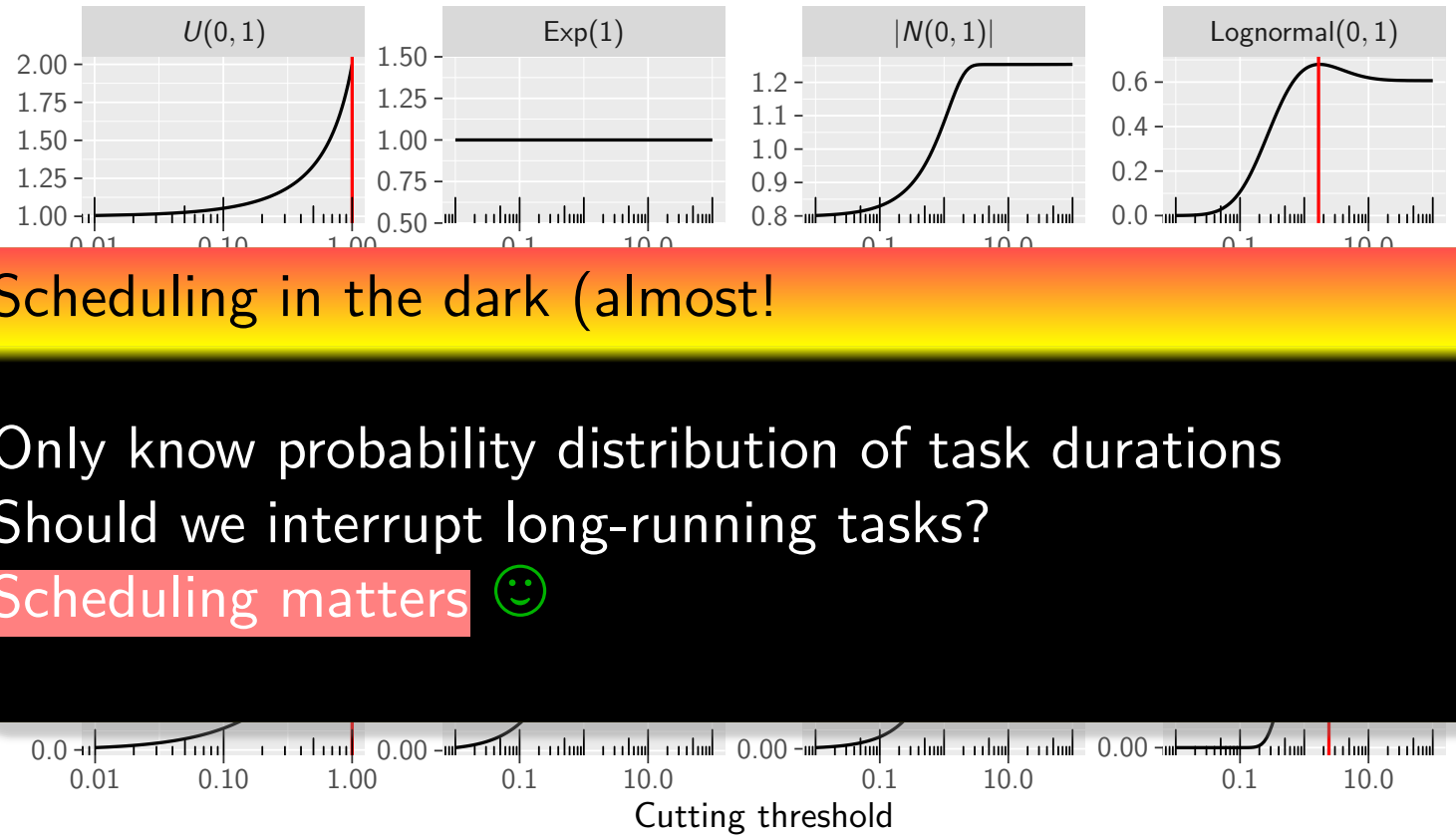
IO gap increases



Stochastic weights



Stochastic weights



Scheduling in the dark (almost!

Only know probability distribution of task durations
Should we interrupt long-running tasks?
Scheduling matters ☺

Outline

Outline

Outline

Definitions

Many types of faults: software error, hardware malfunction, memory corruption

Many possible behaviors: silent, transient, unrecoverable

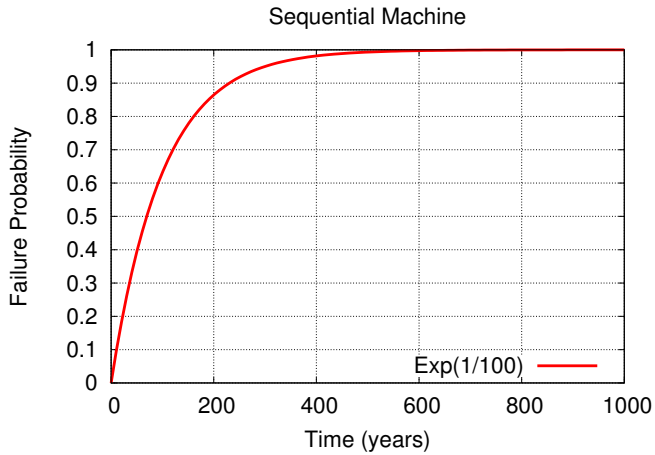
Restrict to faults that lead to application failures

This includes all hardware faults, and some software ones

Will use terms *fault* and *failure* interchangeably

Silent errors (Silent Data Corruptions) addressed later

Failure distributions: (1) Exponential



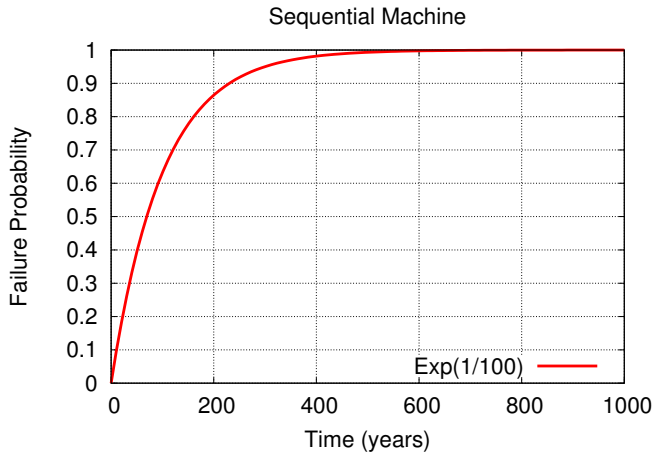
$Exp(\lambda)$: Exponential distribution law of parameter λ:

Pdf: $f(t) = \lambda e^{-\lambda t} dt$ for $t \geq 0$

Cdf: $F(t) = 1 - e^{-\lambda t}$

Mean $= \frac{1}{\lambda}$

Failure distributions: (1) Exponential



X random variable for $Exp(\lambda)$ failure inter-arrival times:

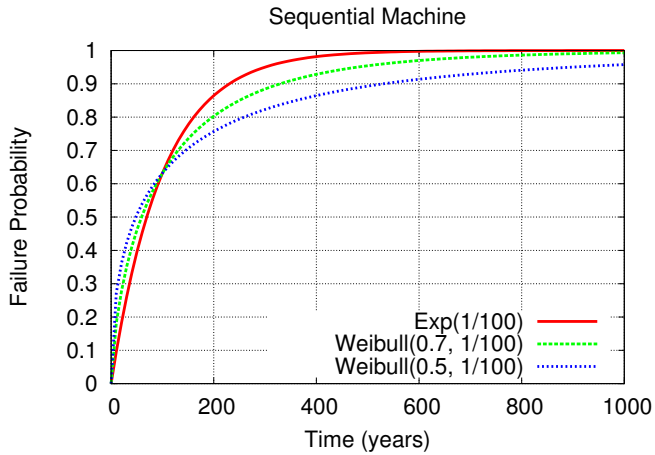
$\mathbb{P}X \leq t = 1 - e^{-\lambda t} dt$ (by definition)

Memoryless property: $\mathbb{P}\left(X \geq t + s \,|\, X \geq s\right) = \mathbb{P}X \geq t$

at any instant, time to next failure does not depend upon time elapsed since last failure

Mean Time Between Failures (MTBF) $\mu = \mathbb{E}\left(X\right) = \frac{1}{\lambda}$

Failure distributions: (2) Weibull



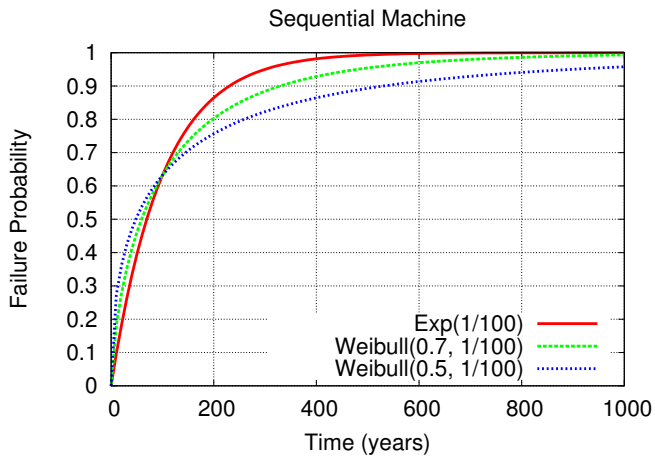
Weibull(k, λ): Weibull distribution law of shape parameter k and scale parameter λ:

Pdf: $f(t) = k\lambda(t\lambda)^{k-1}e^{-(\lambda t)^k}dt$ for $t \geq 0$

Cdf: $F(t) = 1 - e^{-(\lambda t)^k}$

Mean $= \frac{1}{\lambda}\Gamma(1 + \frac{1}{k})$

Failure distributions: (2) Weibull



X random variable for $Weibull(k, \lambda)$ failure inter-arrival times:

If $k < 1$: failure rate decreases with time

"infant mortality": defective items fail early

If $k = 1$: $Weibull(1, \lambda) = Exp(\lambda)$ constant failure time

Platform MTBF

Rebooting only faulty processor

Platform failure distribution

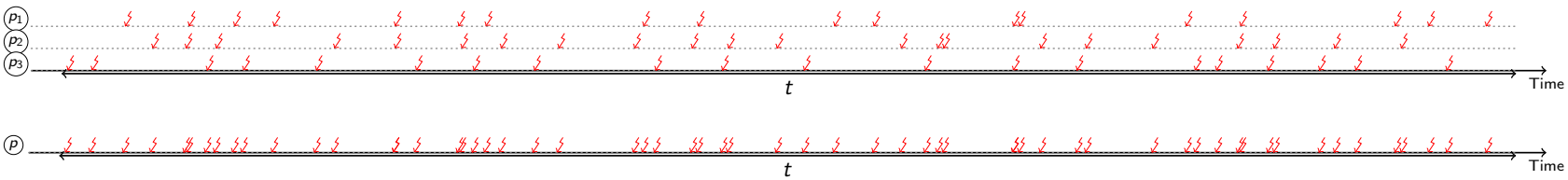
$\Rightarrow$ superposition of p IID processor distributions of MTBF μ

$\Rightarrow$ IID only for Exponential

Define μ_p by

$$\lim_{F \to +\infty} \frac{F)}{n(F)} = \mu_p$$

$n(F)$ = number of platform failures until time F is exceeded



Theorem: $\mu_p = \frac{\mu}{p}$ for arbitrary distributions

Values from the literature

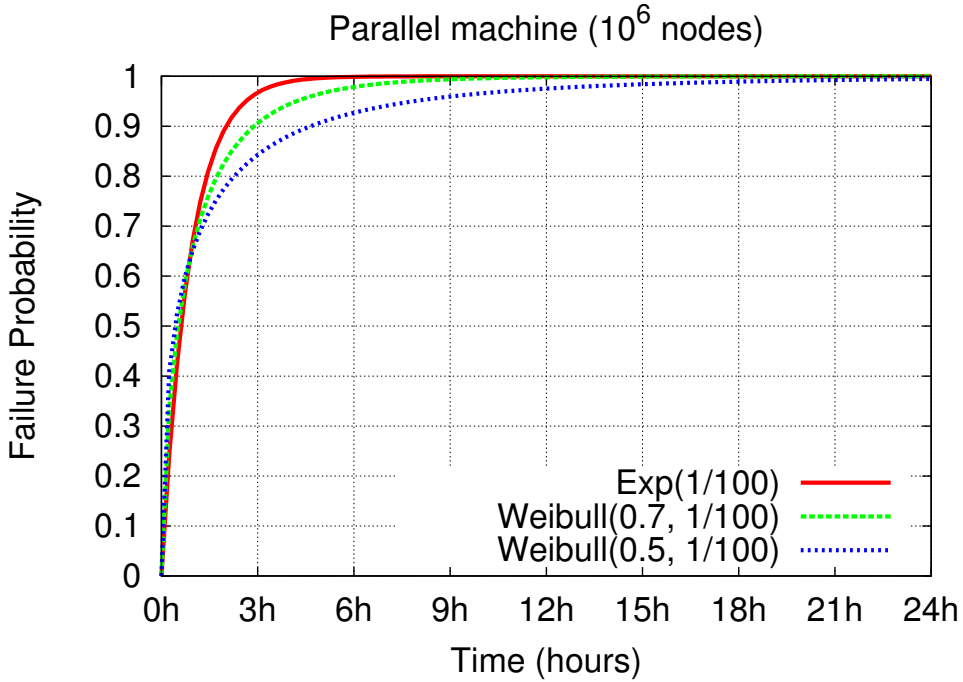
MTBF of one processor: between 1 and 125 years

Shape parameters for Weibull: $k = 0.5$ or $k = 0.7$

Failure trace archive from INRIA
(`http://fta.inria.fr`)

Computer Failure Data Repository from LANL
(`http://institutes.lanl.gov/data/fdata`)

Does it matter?



After infant mortality and before aging,
instantaneous failure rate of computer platforms is almost constant

Summary for the road

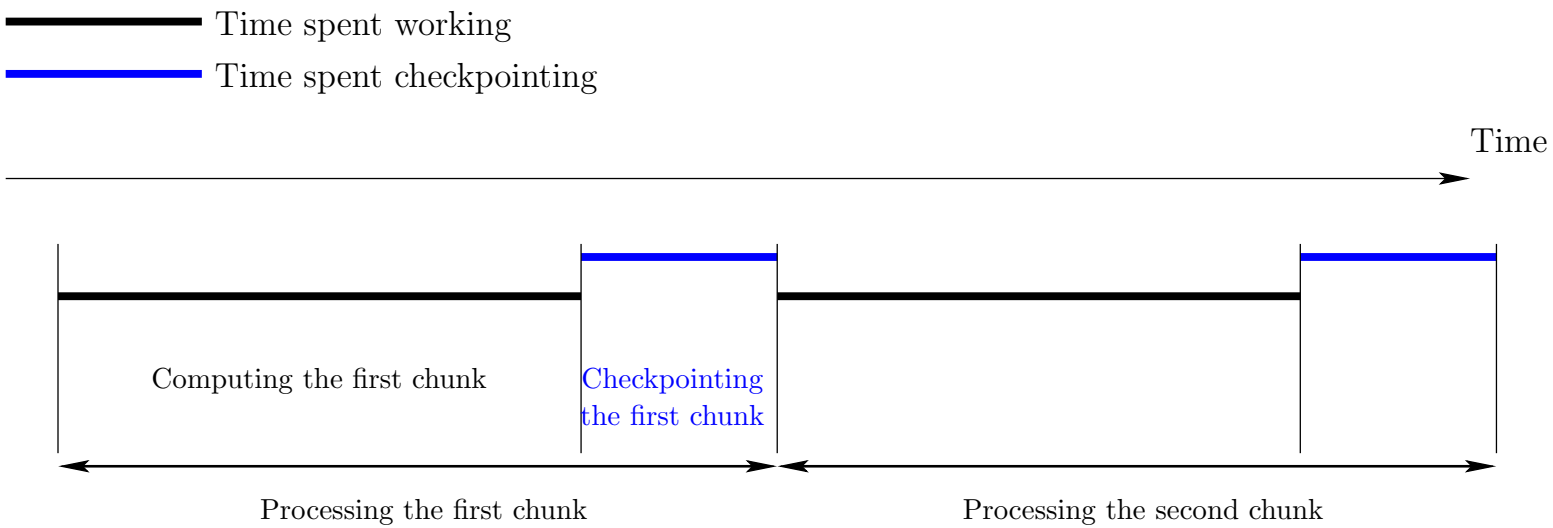
MTBF key parameter and $\mu_p = \frac{\mu}{p}$ ☺

Exponential distribution OK for most purposes ☺

Assume failure independence while not (completely) true ☹

Outline

Periodic checkpointing



Blocking model: while a checkpoint is taken, no computation can be performed

Framework

Periodic checkpointing policy of period $T = W + C$

Independent and identically distributed failures

Applies to a single processor with MTBF $\mu = \mu_{ind}$

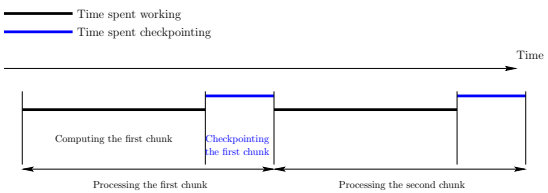
Applies to a platform with p processors and MTBF $\mu = \frac{\mu_{ind}}{p}$

- coordinated checkpointing
- tightly-coupled application
- progress $\Leftrightarrow$ all processors available

$\Rightarrow$ platform = single (powerful, unreliable) processor ☺

Waste: fraction of time not spent for useful computations

Waste in fault-free execution



$\textsc{Time}_{\textsf{base}}$: application base time

$\textsc{Time}_{\textsf{FF}}$: with periodic checkpoints but failure-free

$$\textsc{Time}_{\textsf{FF}} = \textsc{Time}_{\textsf{base}} + \#\mathit{checkpoints} \times C$$

$$\#\mathit{checkpoints} = \left\lceil \frac{\textsc{Time}_{\textsf{base}}}{T - C} \right\rceil \approx \frac{\textsc{Time}_{\textsf{base}}}{T - C} \text{ (valid for large jobs)}$$

$$\textsc{Waste}[\mathit{FF}] = \frac{\textsc{Time}_{\textsf{FF}} - \textsc{Time}_{\textsf{base}}}{\textsc{Time}_{\textsf{FF}}} = \frac{C}{T}$$

Waste due to failures

$\text{TIME}_{\text{base}}$: application base time

TIME_{FF}: with periodic checkpoints but failure-free

$\text{TIME}_{\text{final}}$: expectation of time with failures

$$\text{TIME}_{\text{final}} = \text{TIME}_{\text{FF}} + N_{faults} \times T_{\text{lost}}$$

N_{faults} number of failures during execution

T_{lost}: average time lost per failure

$$N_{faults} = \frac{\text{TIME}_{\text{final}}}{\mu}$$

T_{lost}?

Waste due to failures

$\text{TIME}_{\text{base}}$: application base time

TIME_{FF}: with periodic checkpoints but failure-free

$\text{TIME}_{\text{final}}$: expectation of time with failures

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N_{faults} number of failures during execution
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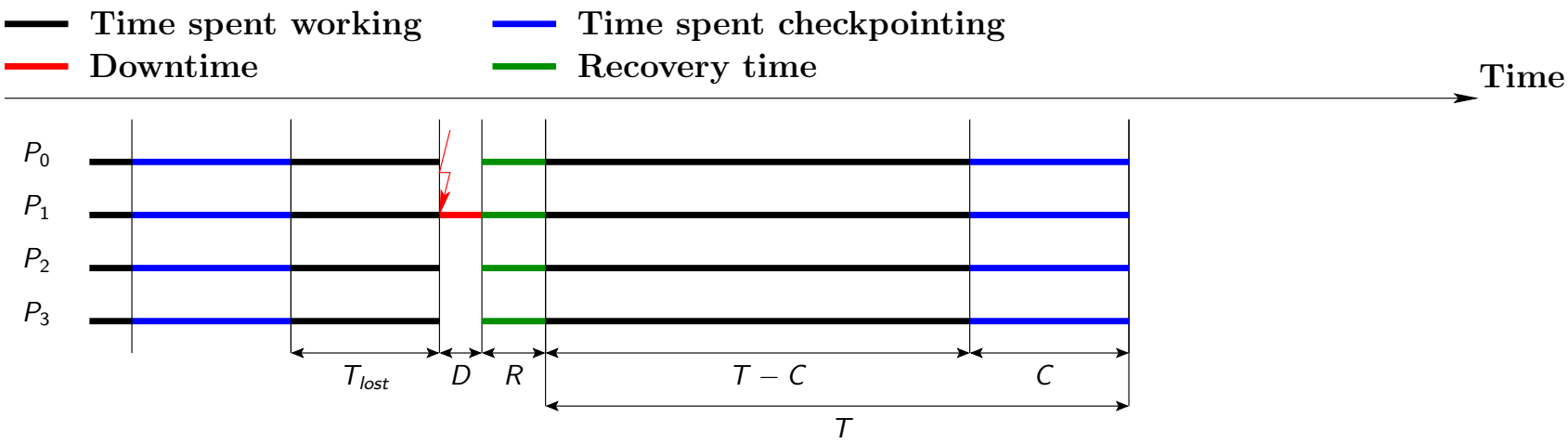
$$N_{faults} = \frac{\text{TIME}_{\text{final}}}{\mu}$$

T_{lost}?

Computing T_{lost}



$$T_{\text{lost}} = D + R + \frac{T}{2}$$

Rationale

$\Rightarrow$ Instants when periods begin and failures strike are independent

$\Rightarrow$ Approximation used for all distribution laws

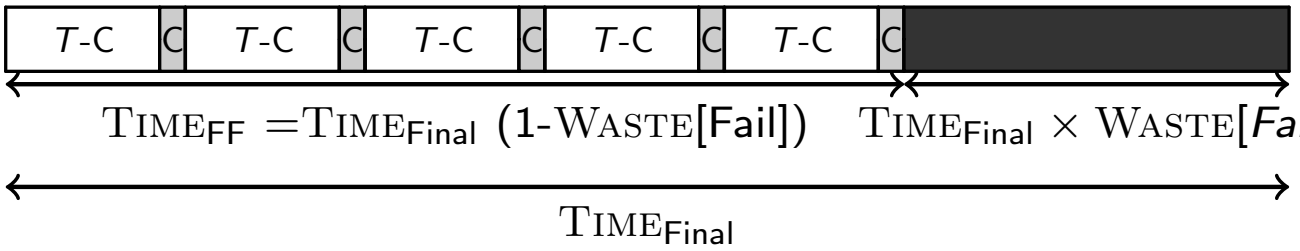
$\Rightarrow$ Exact for Exponential and uniform distributions

Waste due to failures

$$\text{TIME}_{\text{final}} = \text{TIME}_{\text{FF}} + N_{faults} \times T_{\text{lost}}$$

$$\text{WASTE}[fail] = \frac{\text{TIME}_{\text{final}} - \text{TIME}_{\text{FF}}}{\text{TIME}_{\text{final}}} = \frac{1}{\mu}\left(D + R + \frac{T}{2}\right)$$

Total waste

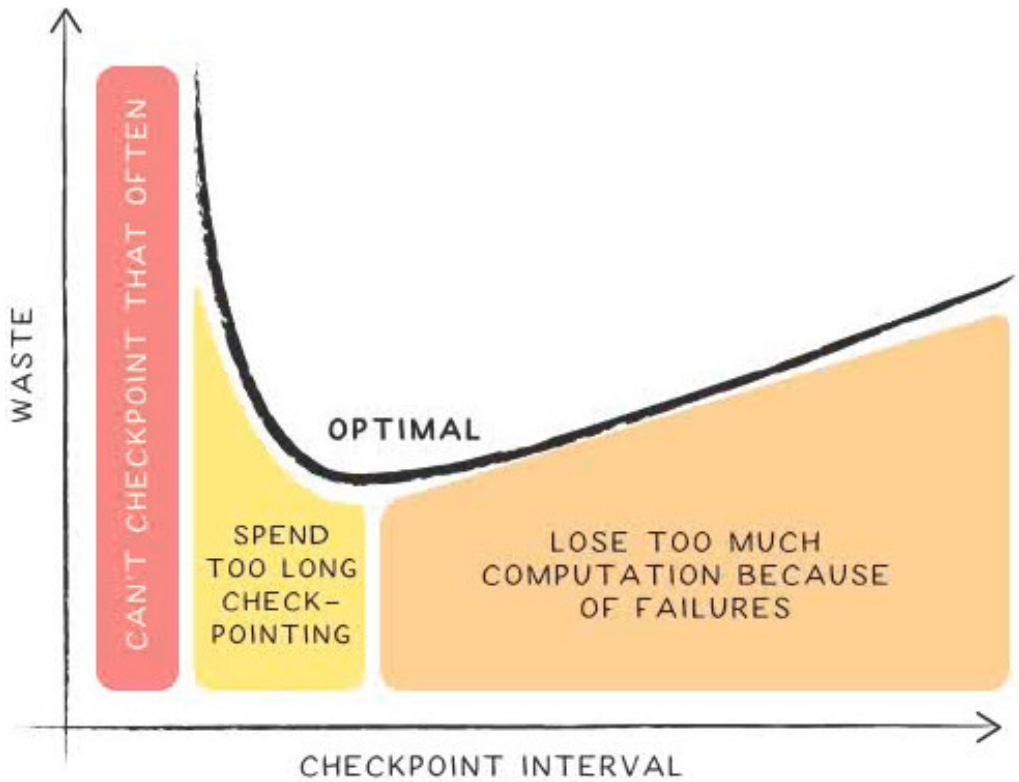


$$\text{WASTE} = \frac{\text{TIME}_{\text{final}} - \text{TIME}_{\text{base}}}{\text{TIME}_{\text{final}}}$$

$$1 - \text{WASTE} = (1 - \text{WASTE}[FF])(1 - \text{WASTE}[fail])$$

$$\text{WASTE} = \frac{C}{T} + \left(1 - \frac{C}{T}\right)\frac{1}{\mu}\left(D + R + \frac{T}{2}\right)$$

Optimal checkpointing interval



Waste minimization

$$\text{WASTE} = \frac{C}{T} + \left(1 - \frac{C}{T}\right)\frac{1}{\mu}\left(D + R + \frac{T}{2}\right)$$

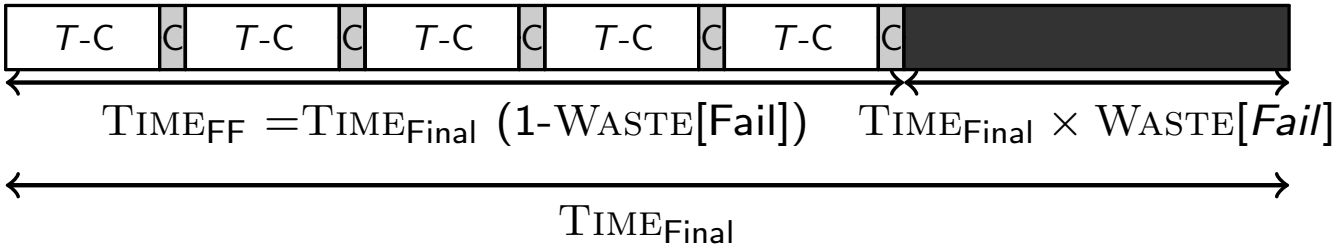
$$\text{WASTE} = \frac{u}{T} + v + wT$$

$$u = C\left(1 - \frac{D+R}{\mu}\right) \qquad v = \frac{D + R - C/2}{\mu} \qquad w = \frac{1}{2\mu}$$

WASTE minimized for $T = \sqrt{\frac{u}{w}}$

$$T = \sqrt{2(\mu - (D+R))C}$$

Comparison with Young/Daly



$$\big(1 - \textsc{Waste}[\mathit{fail}]\big)\,\textsc{Time}_{\mathsf{final}} = \textsc{Time}_{\mathsf{FF}}$$
$$\Rightarrow T = \sqrt{2(\mu - (D + R))C}$$

Daly: $\textsc{Time}_{\mathsf{final}} = \big(1 + \textsc{Waste}[\mathit{fail}]\big)\,\textsc{Time}_{\mathsf{FF}}$
$$\Rightarrow T = \sqrt{2(\mu + (D + R))C} + C$$

Young: $\textsc{Time}_{\mathsf{final}} = \big(1 + \textsc{Waste}[\mathit{fail}]\big)\,\textsc{Time}_{\mathsf{FF}}$ and $D = R = 0$
$$\Rightarrow T = \sqrt{2\mu C} + C$$

Wrap up

Capping periods, and enforcing a lower bound on MTBF
$\Rightarrow$ mandatory for mathematical rigor ☹

Not needed for practical purposes ☺

- actual job execution uses optimal value
- account for multiple faults by re-executing work until success

Approach surprisingly robust ☺

Lesson learnt for fail-stop failures

(Not so) Secret data

- Tsubame 2: 962 failures during last 18 months so $\mu = 13$ hrs
- Blue Waters: 2-3 node failures per day
- Titan: a few failures per day
- Tianhe 2: wouldn't say

$$T_{\text{opt}} = \sqrt{2\mu C} \quad \Rightarrow \quad \text{WASTE}[opt] \approx \sqrt{\frac{2C}{\mu}}$$

Petascale:	$C = 20$ min	$\mu = 24$ hrs	$\Rightarrow \text{WASTE}[opt] = 17\%$
Scale by 10:	$C = 20$ min	$\mu = 2.4$ hrs	$\Rightarrow \text{WASTE}[opt] = 53\%$
Scale by 100:	$C = 20$ min	$\mu = 0.24$ hrs	$\Rightarrow \text{WASTE}[opt] = 100\%$

Lesson learnt for fail-stop failures

(Also) Secret data

- Tsubame: 962 failures during last 18 months so MTBF = 13 hrs
- Blue Waters: 2-3 failures per day
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- Tianhe

Exascale ≠ Petascale ×1000

Need more reliable components

Need to checkpoint faster

Petascale:	$C = 20$ min	$\mu = 24$ hrs	$\Rightarrow$ WASTE$[opt] = 17\%$
Scale by 10:	$C = 20$ min	$\mu = 2.4$ hrs	$\Rightarrow$ WASTE$[opt] = 53\%$
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Lesson learnt for fail-stop failures

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Silent errors:

detection latency $\Rightarrow$ additional problems

Petascale:	$C = 20$ min	$\mu = 24$ hrs	$\Rightarrow$ WASTE$[opt] = 17\%$
Scale by 10:	$C = 20$ min	$\mu = 2.4$ hrs	$\Rightarrow$ WASTE$[opt] = 53\%$
Scale by 100:	$C = 20$ min	$\mu = 0.24$ hrs	$\Rightarrow$ WASTE$[opt] = 100\%$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\mathbb{E}(W) =$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\mathbb{E}(W) = \overbrace{\mathcal{P}_{\text{succ}}(W+C)}^{\substack{\text{Probability} \\ \text{of success}}} (W+C)$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\mathbb{E}(W) = \overbrace{\mathcal{P}_{\text{succ}}(W+C)}^{\text{Time needed to compute the work } W \text{ and checkpoint it}}(W+C)$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\begin{aligned}\mathbb{E}(W) = \quad & \mathcal{P}_{\text{succ}}(W+C)\,(W+C) \\ & + \\ & \underbrace{(1-\mathcal{P}_{\text{succ}}(W+C))}_{\text{Probability of failure}}\,(\mathbb{E}(T_{lost}(W+C)) + \mathbb{E}(T_{rec}) + \mathbb{E}(W))\end{aligned}$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\begin{aligned}\mathbb{E}(W) = & \quad \mathcal{P}_{\text{succ}}(W+C)\,(W+C) \\ & + \\ & (1-\mathcal{P}_{\text{succ}}(W+C))\,(\underbrace{\mathbb{E}(T_{lost}(W+C))}_{\substack{\text{Time elapsed} \\ \text{before failure} \\ \text{stroke}}} + \mathbb{E}(T_{rec}) + \mathbb{E}(W))\end{aligned}$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\begin{aligned}\mathbb{E}((W) = \quad & \mathcal{P}_{\text{succ}}(W+C)\,(W+C) \\ & + \\ & (1-\mathcal{P}_{\text{succ}}(W+C))\,(\mathbb{E}(T_{lost}(W+C)) + \underbrace{\mathbb{E}(T_{rec})}_{\substack{\text{Time needed}\\ \text{to perform}\\ \text{downtime}\\ \text{and recovery}}} + \mathbb{E}(W))\end{aligned}$$

Exponential distributions

Compute the expected time $\mathbb{E}(W)$ to execute a work of duration W followed by a checkpoint of duration C.

Recursive Approach

$$\begin{aligned}\mathbb{E}(W) = \quad & \mathcal{P}_{\text{succ}}(W+C)\,(W+C) \\ & + \\ & (1-\mathcal{P}_{\text{succ}}(W+C))\,(\mathbb{E}(T_{lost}(W+C)) + \mathbb{E}(T_{rec}) + \underbrace{\mathbb{E}(W)}_{\substack{\text{Time needed} \\ \text{to compute } W \\ \text{anew}}})\end{aligned}$$

Computation of $\mathbb{E}(W)$

$$\mathbb{E}(W) = \begin{array}{l} \mathcal{P}_{\text{succ}}(W+C)\,(W+C) \\ + \\ (1-\mathcal{P}_{\text{succ}}(W+C))\,(\mathbb{E}(T_{lost}(W+C)) + \mathbb{E}(T_{rec}) + \mathbb{E}(W)) \end{array}$$

$$\mathbb{P}_{suc}(W+C) = e^{-\lambda(W+C)}$$

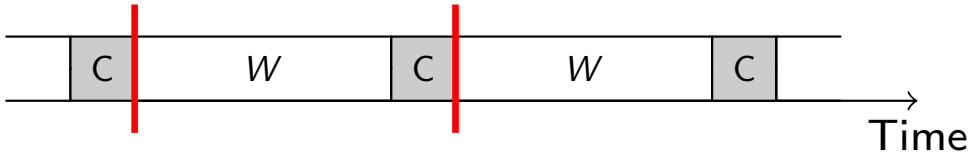
$$\mathbb{E}(T_{lost}(W+C)) = \int_0^{\infty} x\mathbb{P}(X=x|X<W+C)dx = \frac{1}{\lambda} - \frac{W+C}{e^{\lambda(W+C)}-1}$$

$$\mathbb{E}(T_{rec}) = e^{-\lambda R}(D+R)+(1-e^{-\lambda R})(D+\mathbb{E}(T_{lost}(R))+\mathbb{E}(T_{rec}))$$

$$\mathbb{E}(W) = e^{\lambda R}\left(\frac{1}{\lambda}+D\right)\left(e^{\lambda(W+C)}-1\right)$$

Optimal checkpointing interval

Minimize expected execution overhead $H(W) = \frac{\mathbb{E}(W)}{W} - 1$



Exact solution:

$$H(W) = \frac{e^{\lambda R}(\frac{1}{\lambda} + D)e^{\lambda(W+C)}}{W} - 1, \text{use Lambert function}$$

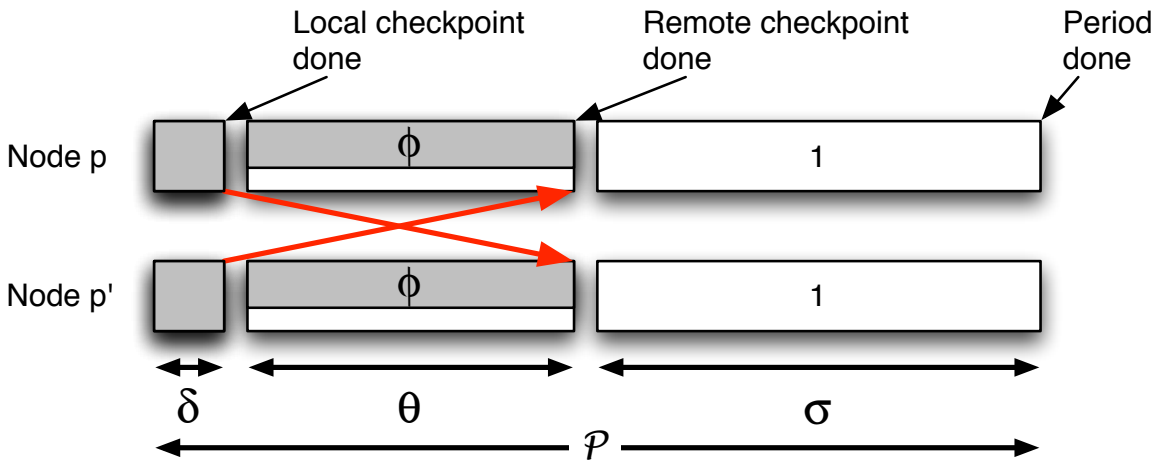
First-order approximation [Young/Daly]:

$$W_{\text{opt}} = \sqrt{\frac{2C}{\lambda}} = \sqrt{2C\mu}$$

$$H_{\text{opt}} = \sqrt{2\lambda C} + \Theta(\lambda)$$

Outline

Double checkpoint algorithm (Kale et al., UIUC)

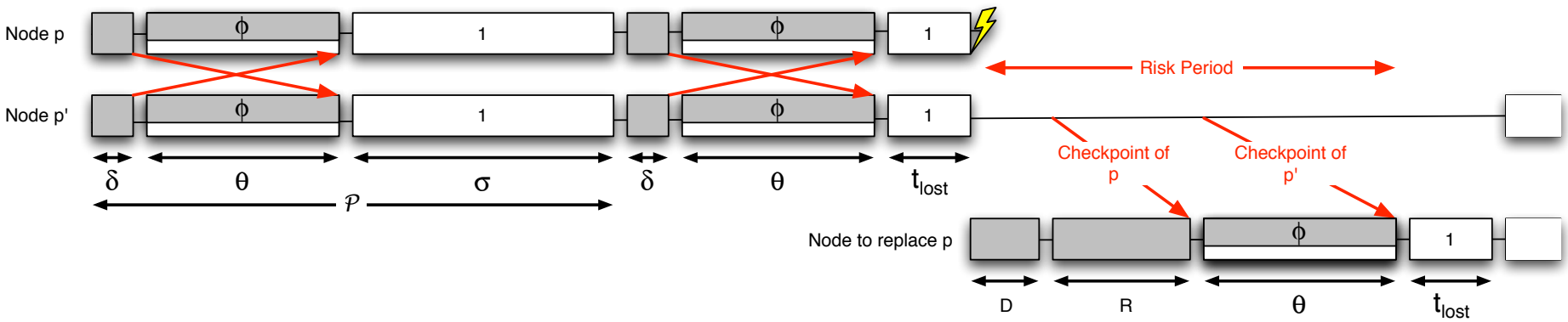


Platform nodes partitioned into pairs

Each node in a pair exchanges its checkpoint with its *buddy*

Each node saves two checkpoints:
- one locally: storing its own data
- one remotely: receiving and storing its buddy's data

Failures

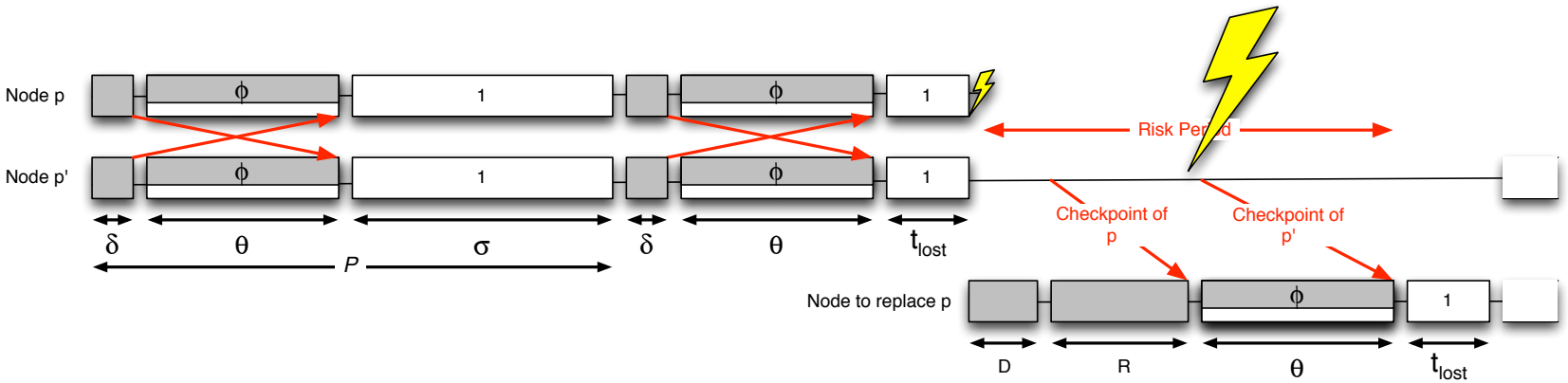


After failure: downtime D and recovery from buddy node

Two checkpoint files lost, must be re-sent to faulty processor

Best trade-off between performance and risk?

Failures



After failure: downtime D and recovery from buddy node

Two checkpoint files lost, must be re-sent to faulty processor

Application at risk until complete reception of both messages

Best trade-off between performance and risk?

Outline

Multi-level checkpointing

Coordinated checkpointing
$\Rightarrow$ **Scalability problem for large-scale platforms**

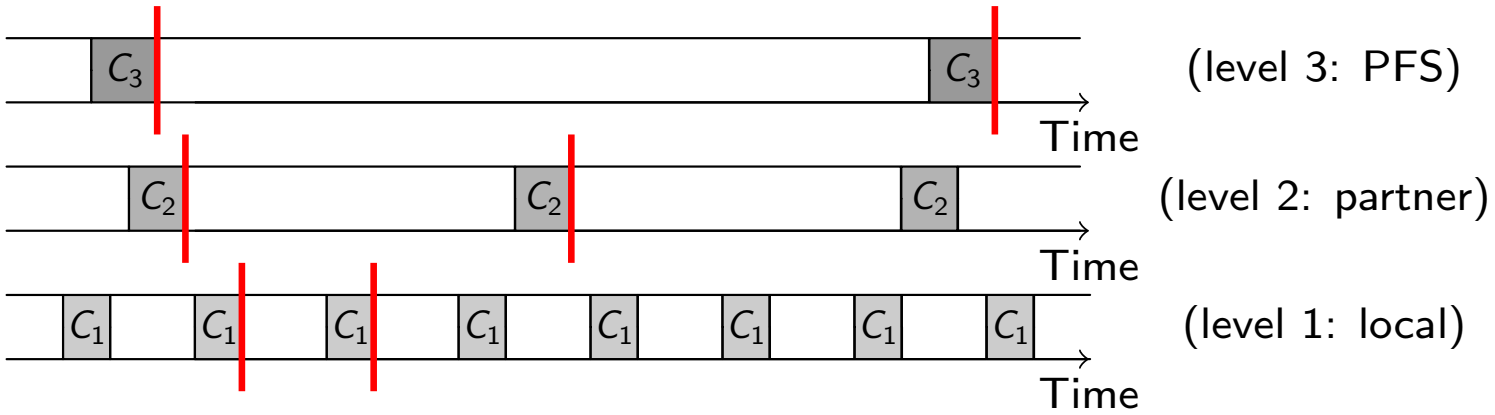
Multiple technologies to cope with different failure types:

- Local memory/SSD
- Partner copy/XOR
- Reed-Solomon coding
- Parallel file system

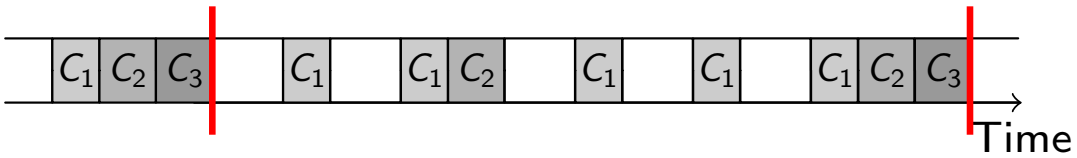
Scalable Checkpoint/Restart (SCR) library
Fault Tolerance Interface (FTI)

Simplified model

Independent checkpointing:

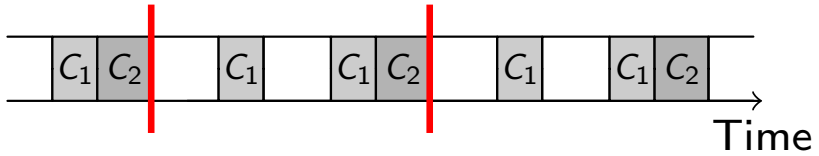


Synchronized checkpointing:



Two Levels

Easier because pattern repeats (memoryless property)



Exact solution: very complicated (which error type occurs first?), equal-length chunks, see [1]

First-order approximation:

$$H_{\text{opt}} = \sqrt{2\lambda_1 C_1} + \sqrt{2\lambda_2 C_2} + \Theta(\lambda)$$

(obtained for some optimal pattern)

[1] S. Di, Y. Robert, F. Vivien, F. Cappello. Toward an optimal online checkpoint solution under a two-level HPC checkpoint model, *IEEE TPDS*, 2017.

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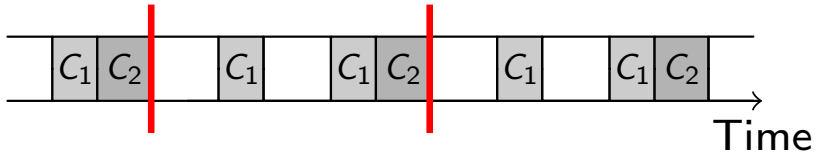

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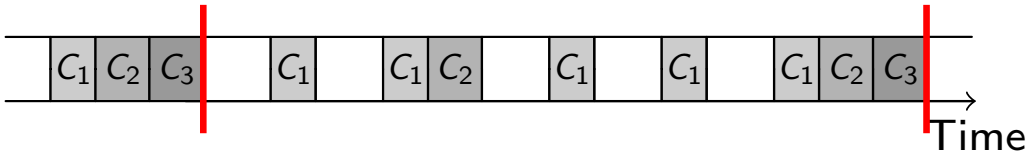
Three Levels

Difficult because sub-patterns may differ



Exact solution: unknown

First-order approximation:

$$H_{\text{opt}} = \sqrt{2\lambda_1 C_1} + \sqrt{2\lambda_2 C_2} + \sqrt{2\lambda_3 C_3} + \Theta(\lambda)$$

Choose optimal set of levels:

Level	Overhead
$1, 2, 3$	$\sqrt{2C_1\lambda_1} + \sqrt{2C_2\lambda_2} + \sqrt{2C_3\lambda_3}$
$1, 3$	$\sqrt{2C_1\lambda_1} + \sqrt{2C_3(\lambda_2 + \lambda_3)}$
$2, 3$	$\sqrt{2C_2(\lambda_1 + \lambda_2)} + \sqrt{2C_3\lambda_3}$
3	$\sqrt{2C_3(\lambda_1 + \lambda_2 + \lambda_3)}$

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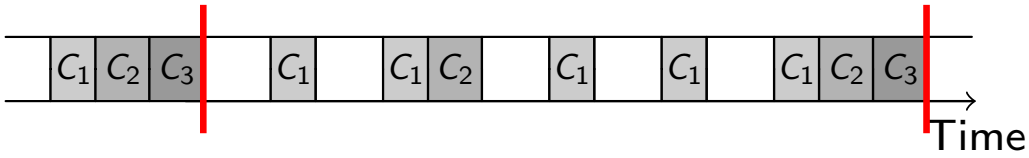

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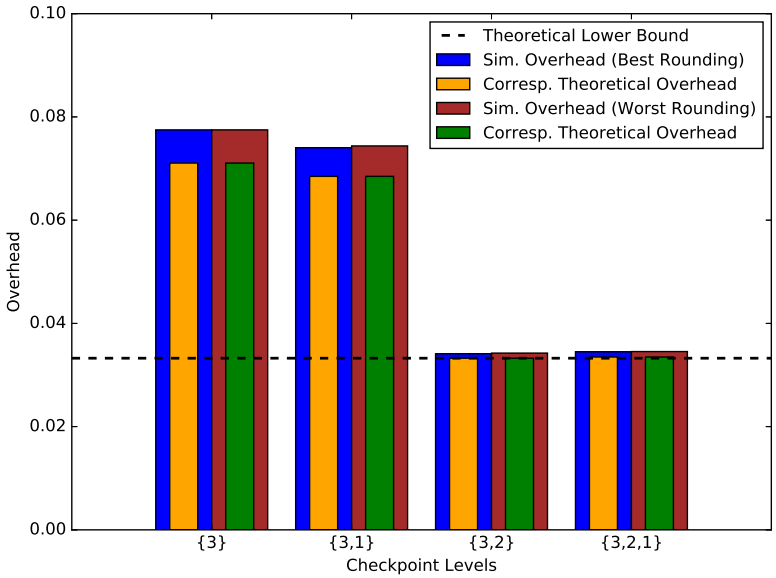
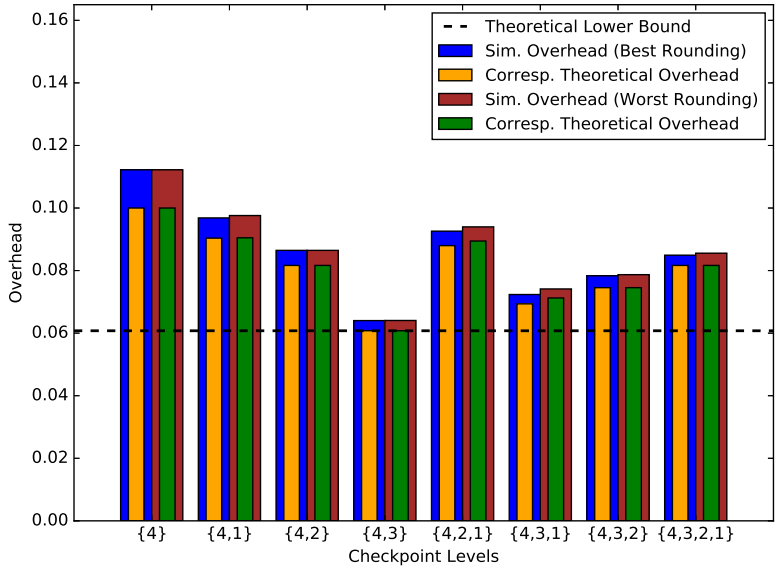
Simulations

Set	Source	Level	1	2	3	4
(A)	Moody et al. [1]	C (s)	0.5	4.5	1051	-
		MTBF (s)	5.00e6	5.56e5	2.50e6	-
(B)	Balaprakash et al. [2]	C (s)	10	20	20	100
		MTBF (s)	3.60e4	7.20e4	1.44e5	7.20e5



(A)



(B)

[1] A. Moody, G. Bronevetsky, K. Mohror, and B. R. de Supinski. Design, modeling, and evaluation of a scalable multi-level checkpointing system. *Supercomputing*, 2010.

[2] P. Balaprakash, L. A. Bautista-Gomez, M.-S. Bouguerra, S. M. Wild, F. Cappello, and P. D. Hovland. Analysis of the tradeoffs between energy and run time for multilevel checkpointing. *PMBS*, 2014.

Takeaway

Explicit formulas for (almost) optimal multi-level checkpointing

$$H_{\text{opt}} = \sum_{\ell=1}^{k} \sqrt{2\lambda_\ell C_\ell} + \Theta(\lambda)$$

Limitations:

- First-order accurate for platform MTBF in hours
  $\Longleftrightarrow$ 10,000s of nodes. Beyond?
- Independent errors ☹
  Correlated failures across levels?

---

[1] A. Benoit, A. Cavelan, Y. Robert and H. Sun. Towards optimal multi-level checkpointing, *IEEE TC*, 2017.

Outline

Definitions

Instantaneous error detection ⇒ fail-stop failures, e.g. resource crash

Silent errors (data corruption) ⇒ detection latency

Silent error detected only when the corrupt data is activated

Includes some software faults, some hardware errors (soft errors in L1 cache), double bit flip

Cannot always be corrected by ECC memory

Probability distributions for silent errors

Theorem: $\mu_p = \dfrac{\mu_{\text{ind}}}{p}$ for arbitrary distributions

Probability distributions for silent errors

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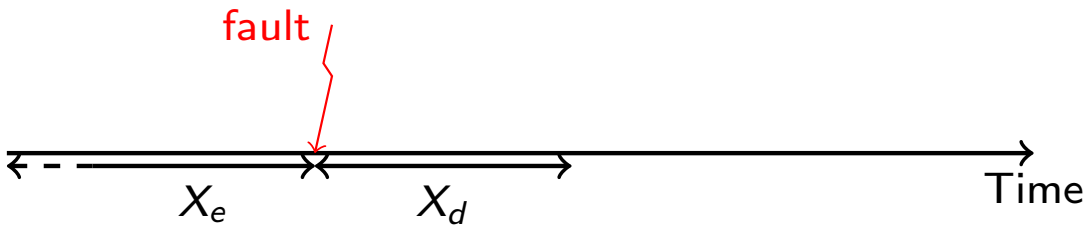
General-purpose approach



Error and detection latency

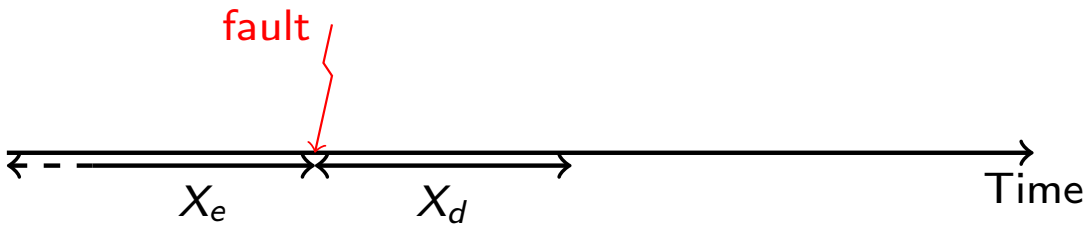
Last checkpoint may have saved an already corrupted state

Saving k checkpoints (Lu, Zheng and Chien):

① Critical failure when all live checkpoints are invalid

② Which checkpoint to roll back to?

General-purpose approach



Error and detection latency

Last checkpoint may have saved an already corrupted state

Saving k checkpoints (Lu, Zheng and Chien):

1. Critical failure when all live checkpoints are invalid
   Assume unlimited storage resources
2. Which checkpoint to roll back to?
   Assume verification mechanism

Limitation of the model

It is not clear how to detect when the error has occurred (hence to identify the last valid checkpoint) ☹ ☹ ☹

Need a verification mechanism to check the correctness of the checkpoints. This has an additional cost!

Coupling checkpointing and verification

Verification mechanism of cost V

Silent errors detected only when verification is executed

Approach agnostic of the nature of verification mechanism (checksum, error correcting code, coherence tests, etc)

Fully general-purpose
(application-specific information, if available, can always be used to decrease V)

On-line ABFT scheme for PCG

1 : Compute $r^{(0)} = b - Ax^{(0)}, z^{(0)} = M^{-1}r^{(0)}, p^{(0)} = z^{(0)}$, and $\rho_0 = {r^{(0)}}^T z^{(0)}$ for some initial guess $x^{(0)}$
2 : checkpoint: A, M, and b
3 : for $i = 0, 1, \ldots$
4 : if ((i>0) and $(i\%d = 0)$)
5 : if ($\frac{{p^{(i+1)}}^T q^{(i)}}{||p^{(i+1)}||.||q^{(i)}||} > 10^{-10}$ or $\frac{||r^{(i+1)} + Ax^{(i+1)} - b||}{||b||.||A||} > 10^{-10}$)
6 : recover: A, M, b, i, ρ_i, $p^{(i)}$, $x^{(i)}$, and $r^{(i)}$.
7 : else if ($i\%(cd) = 0$)
8 : checkpoint: i, ρ_i, $p^{(i)}$, and $x^{(i)}$
9: endif
10: endif
11: $q^{(i)} = Ap^{(i)}$
12: $\alpha_i = \rho_i / {p^{(i)}}^T q^{(i)}$
13: $x^{(i+1)} = x^{(i)} + \alpha_i p^{(i)}$
14: $r^{(i+1)} = r^{(i)} - \alpha_i q^{(i)}$
15: solve $Mz^{(i+1)} = r^{(i+1)}$, where $M = M^T$
16: $\rho_{i+1} = {r^{(i+1)}}^T z^{(i+1)}$
17: $\beta_i = \rho_{i+1} / \rho_i$
10: $p^{(i+1)} = z^{(i+1)} + \beta_i p^{(i)}$
19: check convergence; continue if necessary
20: end

Zizhong Chen, PPoPP'13

Iterate PCG
Cost: SpMV, preconditioner solve, 5 linear kernels

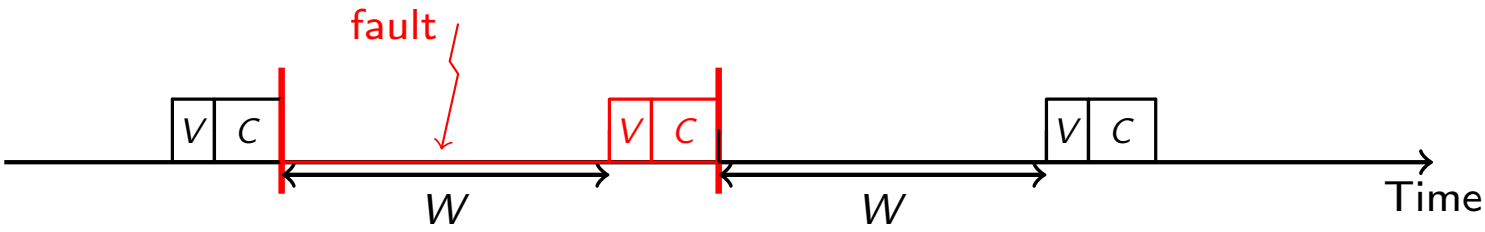
Detect soft errors by checking orthogonality and residual

Verification every d iterations
Cost: scalar product+SpMV

Checkpoint every c iterations
Cost: three vectors, or two vectors + SpMV at recovery

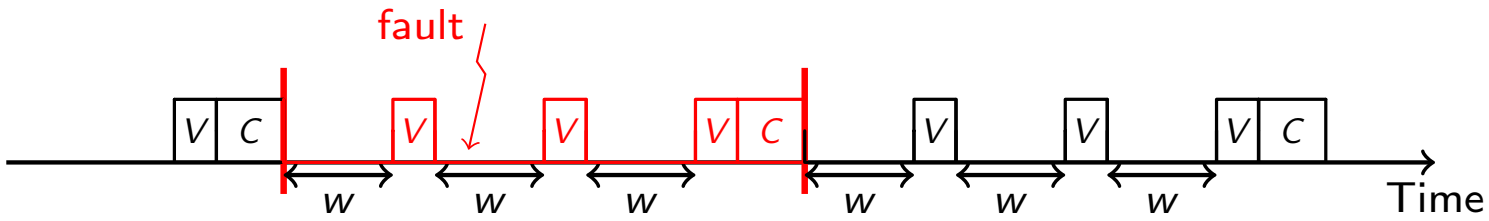
Experimental method to choose c and d

Base pattern (and revisiting Young/Daly)



	Fail-stop (classical)	Silent errors
Pattern	$T = W + C$	$S = W + V + C$
$\text{WASTE}[FF]$	$\frac{C}{T}$	$\frac{V+C}{S}$
$\text{WASTE}[fail]$	$\frac{1}{\mu}(D + R + \frac{W}{2})$	$\frac{1}{\mu}(R + W + V)$
Optimal	$T_{\text{opt}} = \sqrt{2C\mu}$	$S_{\text{opt}} = \sqrt{(C+V)\mu}$
$\text{WASTE}[opt]$	$\sqrt{\frac{2C}{\mu}}$	$2\sqrt{\frac{C+V}{\mu}}$

With $p = 1$ checkpoint and $q = 3$ verifications



Base Pattern	$p = 1, q = 1$	$\text{WASTE}[opt] = 2\sqrt{\frac{C+V}{\mu}}$
New Pattern	$p = 1, q = 3$	$\text{WASTE}[opt] = 2\sqrt{\frac{4(C+3V)}{6\mu}}$

Application-specific methods (1/2)

ABFT: dense matrices / fail-stop, extended to sparse / silent. Limited to one error detection and/or correction in practice

Asynchronous (chaotic) iterative methods (old work)

Partial differential equations: use lower-order scheme as verification mechanism (detection only, Benson, Schmit and Schreiber)

FT-GMRES: inner-outer iterations (Hoemmen and Heroux)

PCG: orthogonalization check every k iterations, re-orthogonalization if problem detected (Sao and Vuduc)

Algorithm-based focused recovery: use application data-flow to identify potential error source and corrupted nodes (Fang and Chien 2014)

Application-specific methods (2/2)

Dynamic monitoring of datasets based on physical laws (e.g., temperature/speed limit) and space or temporal proximity (Bautista-Gomez and Cappello)

Time-series prediction, spatial multivariate interpolation (Di et al.)

Offline training, online detection based on SDC signature for convergent iterative applications (Liu and Agrawal)

Spatial regression based on support vector machines (Subasi et al.)

Many others ata-analytics/machine learning approaches

Application-specific detectors

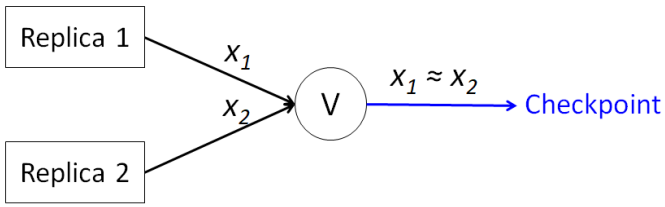
Do you believe it?

- Detectors are not perfect
- High recall is expensive if at all achievable
- With higher error rates, it would be good to correct a few errors

Replication mandatory at scale? ☹

Why Is Replication Useful?

Error detection (duplication):



Error correction (triplication):

Why Is Replication Useful?

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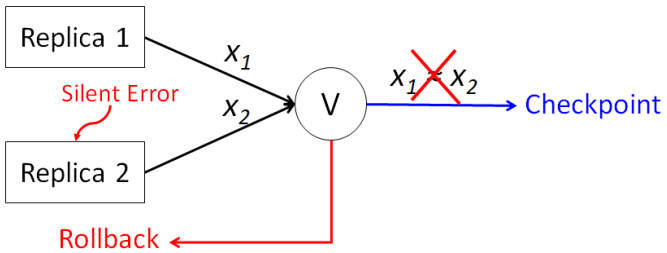

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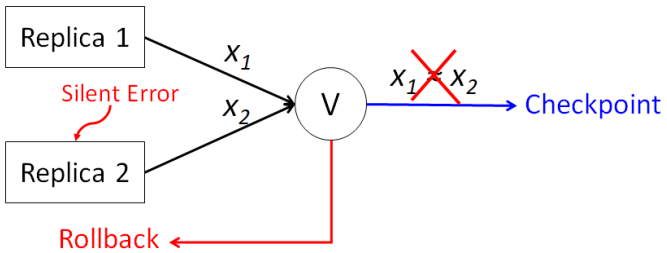

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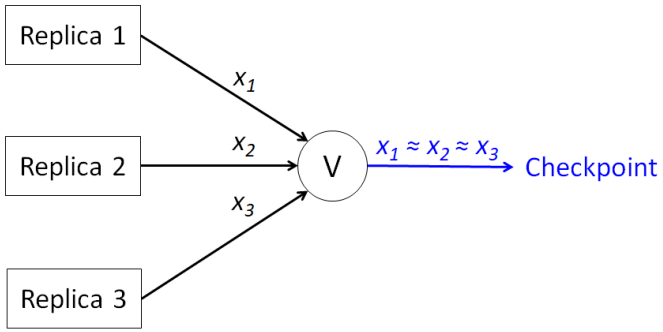

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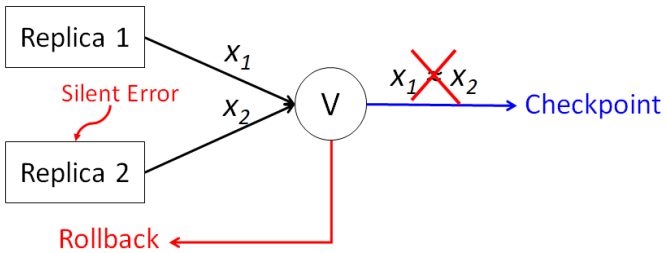

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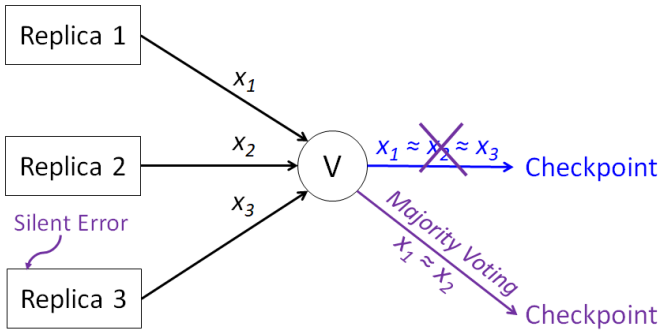

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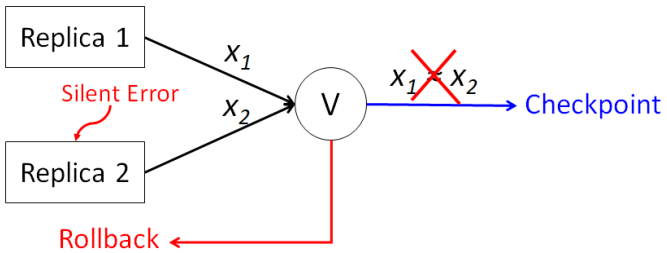

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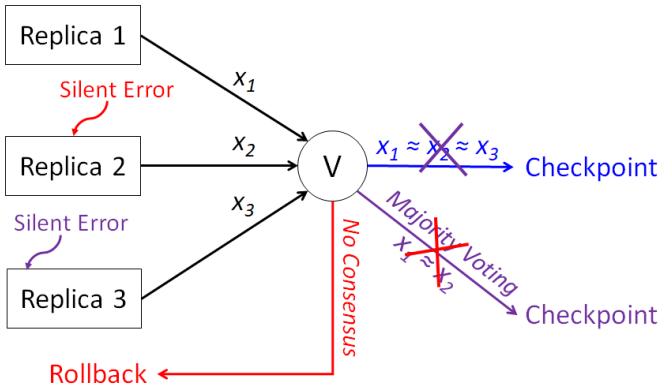
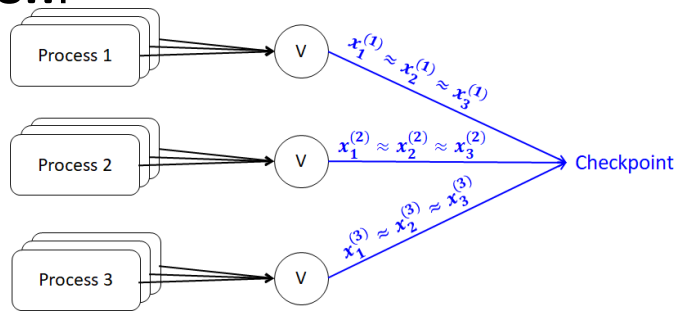


Two Replication Modes

Process Replication:



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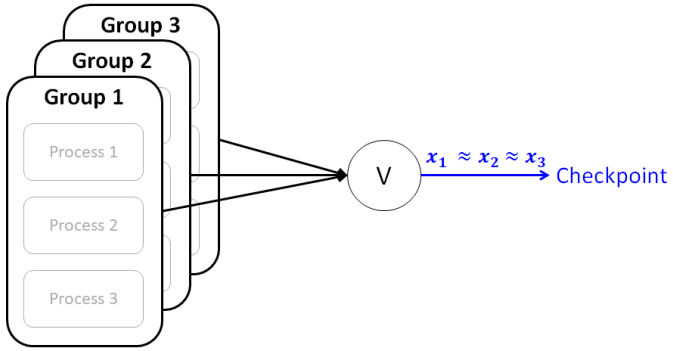

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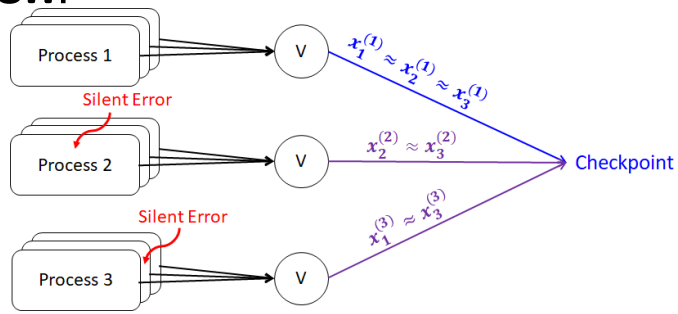

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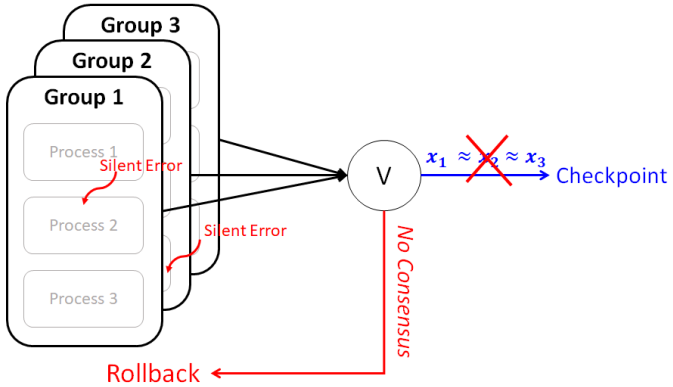

A few questions

Silent errors

- Error rate? MTBE?
- Selective reliability?
- New algorithms beyond iterative? matrix-product, FFT, ...
- Multi-level patterns for both fail-stop and silent errors

Resilient research on resilience

Models needed to assess techniques at scale without bias ☺

A few questions

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Resilient research on resilience

Models needed to assess techniques at scale
without bias ☺

Outline

IO contention

Space-sharing prevalent in HPC platforms

Application instances:

- have dedicated computational nodes
- share interconnect links and storage partition (PFS)
- checkpoint (to stable storage) independently

$\Rightarrow$ network and storage contention

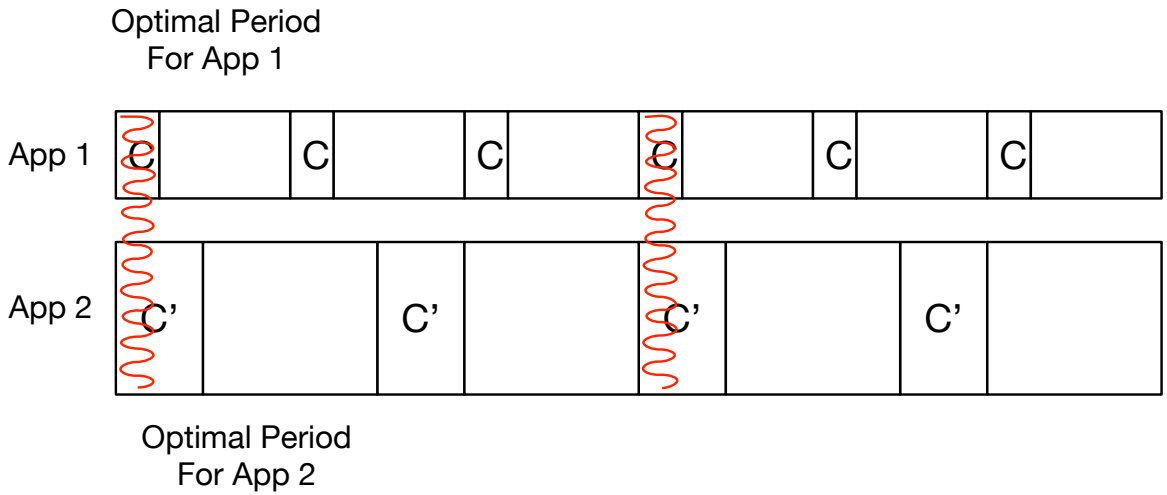
Checkpointing 101 (revisited)

When do applications checkpoint on HPC systems?

State-of-the-art: Young/Daly period

Standard practice: every hour ☹

How long does it take to checkpoint?



Optimal period computed assuming fixed checkpoint cost

Interferences between checkpointing I/O of App 1 and App 2 change their checkpoint time

$\Rightarrow$ Applications checkpoint too often

When to checkpoint in a shared environment,
since checkpoint cost is not predictable?

Model

Platform

I/O subsystem time-shared (contended)

Linear interference model

Workload

Many applications but only a few classes (sets of applications with similar sizes, durations, footprints and I/O needs)

Initialization and finalization I/O at max bandwidth; regular (non-CR) I/O evenly distributed over execution

Job makespans known a priori

Simulations based on APEX workflow / Cielo platform

Checkpoint

Fixed: 1 hour (unless otherwise specified)

Daly: uses Young/Daly application period $\sqrt{2C_{app}\mu_{app}}$

Outline

I/O Scheduling Algorithms

Oblivious (Fixed / Daly)

No scheduling of any I/O: when overlapping, interfere linearly

⇒ Risk of I/O Inefficiency

Ordered (Fixed / Daly)

I/O (checkpoint or init/final) served First Come - First Served

If another application is being served, wait in turn

⇒ Risk of delayed I/O and checkpoints, increasing waste

Ordered-NB (Fixed / Daly)

I/O (checkpoint or init/final) served First Come - First Served

In case of checkpoints, continue working until served

⇒ Risk of extra re-execution due to delayed checkpoints

Least-Waste

Serve I/O request that minimizes potential waste

⇒ Checkpoints are non-blocking: continue working until they are served

⇒ Daly period embedded in scheduling (prevent from checkpointing too often)

Oblivious

Jobs fill up the system based on processor availability

I/O workloads (including CR activities) not coordinated

Each I/O stream given decrease in bandwidth linearly proportional to the number of competing operations

Subsequent checkpoint scheduled to start after $P_i - C_i$
$\Rightarrow$ Resultant checkpoint period may be longer than P_i

Ordered

Blocking FCFS I/O Scheduling

I/O requests performed sequentially, in request arrival order

Jobs with outstanding I/O requests blocked until their requests are completed

With two jobs simultaneously requesting I/O of volume V_1, V_2:

Oblivious: Linear interference (both jobs I/O are slowed down) until the smallest of (V_1, V_2) is transferred

Ordered:

- first scheduled job takes $\frac{V_1}{\beta_{\text{avail}}}$
- second job waits $\frac{V_1}{\beta_{\text{avail}}}$ then takes $\frac{V_2}{\beta_{\text{avail}}}$

Resultant checkpoint period may be longer than P_i

Ordered-NB

Non-Blocking FCFS I/O Scheduling

Refactor code to continue computing while awaiting **checkpoint** I/O

Previous checkpoint ends at time t_{now}
$\Rightarrow$ tentative time for next checkpoint $t_{req} = t_{now} + P_i - C_i$

At t_{req}, make non-blocking I/O request (I/O token still FCFS)

Job continues until I/O token is available

At this point, job generates its checkpoint data

Use existing APIs in SCR or FTI to regularly poll if a checkpoint should be taken at this time

Postponed checkpoint $\Rightarrow$ increased risk exposure

Least-Waste

Non-Blocking least waste I/O Scheduling

When an I/O request completes at time t,
select best candidate from pool:

IO-Candidate $\mathcal{C}_{IO}$
Job J_i, $1 \leq i \leq r$ with an (input, output or recovery) I/O request of length v_i seconds, has q_i processors, initiated its I/O request d_i seconds ago (idle since)

Ckpt-Candidate $\mathcal{C}_{Ckpt}$
Job J_i, $r+1 \leq i \leq r+s$, with a checkpoint duration of C_i seconds and q_i processors, took its last checkpoint d_i seconds ago and keeps executing, with $d_i \geq P_{Daly}(J_i)$

Job selection

$J_i \in \mathcal{C}_{IO}$ uses the I/O resource for v_i seconds

For $J_j \in \mathcal{C}_{IO}$, $W_i(j) = q_j(d_j + v_i)$

For $J_j \in \mathcal{C}_{Ckpt}$, $W_i(j) = \frac{v_i}{\mu_{\text{ind}}} q_j^2 (R_j + d_j + \frac{v_i}{2})$

Expected waste $W_i = \sum_{J_j \in \mathcal{C}_{IO}, j \neq i} W_i(j) + \sum_{J_j \in \mathcal{C}_{Ckpt}} W_i(j)$

$J_i \in \mathcal{C}_{Ckpt}$ uses the I/O resource for C_i seconds

Similar equations ...

Select job $J_i \in \mathcal{C}_{IO} \cup \mathcal{C}_{Ckpt}$ whose waste W_i is minimal

Feasibility of Cooperative Strategies

Ordered, *Ordered-NB*, *Least-Waste* require synchronization

Ordered
at filesystem level

Ordered-NB and *Least-Waste*:
modify apps to continue working until access is granted
$\Rightarrow$ implementation in checkpointing library SCR or FTI

Memory hierarchy:
- checkpoint process memory on unreliable (but fast) media
- upload checkpoints in the background,
while the application proceeds to compute

Steady-state

n_i jobs of class A_i, q_i nodes, $C_i = \frac{size_i}{\beta_{avail}}$

Waste of J_i with checkpoint period P_i:

$$W_i = W_i(P_i) = \frac{C_i}{P_i} + \frac{q_i}{\mu}(\frac{P_i}{2} + R_i)$$

MINIMIZE

$$W = \sum_i \frac{n_i q_i}{\mathcal{N}} \left(\frac{C_i}{P_i} + \frac{q_i}{\mu}(\frac{P_i}{2} + R_i) \right)$$

SUBJECT TO

$$F = \sum_i \frac{n_i C_i}{P_i} \leq 1$$

Lower bound

KKT

$$P_i = \sqrt{\frac{2\mu\mathcal{N}}{q_i^2}\left(\frac{q_i}{\mathcal{N}} + \lambda\right) C_i}$$

Choose λ minimal s.t. $F \leq 1$ (solve numerically)

$\lambda = 0 \Rightarrow$ Young/Daly

I/O constraint not sufficient

$\Rightarrow$ orchestrate checkpoints into periodic repeating pattern

$\Rightarrow$ lower bound of $W = \sum_i \frac{n_i q_i}{\mathcal{N}} W_i(P_i)$

Outline

LANL Workloads from the APEX Workflows report

Workflow	EAP	LAP	Silverton	VPIC
Workload percentage	66	5.5	16.5	12
Work time (h)	262.4	64	128	157.2
Number of cores	16384	4096	32768	30000
Initial Input (% of memory)	3	5	70	10
Final Output (% of memory)	105	220	43	270
Checkpoint Size (% of memory)	160	185	350	85

Cielo

1.37 Petaflops capability system at LANL (2010-2016)

143,104 cores, 286 TB main memory

PFS with theoretical maximum capacity 160GB/s

Simulation Framework

Random selection of jobs according to class ratios

Duration uniformly distributed between $0.8w$ and $1.2w$

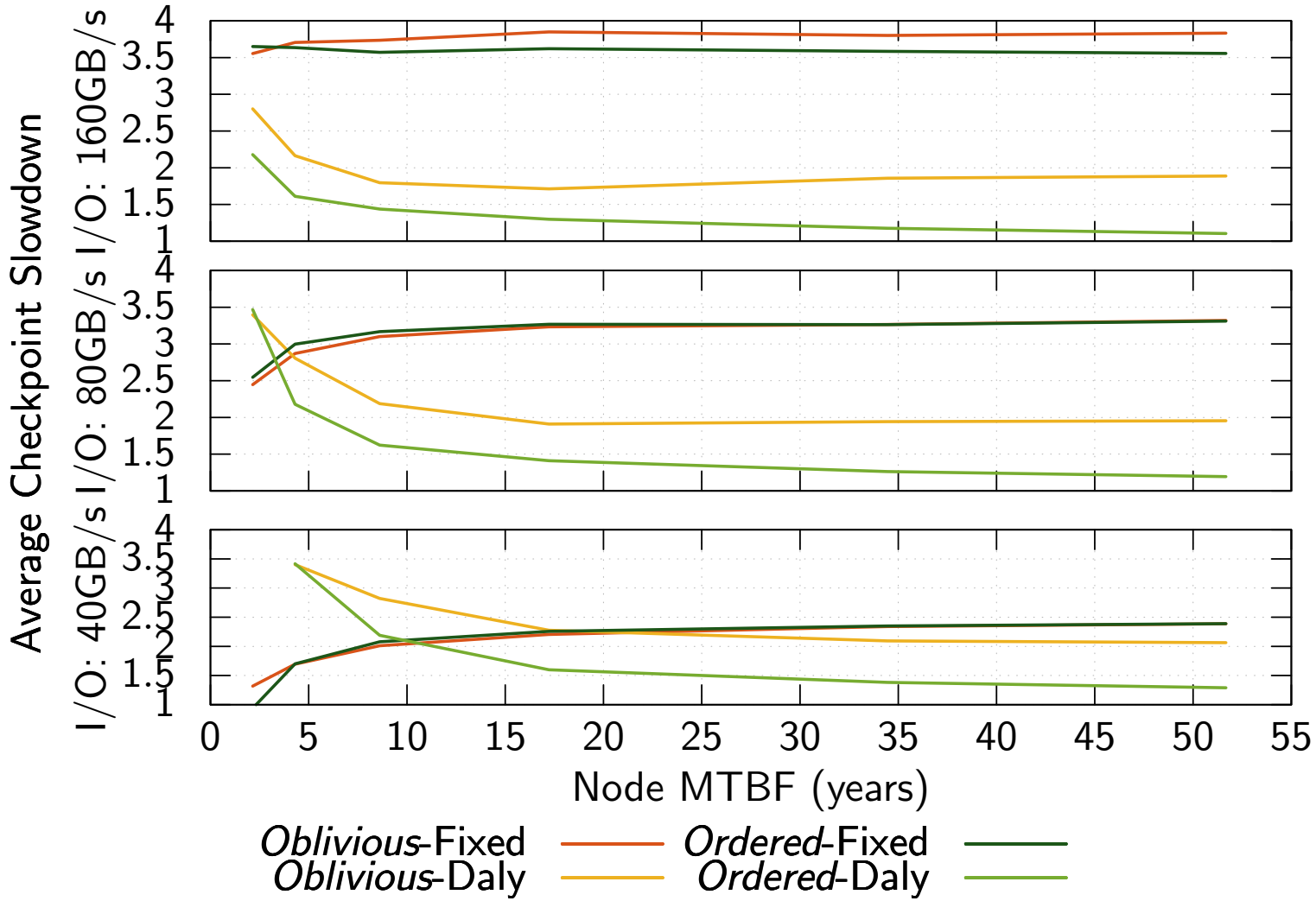
Generation of node failures with Exponential distributions

First-fit strategy (job characteristics, job priority, resource availability)

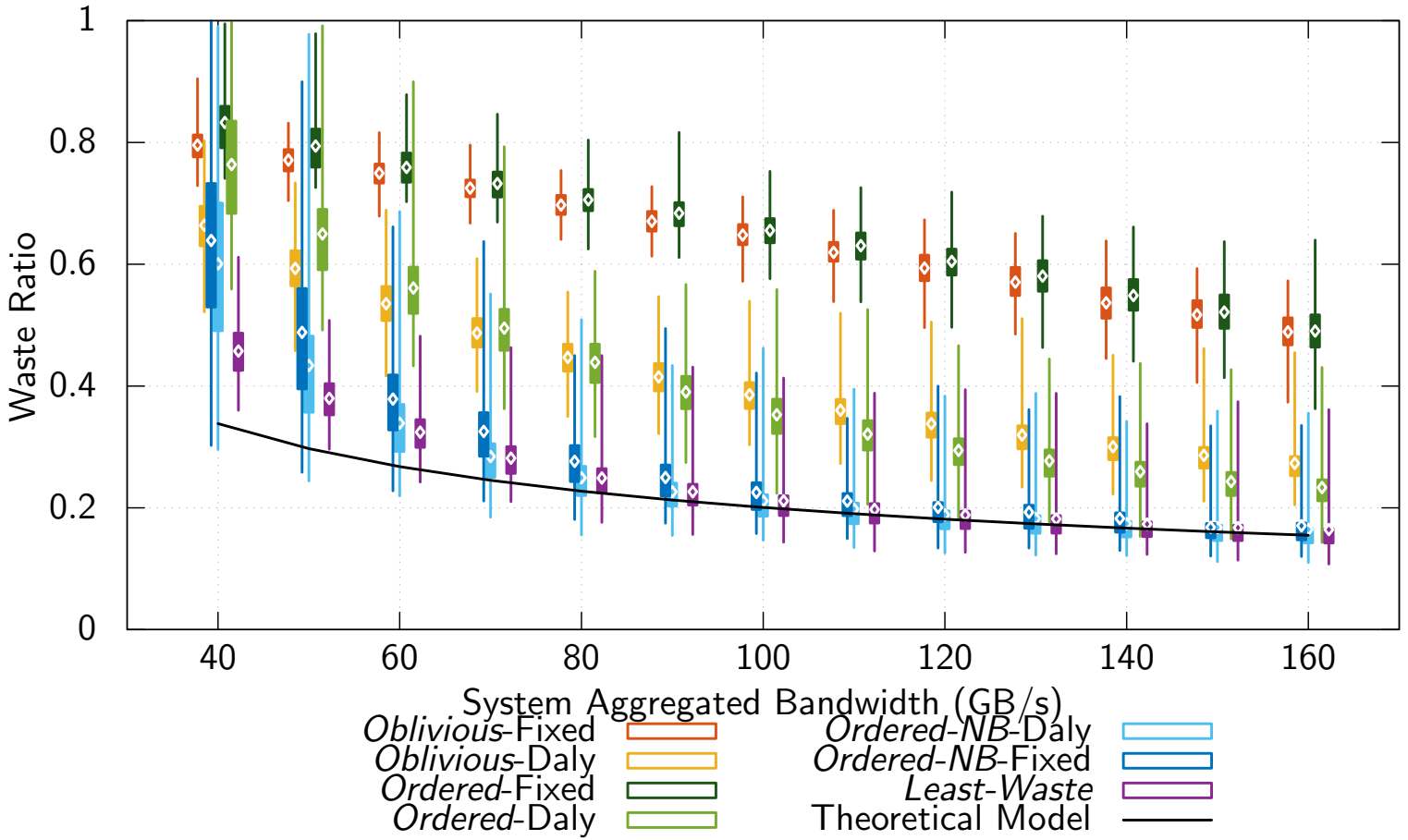
Simulate online scheduling

Restarted jobs set to highest priority

Slowdown of checkpoints

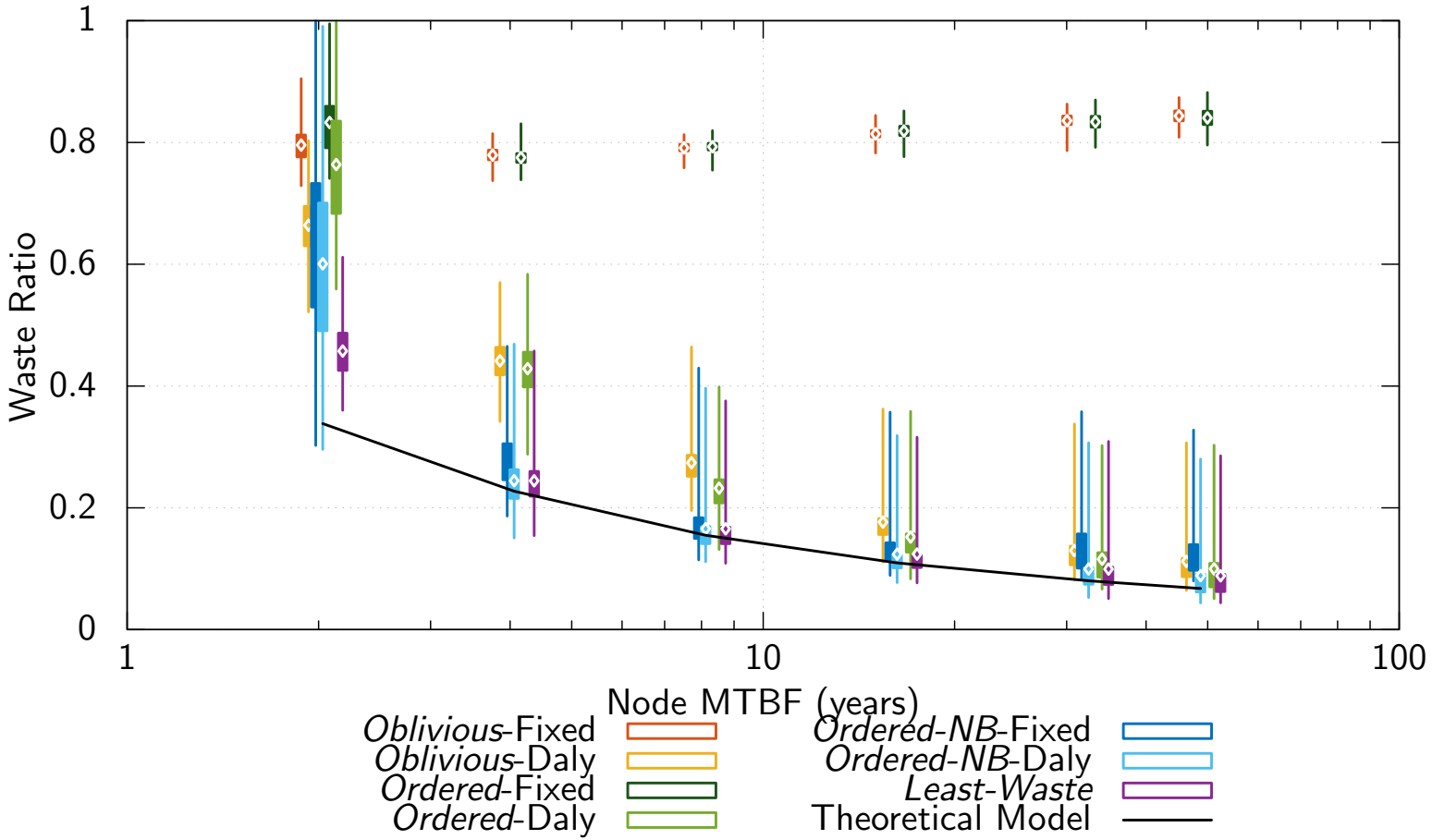


Waste as a function of system bandwidth



$\mu_{\text{ind}} = 2\text{years}, \ \mu = 1\text{hour}$

Waste as a function of system MTBF



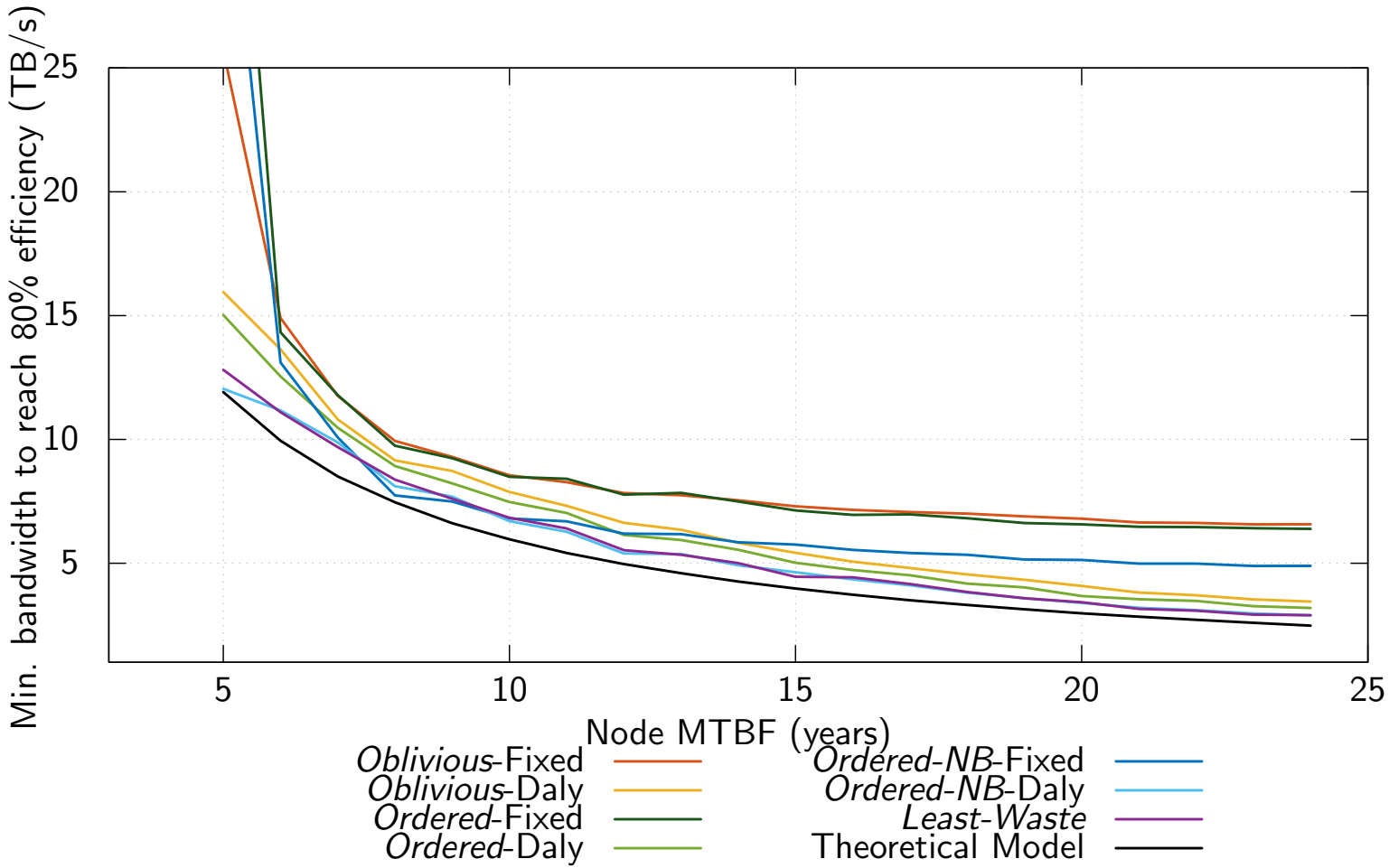
$\beta_{\text{avail}} = 40\text{GBs}$

Prospective system (1/2)

Aurora-like
7PB of main memory and 50,000 compute nodes

Scale APEX workflow
accordingly to Aurora/Celio memory size increase

Prospective system (2/2)



Minimum aggregated filesystem bandwidth to reach 80% efficiency

Burst buffers

Dedicated

- Same throughput constraint
- Schedule according to priority
- Allows for some slack (shift checkpoints)

Shared

- Hierarchical system
- Same contention problem at subsystem level

See paper

To appear in IJNC. Also RR Inria.

Outline

A little scheduling problem

Independent tasks, IID execution times with distribution $\mathcal{D}$

Platform: identical processors, unit speed, unit cost

User: limited budget b and execution deadline d

Objective: maximize expected number of tasks completed

Motivation

Imprecise computations: tasks have a mandatory part and optional part, maximize optional parts with leftover time and budget

A little scheduling problem (2/2)

Scheduling policy

Decide how many processors to launch & stop at each second

Processors interrupted when deadline or budget is exceeded

Each task can be deleted at any instant before completion

Non-preemptive execution:

- interrupted tasks cannot be relaunched
- time/budget spent until interruption: completely lost

Outline

Simple instance

One processor

Unlimited budget, no deadline

Discrete distribution:

Probability	Execution time
p_1	w_1
p_2	w_2
p_3	w_3

Objective: maximize success rate per time/budget unit $\mathcal{R}$

Simple instance

One processor

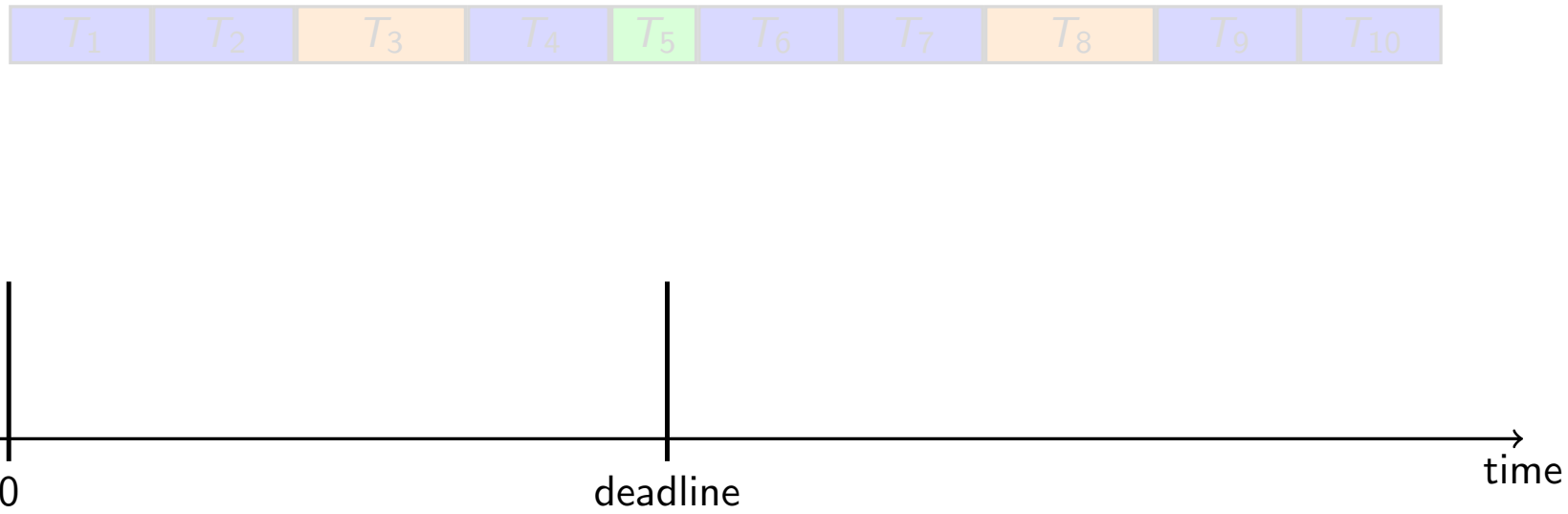
Unlimited budget, no deadline

Discrete distribution:

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 6$

Objective: maximize success rate per time/budget unit $\mathcal{R}$

Illustrating example



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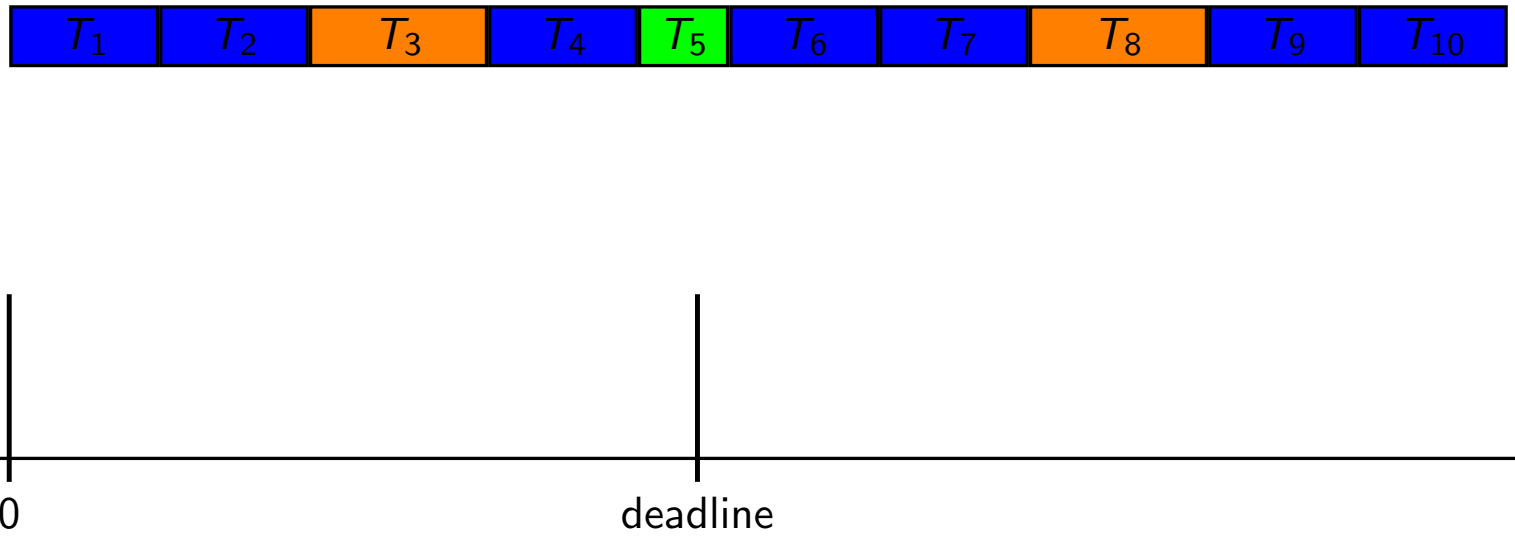

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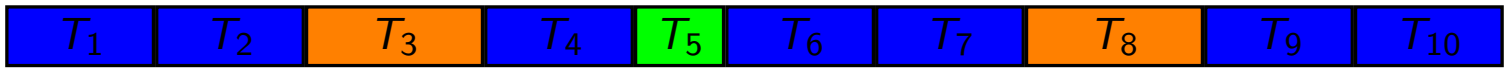

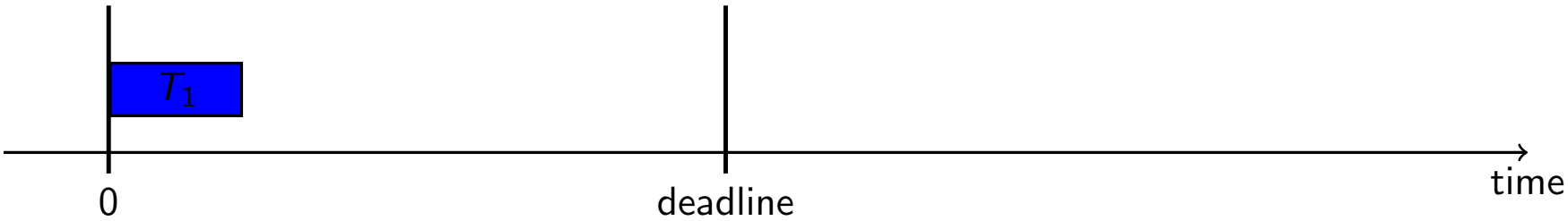

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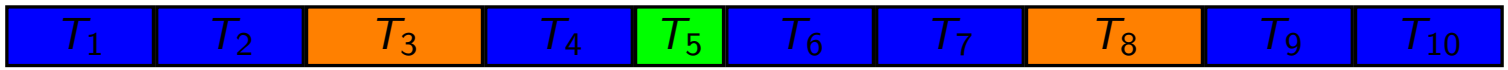

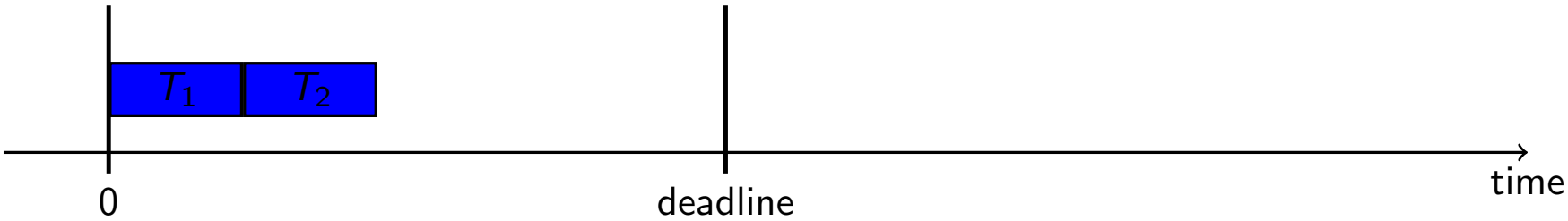

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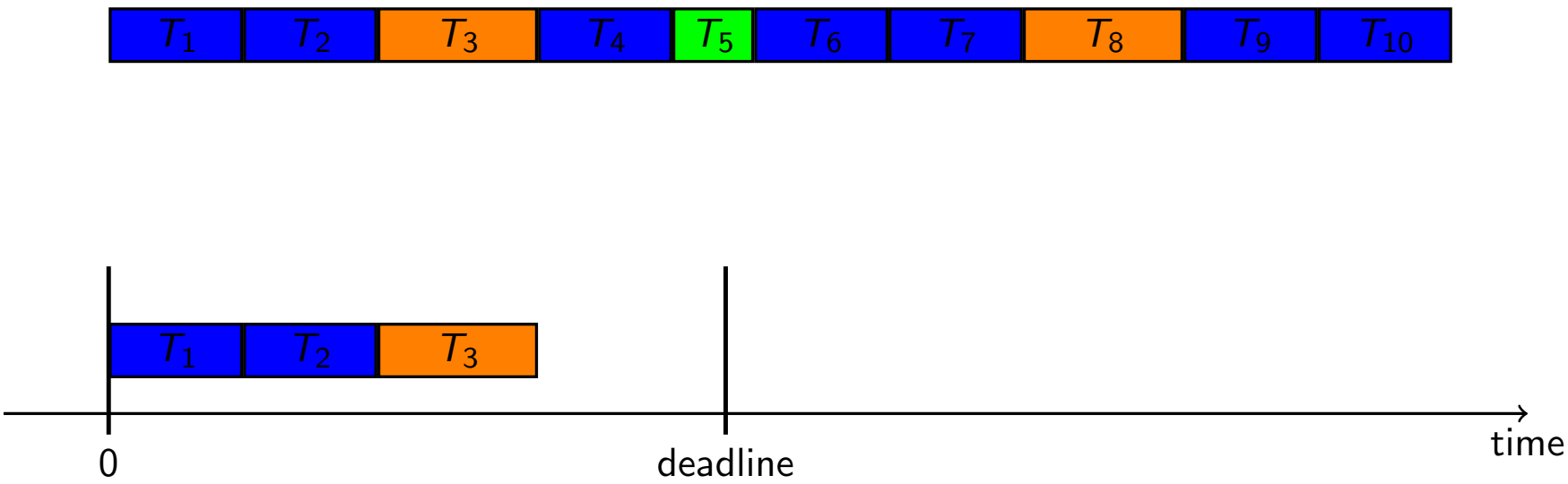

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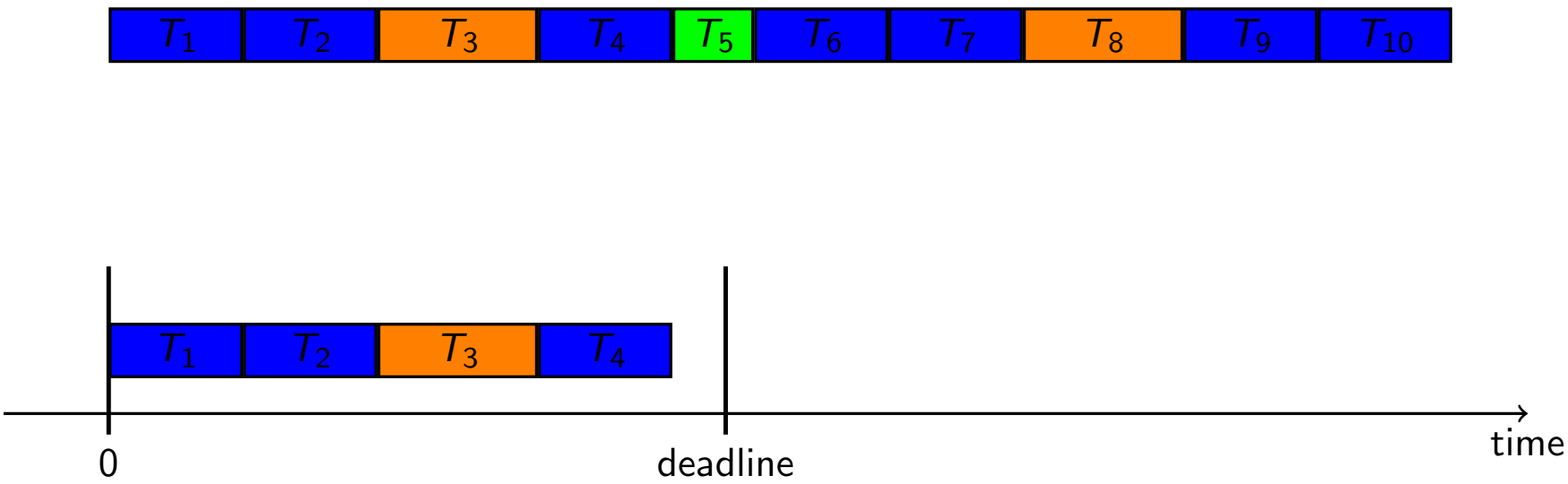

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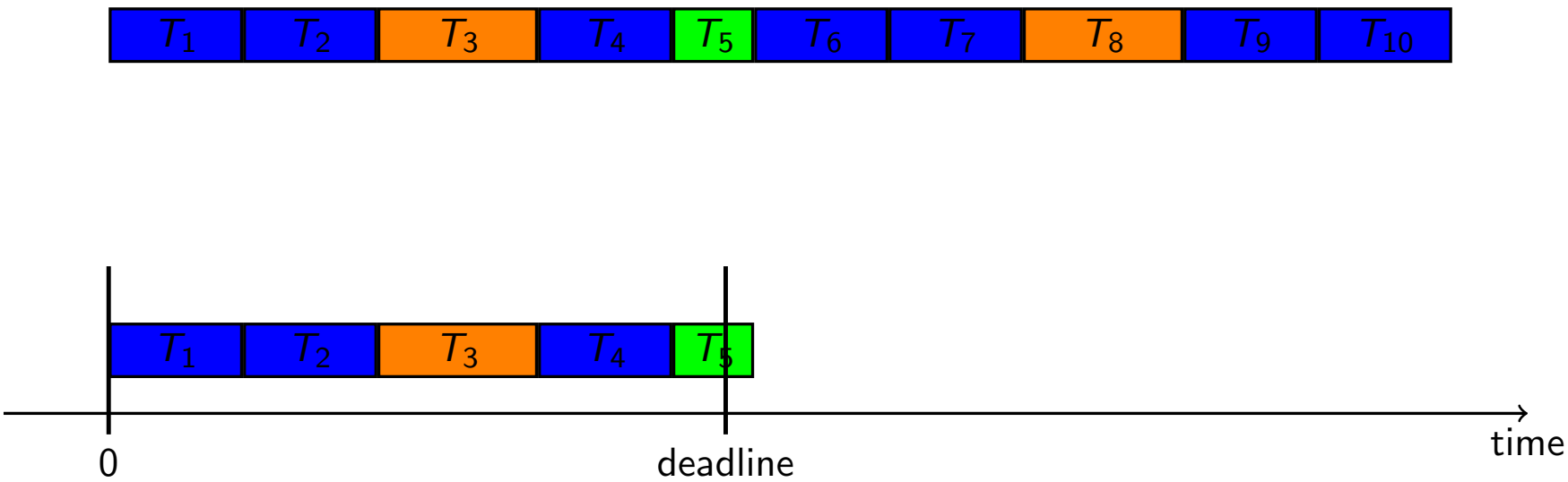

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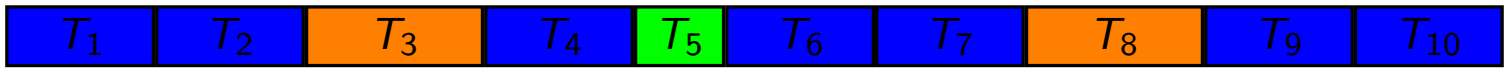

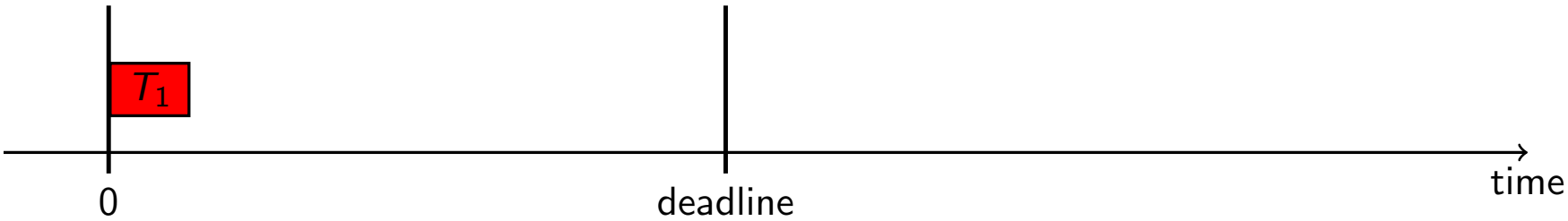

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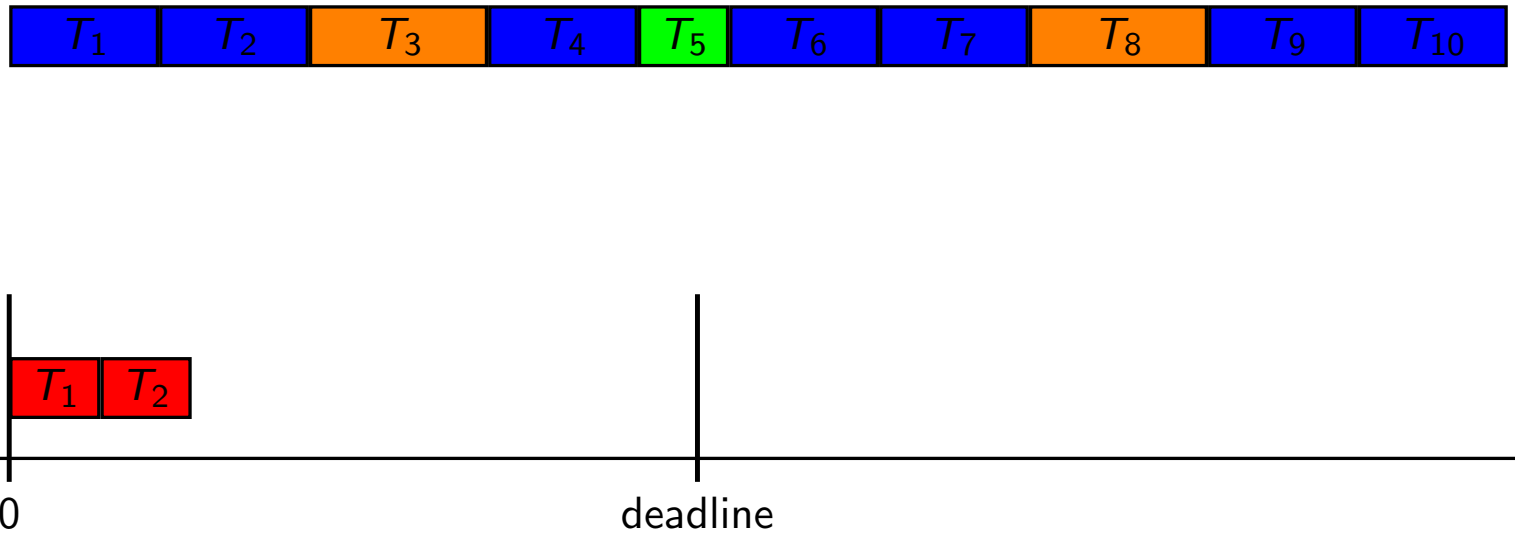

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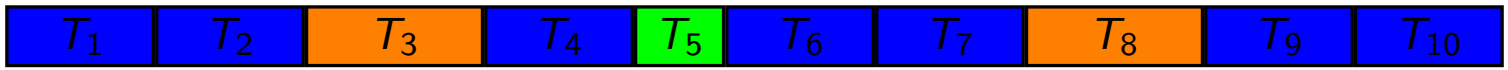

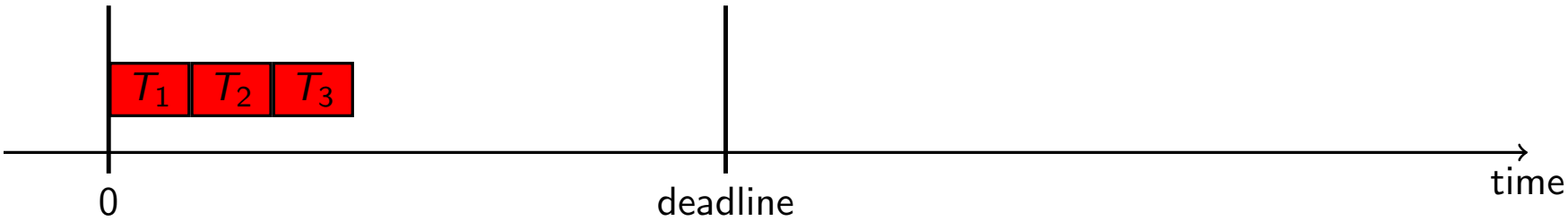

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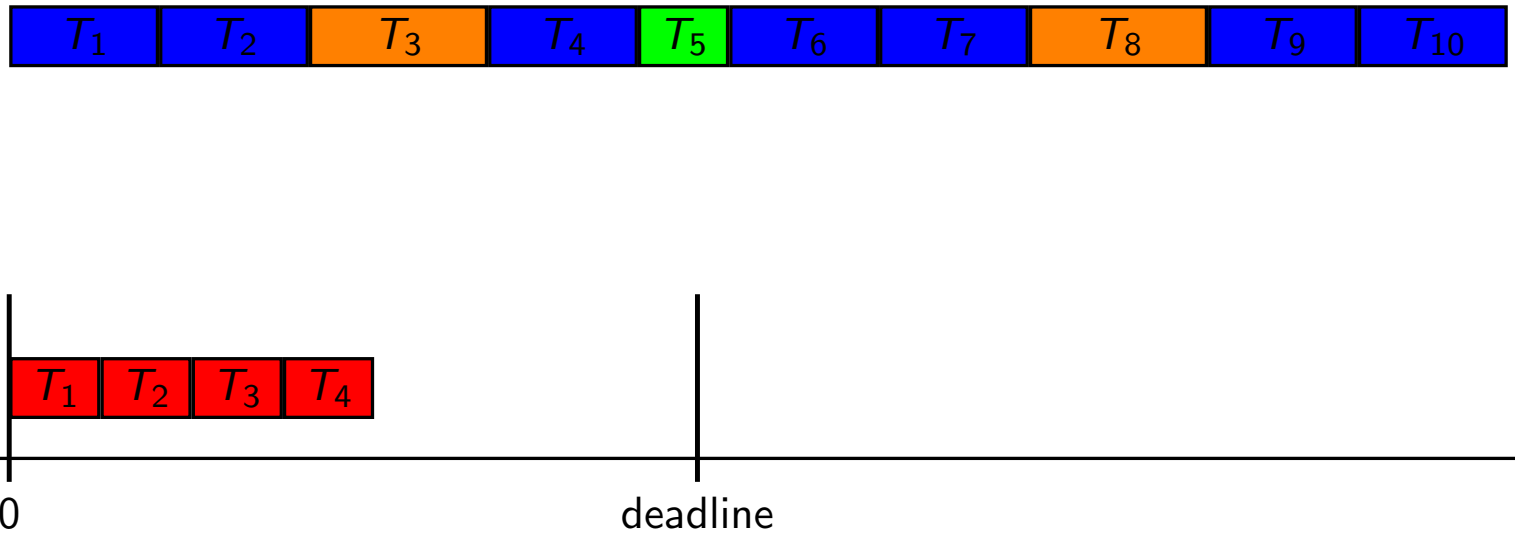

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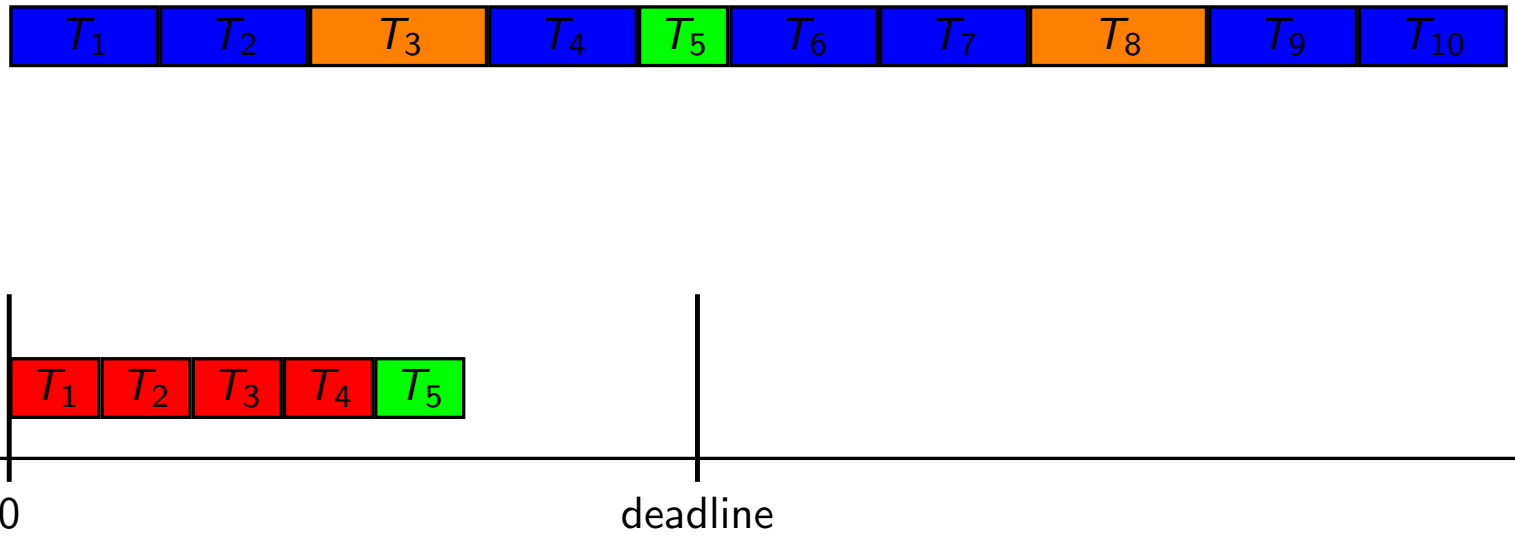

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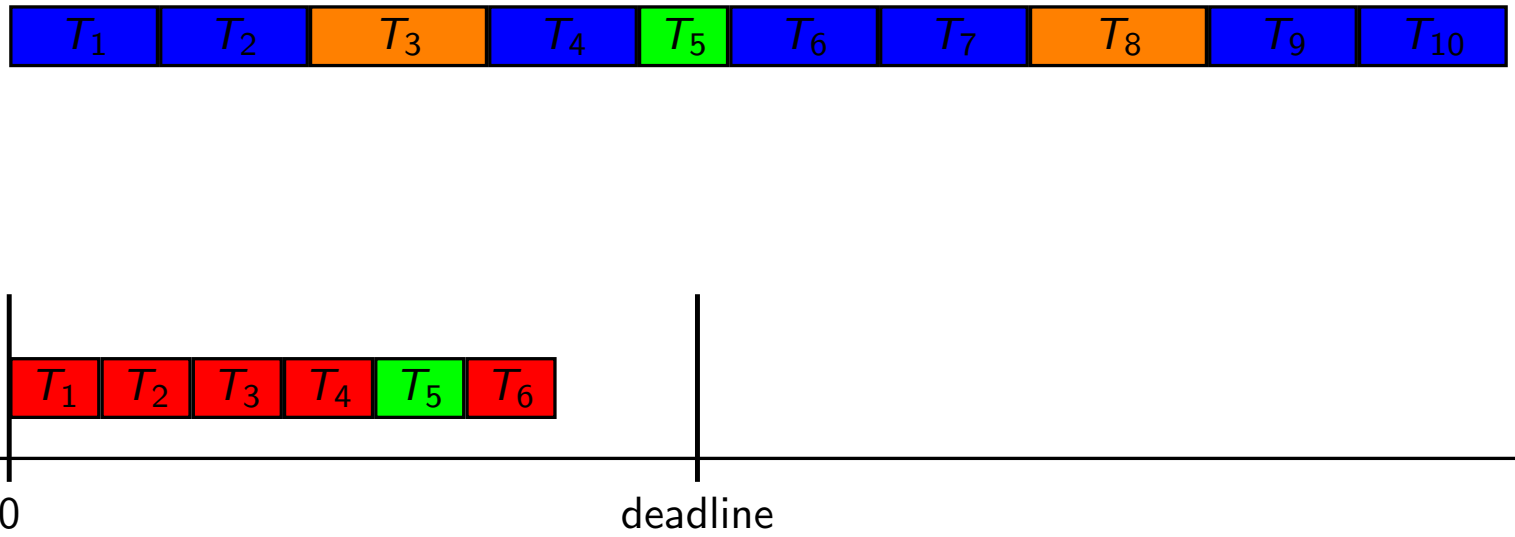

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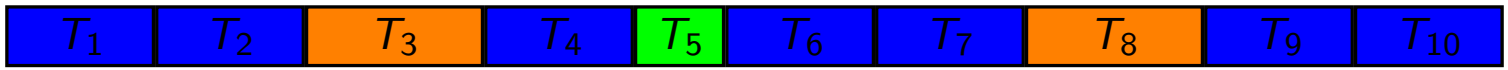

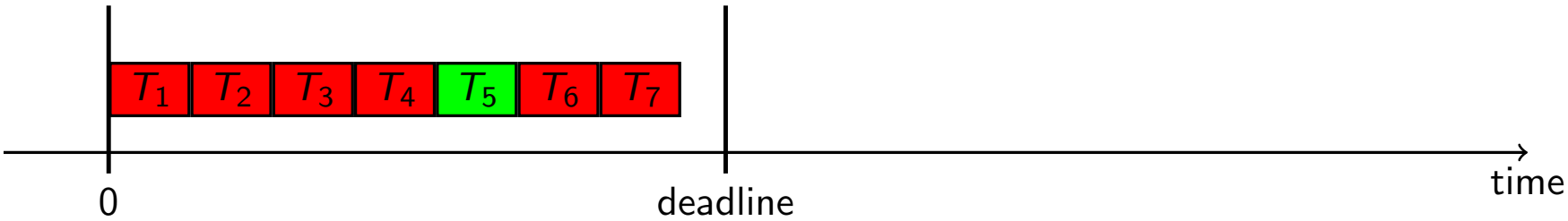

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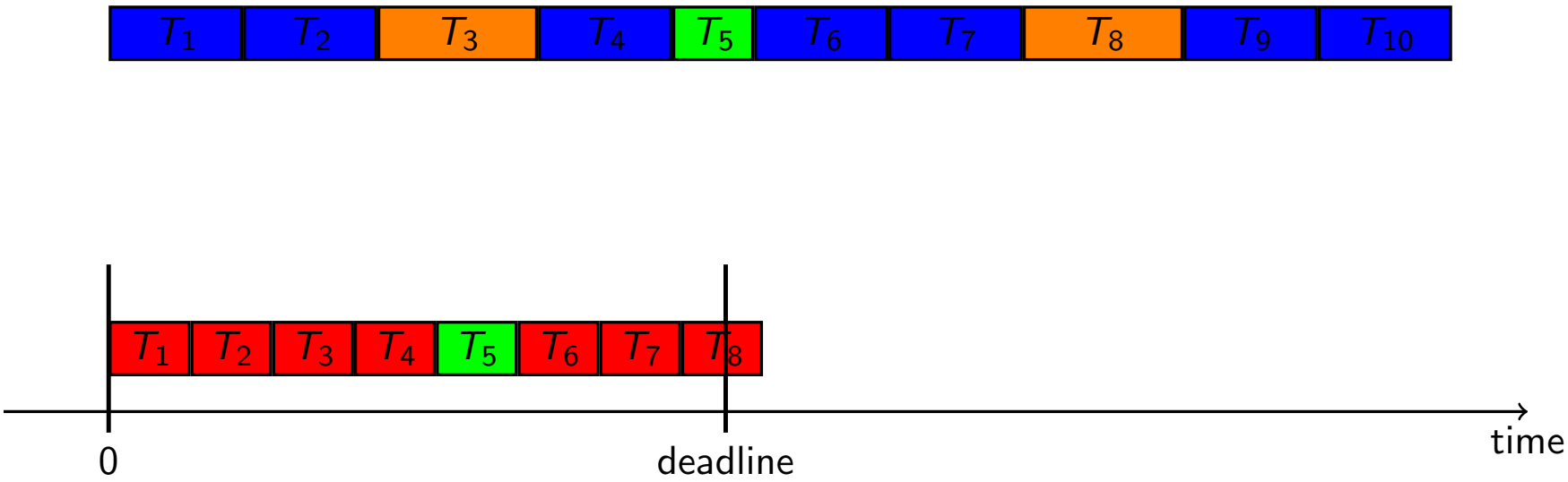

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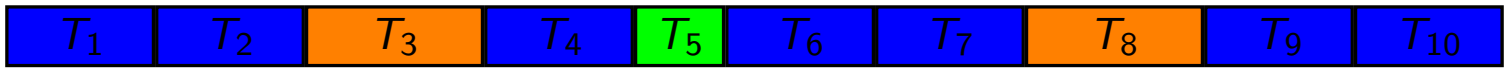

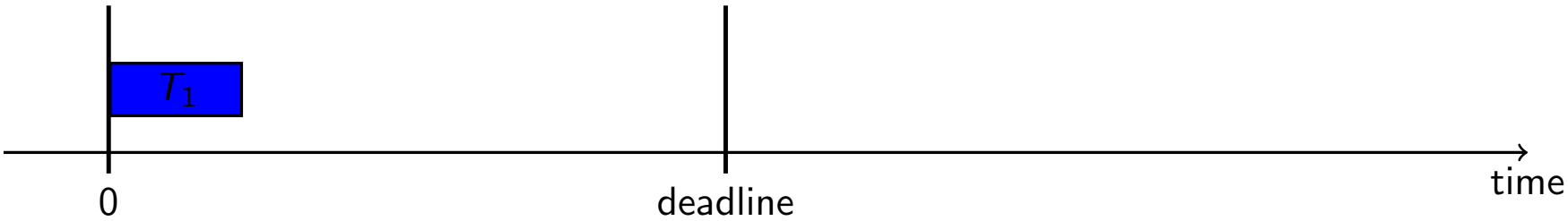

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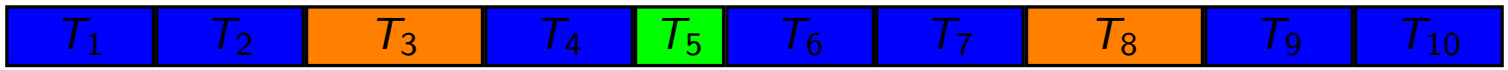

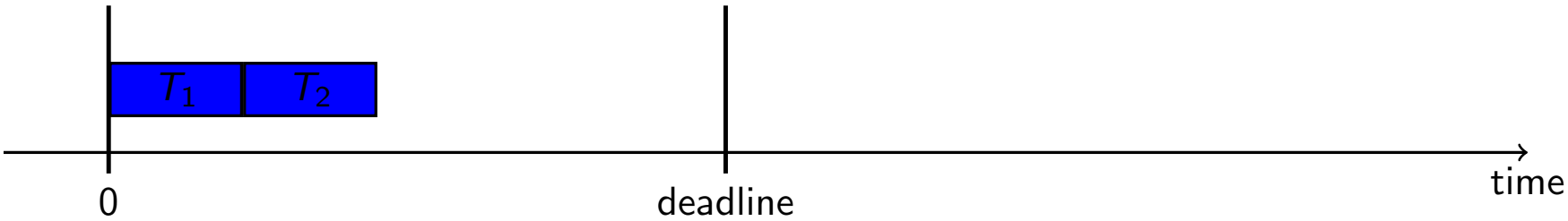

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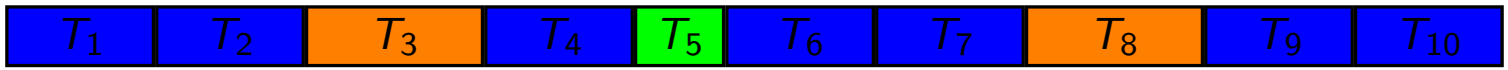

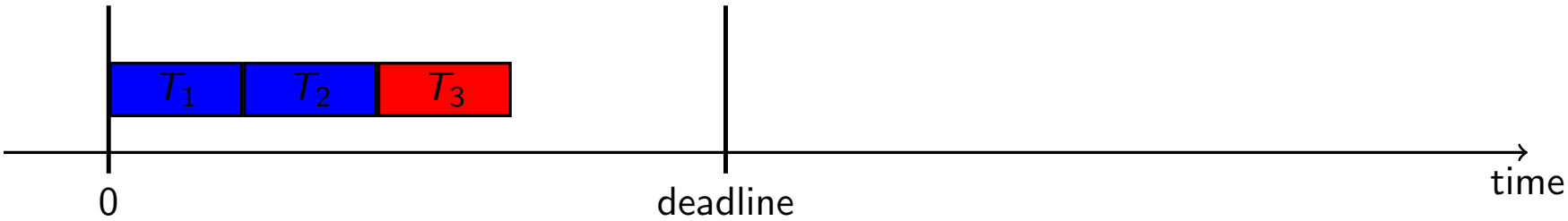

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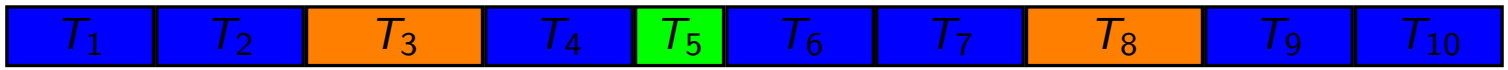

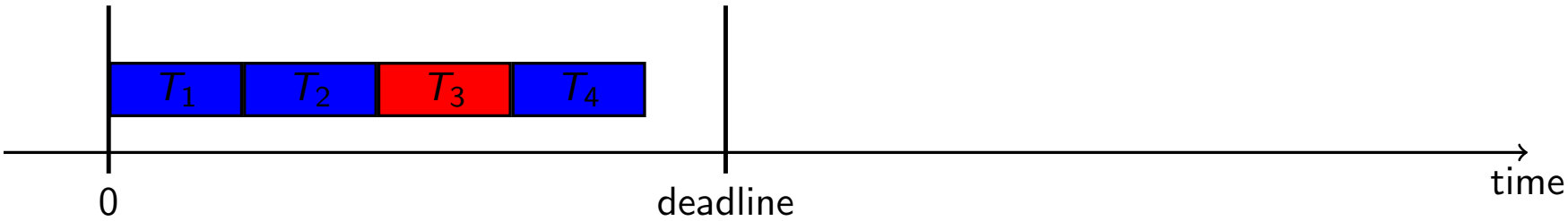

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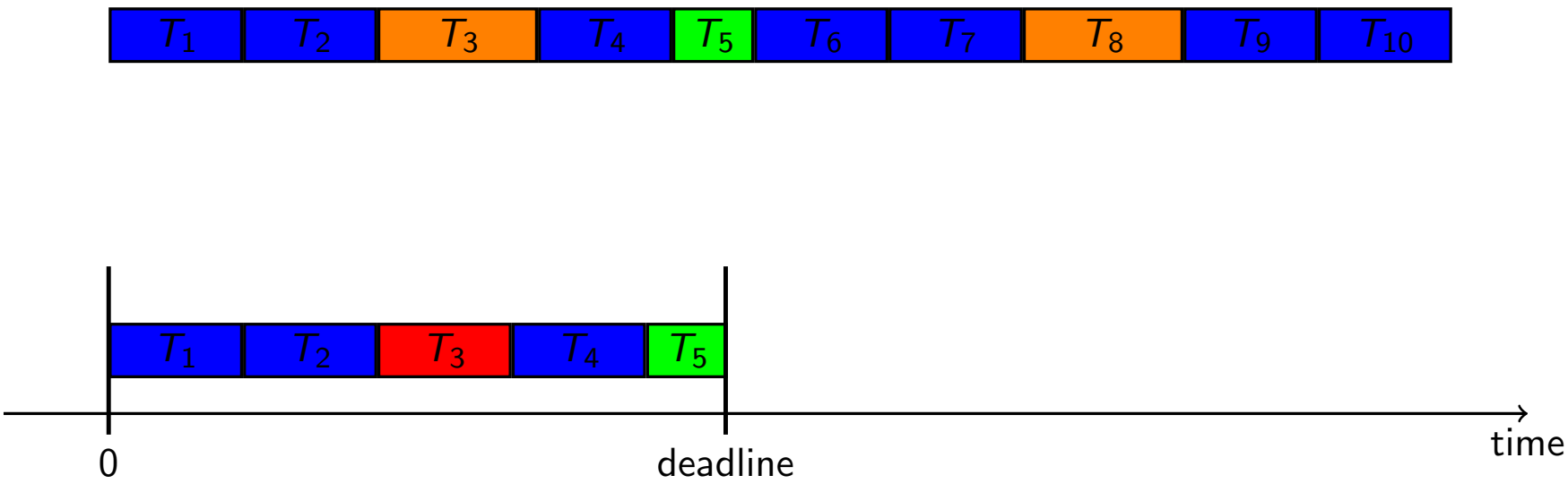

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Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 6$

Stop all tasks after w_1: $\mathcal{R}_1 = \frac{p_1}{w_1} = \frac{1}{30}$

Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 6$

Stop all tasks after w_1: $\mathcal{R}_1 = \frac{p_1}{w_1} = \frac{1}{30}$

Stop all tasks after w_2: $\mathcal{R}_2 = \frac{p_1+p_2}{p_1 w_1+(1-p_1)w_2} = \frac{1}{6}$

Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
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Stop all tasks after w_1: $\mathcal{R}_1 = \frac{p_1}{w_1} = \frac{1}{30}$

Stop all tasks after w_2: $\mathcal{R}_2 = \frac{p_1+p_2}{p_1 w_1+(1-p_1)w_2} = \frac{1}{6}$

Stop half unsuccessful tasks after w_1 and one-third after w_2: $\mathcal{R} = ?$

Optimal strategy

Theorem

Best strategy is to stop all tasks at some threshold

Strategy

Find i maximizing

$$\mathcal{R}_i \stackrel{\text{def}}{=} \frac{\sum_{j=1}^{i} p_j}{\sum_{j=1}^{i} p_j w_j + (1 - \sum_{j=1}^{i} p_j) w_i}$$

If ties, pick smallest index

Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 6$

Stop all tasks after w_1: $\mathcal{R}_1 = \frac{p_1}{w_1} = \frac{1}{30}$

Stop all tasks after w_2: $\mathcal{R}_2 = \frac{p_1+p_2}{p_1 w_1+(1-p_1)w_2} = \frac{1}{6}$

Stop all tasks after w_3: $\mathcal{R}_3 = \frac{1}{p_1 w_1+p_2 w_2+p_3 w_3} = \frac{1}{5}$

Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 6$

Question?

So in the end you should not interrupt anything, right?

Pfhhh these scheduling guys ☹

Stop all tasks after w_2: $\mathcal{R}_2 = \frac{p_1+p_2}{p_1 w_1+(1-p_1)w_2} = \frac{1}{6}$

Stop all tasks after w_3: $\mathcal{R}_3 = \frac{1}{p_1 w_1+p_2 w_2+p_3 w_3} = \frac{1}{5}$

Scheduling strategies

Probability	Execution time
$p_1 = 0.1$	$w_1 = 3$
$p_2 = 0.7$	$w_2 = 5$
$p_3 = 0.2$	$w_3 = 101$

Stop all tasks after w_1: $\mathcal{R}_1 = \frac{p_1}{w_1} = \frac{1}{30}$

Stop all tasks after w_2: $\mathcal{R}_2 = \frac{p_1+p_2}{p_1 w_1+(1-p_1)w_2} = \frac{1}{6}$

Stop all tasks after w_3: $\mathcal{R}_3 = \frac{1}{p_1 w_1+p_2 w_2+p_3 w_3} = \frac{1}{24}$

From discrete to continuous distributions

$f(x)$ probability density, $F(x)$ cumulative distribution

Expected value μ_D, variance, σ_D^2

$$\arg\max_i \mathcal{R}_i \stackrel{\text{def}}{=} \frac{\sum_{j=1}^{i} p_j}{\sum_{j=1}^{i} p_j w_j + (1 - \sum_{j=1}^{i} p_j) w_i}$$

$$\arg\max_I \mathcal{R}(I) \stackrel{\text{def}}{=} \frac{F(I)}{\int_0^I x f(x) dx + (1 - F(I)) I}$$

No more a theorem, but hopefully a good heuristic ...

Best cutting threshold

$$\mathcal{D} = \textsc{Exp}(\lambda)$$

Best cutting threshold

$\mathcal{D} = \text{Exp}(\lambda)$

Interrupt at any instant ($\mathcal{R}_l$ constant)

$\mathcal{D} = \text{Uniform}[a, b]$

Best cutting threshold

$\mathcal{D} = \text{Exp}(\lambda)$
Interrupt at any instant ($\mathcal{R}_l$ constant)

$\mathcal{D} = \text{Uniform}[a, b]$
Never interrupt ($\mathcal{R}_l$ maximal for $l = b$)

Best cutting threshold

Question?

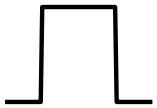
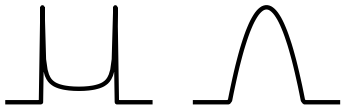
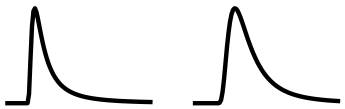
Well, do you know any important distribution for which it is really worth interrupting tasks?

Pfhhh these scheduling guys ☹

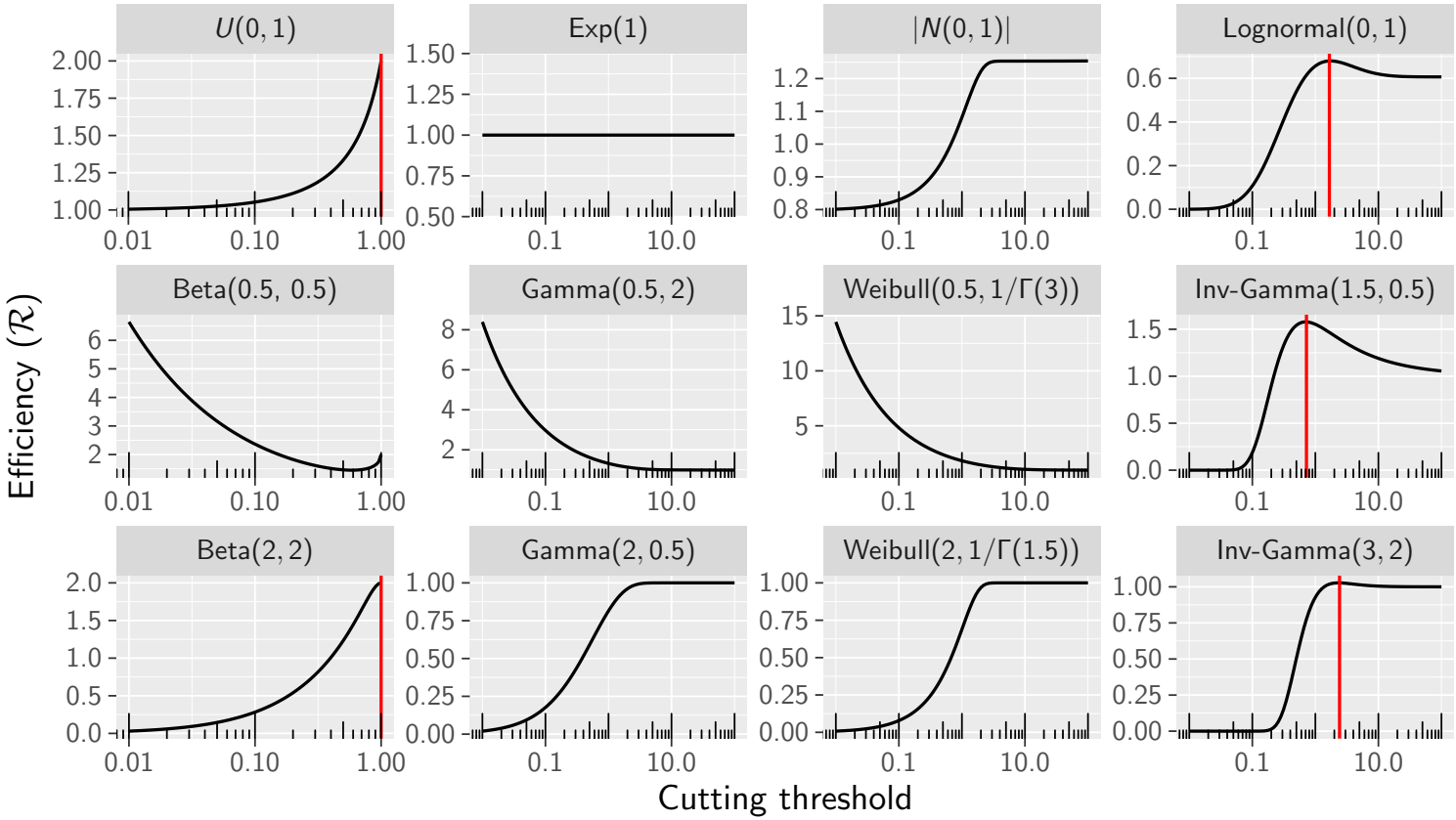
$D = \text{UNIFORM}[a, b]$

Never interrupt ($\mathcal{R}_l$ maximal for $l = b$)

A few distributions . . .

Name	PDF	Density
Uniform	$\frac{1}{b-a}$	
Exponential	$\lambda e^{-\lambda x}$	
Half-normal	$\frac{\sqrt{2}}{\theta\sqrt{\pi}} e^{-\frac{x^2}{2\theta^2}}$	
Lognormal	$\frac{1}{x\beta\sqrt{2\pi}} e^{-\frac{(\log(x)-\alpha)^2}{2\beta^2}}$	
Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$	
Gamma	$\frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$	
Weibull	$\frac{k}{\theta^k} x^{k-1} e^{-(\frac{x}{\theta})^k}$	
Inverse-gamma	$\frac{\theta^k}{\Gamma(k)} x^{-k-1} e^{-\frac{\theta}{x}}$	

. . . and their optimal cutting threshold



Outline

Heuristics with 1 processor

MeanVariance(x): kill a task as time $\mu_D + x\sigma_D$, with x some constant

Quantile(x): kill a task when execution time reaches the x-quantile of $\mathcal{D}$, with $0 \leq x \leq 1$

OptRatio: optimal cutting threshold

Heuristics with many processors

With budget b and deadline d, enroll $\lceil \frac{b}{d} \rceil$ processors

Run previous heuristics in parallel

Experiments

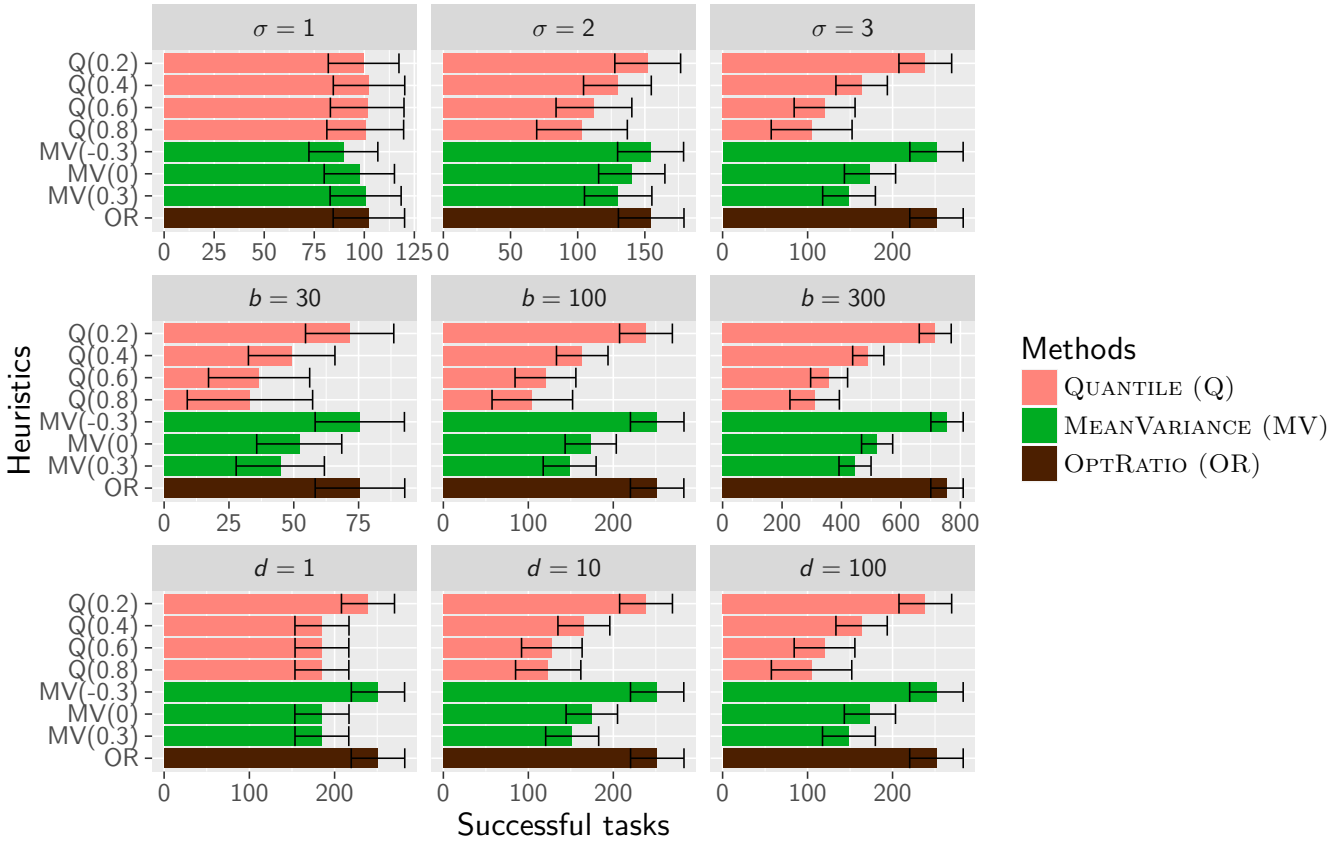
Normalized for $\mu = 1$, budget and deadline $b = d = 100$

Exponential: $\lambda = 1$, $l_{\text{opt}} = 2$ (arbitrarily)

Uniform: $a = 0$, $b = 2$, $l_{\text{opt}} = 2$

Lognormal: $\alpha \approx -1.15$, $\beta \approx 1.52$, $\mu = 1$, $\sigma = 3$, $l_{\text{opt}} \approx 0.1$

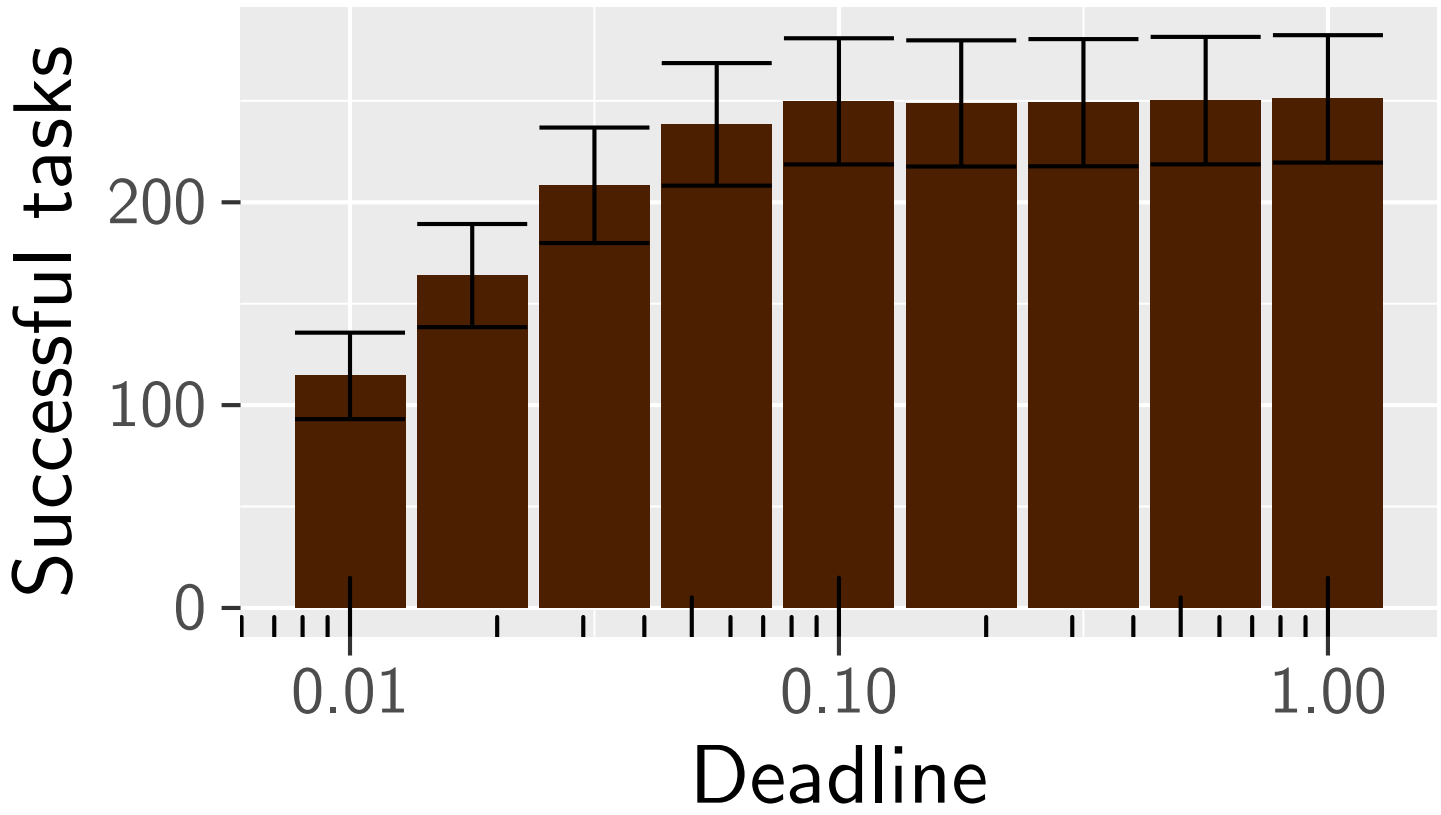
Focus on LogNormal



Lognormal: $\alpha \approx -1.15$, $\beta \approx 1.52$, $\mu = 1$, $\sigma = 3$, $l_{\text{opt}} \approx 0.1$

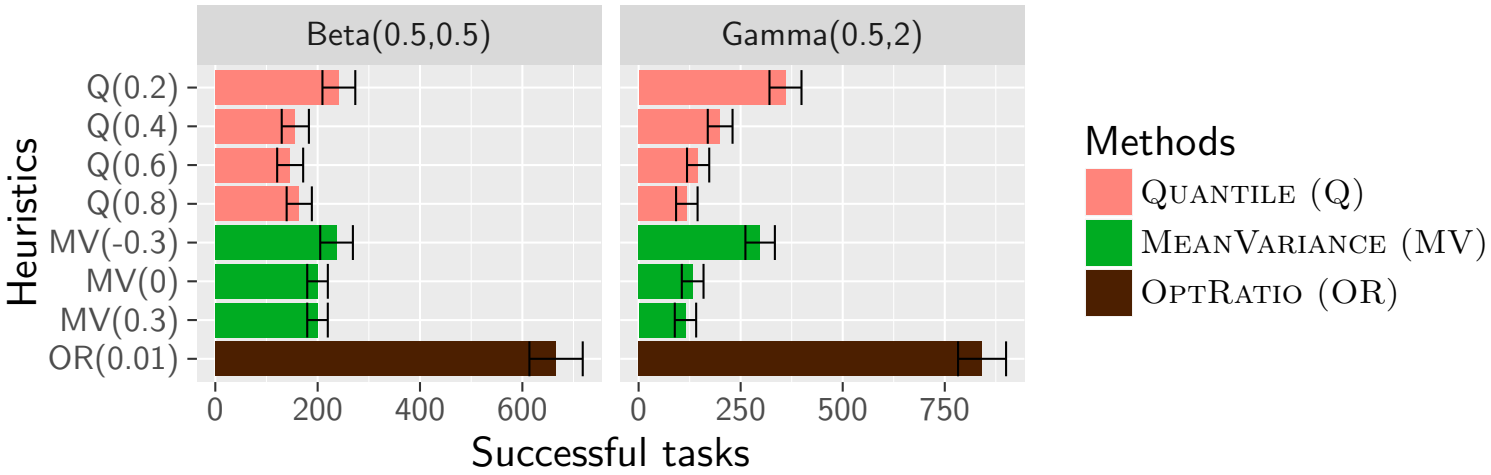
First row $b = d = 100$, second row $b = d$, third row $b = 100$ (hence $\lceil \frac{b}{d} \rceil$ processors)

With small deadlines



Budget $b = 100$, varying deadline (hence number of processors)
Lognormal: $\alpha \approx -1.15$, $\beta \approx 1.52$, $\mu = 1$, $\sigma = 3$, $l_{\text{opt}} \approx 0.1$

Cut them short

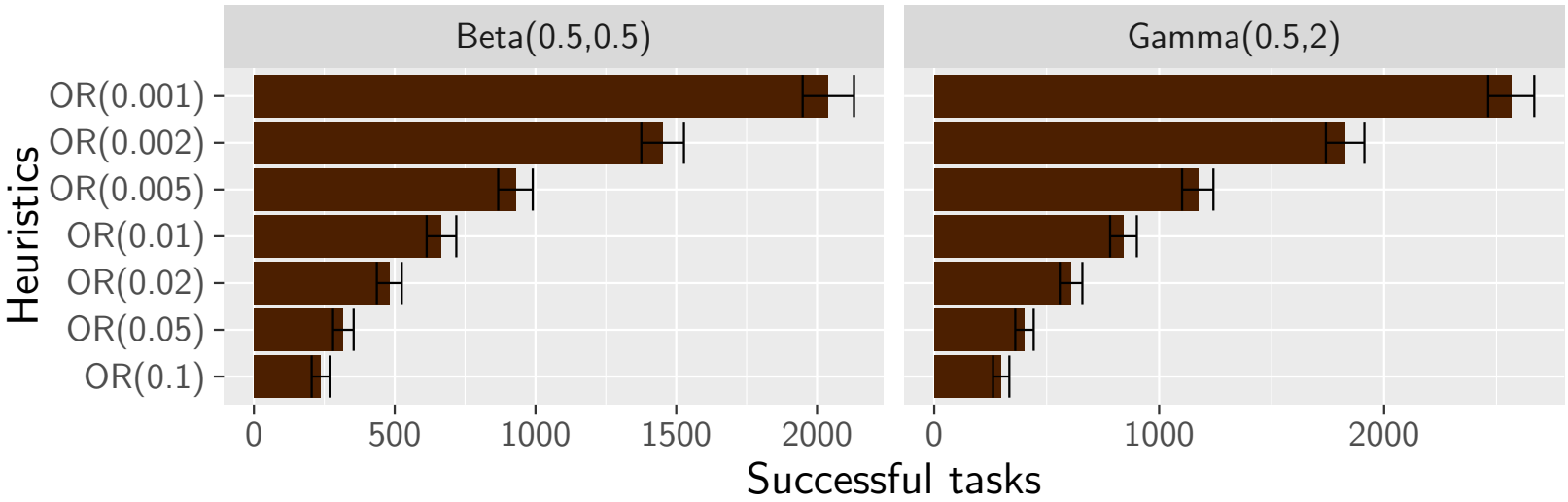


$\mu = 0.5$ for Beta, $\mu = 1$ for Gamma

cutting threshold is 0.01 for OR in both plots

$b = d = 100$

Zoom on threshold impact



$\mu = 0.5$ for BETA, $\mu = 1$ for GAMMA

$b = d = 100$

Nice little (albeit technical) example

Probability	Execution time
$p_1 = 0.4$	$w_1 = 2$
$p_2 = 0.15$	$w_2 = 3$
$p_3 = 0.45$	$w_3 = 7$

Budget $b = 6$, no deadline (say $d = 6$)

Optimal schedule with 1 processor

Probability	Execution time
$p_1 = 0.40$	$w_1 = 2$
$p_2 = 0.15$	$w_2 = 3$

$E(1) = 0$, $E(2) = p_1 = 0.4$

$E(3) = (p_1 + p_2) = 0.55$ (pointless to kill an unsuccessful task at time 2)

$E(4) = \max\{p_1 + E(2), p_1(1 + E(2)) + p_2(1 + E(1)) + p_3(0 + E(1))\} = 0.8$
Either kill the first task (if not completed) at time 2
or continue up to time 3 (if not completed) and then kill

$E(6) = \max\{p_1 + E(4), p_1(1 + E(4)) + p_2(1 + E(3))\} = 1.2$

An efficient schedule with 2 processors

Probability	Execution time
$p_1 = 0.40$	$w_1 = 2$
$p_2 = 0.15$	$w_2 = 3$

Two processors, each starting a task in parallel

If none completes by time 2, let them run up to time 3

Otherwise, kill at time 2 any not-yet completed task and start a new task

	Processor 1		
Processor 2	w_1	w_2	w_3
w_1	$2 + p_1$	$1 + p_1$	$1 + p_1$
w_2	$1 + p_1$	2	1
w_3	$1 + p_1$	1	0

With probability $p_1 p_2$, 1st task completes, 2nd task is killed, 2 units remain for the newnone, expected number of completed tasks in this configuration is $1 + p_1$

$$E_{//} = p_1^2(2 + p_1) + 2p_1(p_2 + p_3)(1 + p_1) + 2p_2^2 + 2p_2 p_3 = 1.236$$

An efficient schedule with 2 processors

Probability	Execution time
$p_1 = 0.40$	$w_1 = 2$
$p_2 = 0.15$	$w_2 = 3$

Two processors, each starting a task in parallel

If none completes by time 2, let them run up to time 3

Otherwise, kill at time 2 any not-yet completed task and start a new task

Question?

No kidding? You win 0.036 tasks in the end
and you are proud of you?!
Time to finish your talk, dude!
Pfhhh these scheduling guys ☹

Outline

Conclusion

This talk

A few (simple) scheduling problems

Conclusion

This talk

A few (simple) scheduling problems

Future work

Multi-criteria scheduling problems

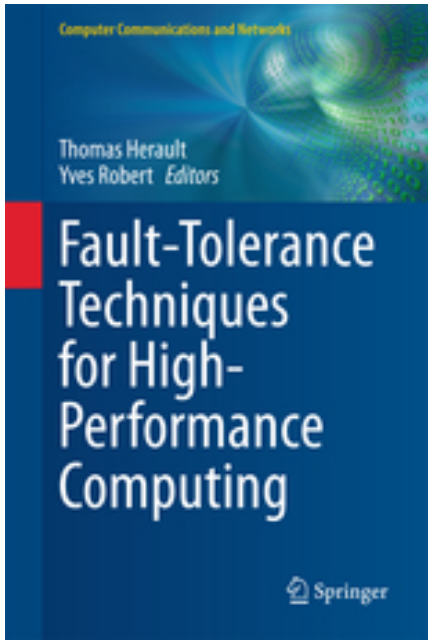
execution time/energy/reliability

add replication

best resource usage (performance trade-offs)

Several challenging algorithmic/scheduling problems ☺

Bibliography



First chapter = comprehensive survey, freely available
LAWN 289 (LApack Working Note)