Notes on scheduling radio-astronomical observations

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Outline

Outline

Observing from the Earth

Figure: Time lapse photography (Animalia Life)

Altitude over time

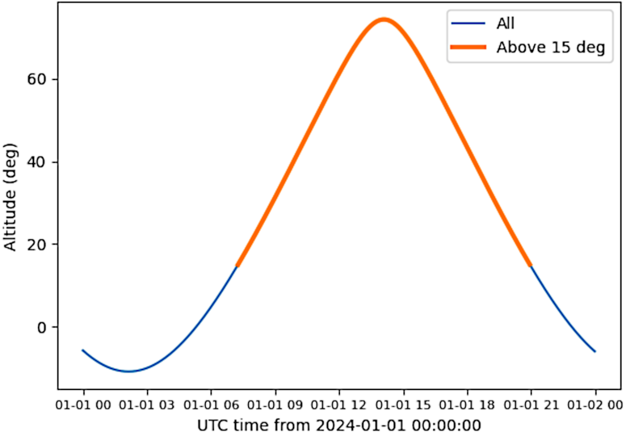

Figure: Altitude of star KELT 16 from the NenuFAR radio-telescope on 1 January 2024.

Altitude over time

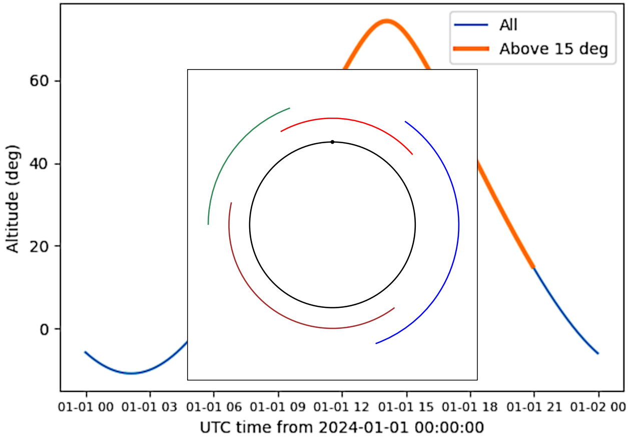

Figure: Altitude of star KELT 16 from the NenuFAR radio-telescope on 1 January 2024.

Postdoc: "Observation Scheduling"

ROMA team, LIP, ENS de Lyon Nov. 2024 - Now

- *Topic:* Scheduling radio-astronomical observations.
  - Joint work with Frédéric Vivien (and Anne Benoit).
  - Partnership with the ECLAT laboratory (HPC & AI for astronomy & astrophysics).
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- *Model:* **“Observation Scheduling”**.
  - Day length $L \in \mathbb{R}_{>0}$.
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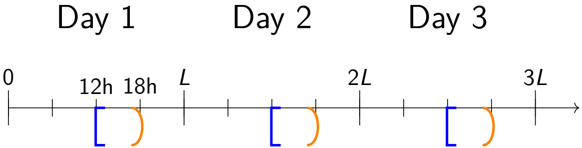

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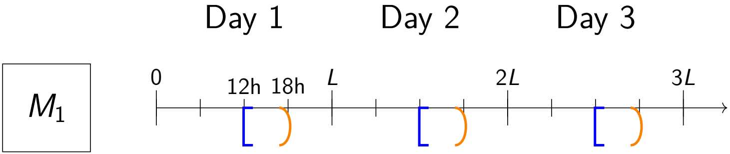
  - Single machine, non-preemptive, minimizing idle time (i.e., makespan).



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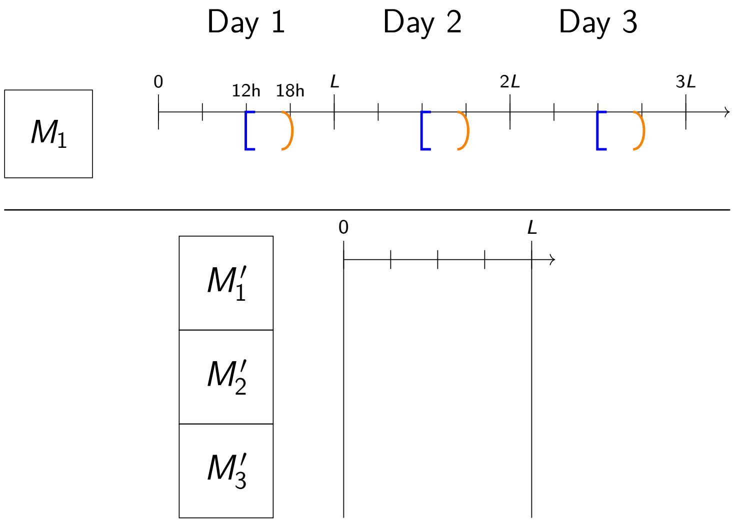

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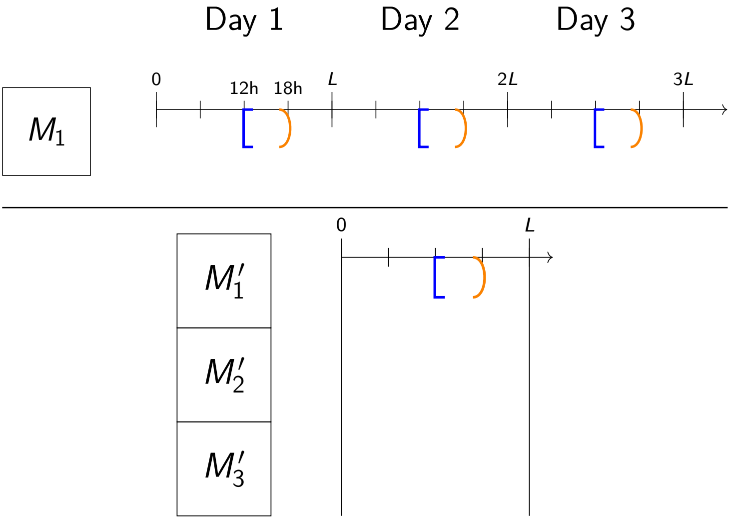

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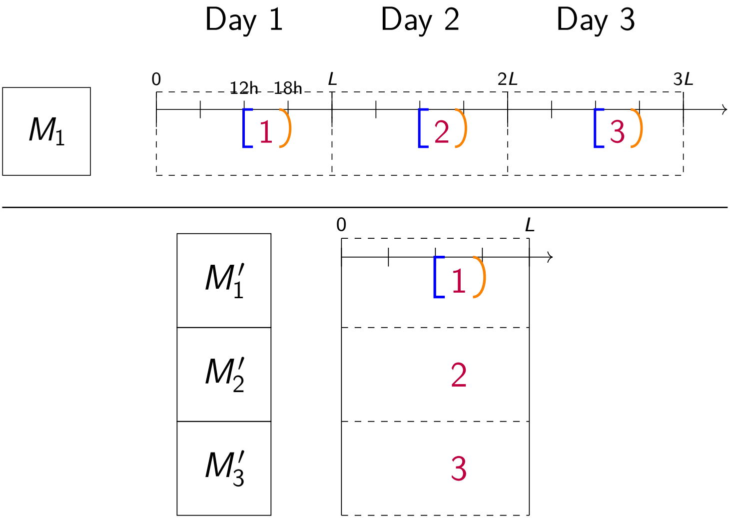

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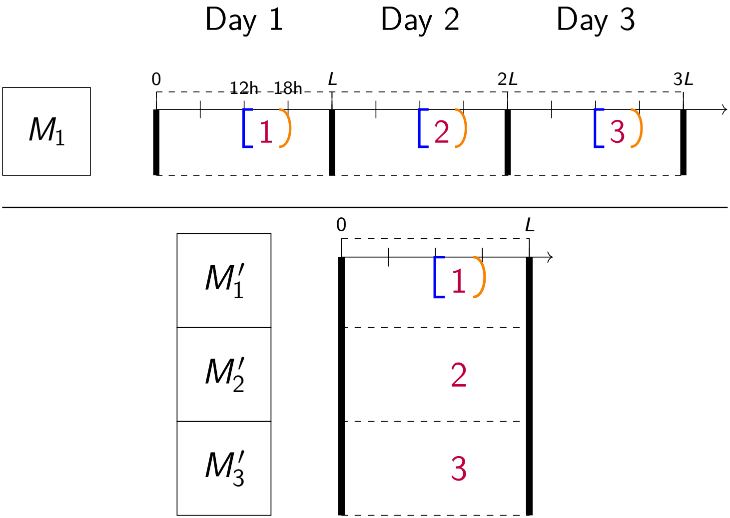
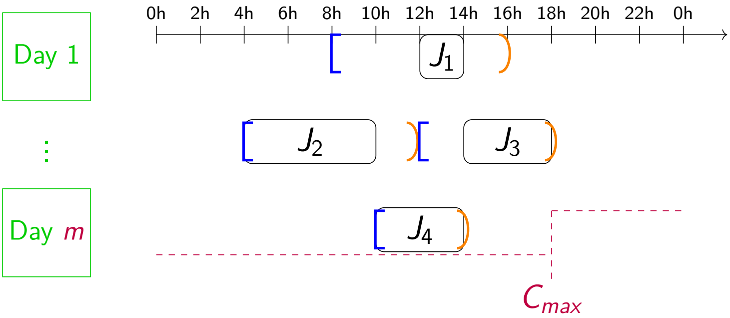
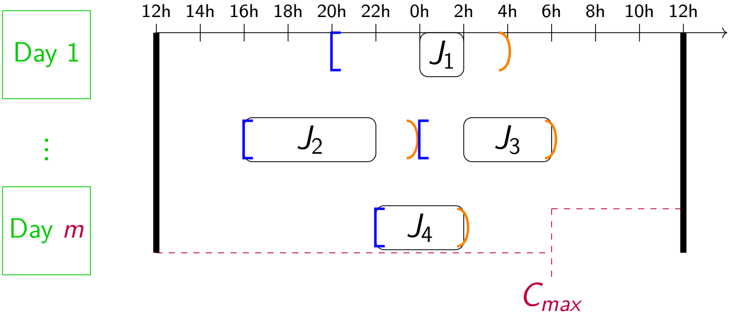
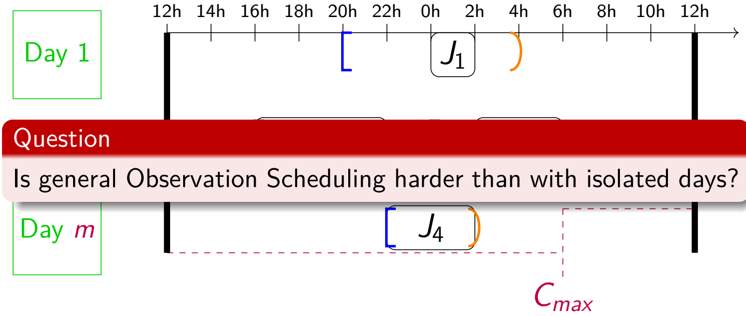


Scheduling observations





With isolated days: $P|r_j, d_j|\min(m)$.



Question

Is general Observation Scheduling harder than with isolated days?

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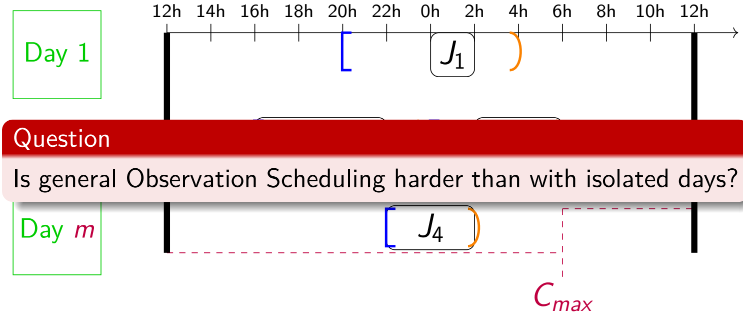

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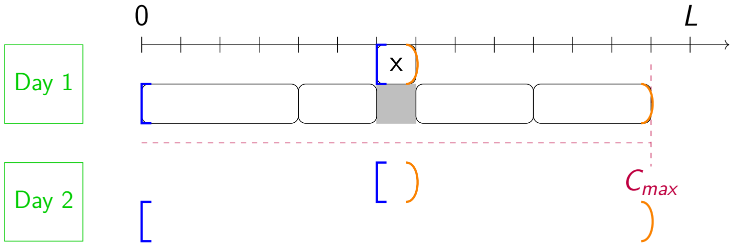
One day: strongly NP-complete [Lenstra et al., 1977].

Outline

$(2-\varepsilon)$-inapproximability



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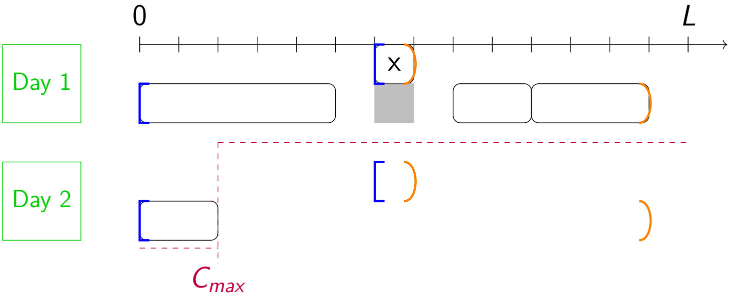

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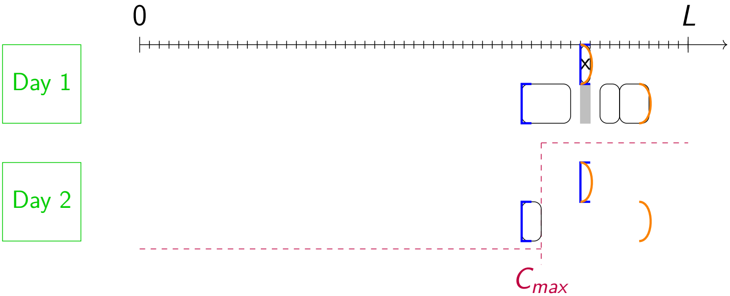

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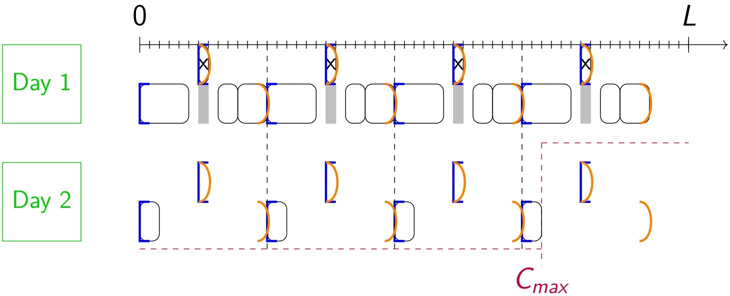

Observation Scheduling with isolated days.

- Goal: minimize the number of days.
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Job Interval Selection problem.

- Goal: maximize the number of scheduled jobs.

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- 1/2-approximation: "Earliest Completion Time". [Spieksma, 1999]

$\mathcal{O}(\log(n))$-approximation (cont.)

Remark

W.l.o.g: $\forall j, p_j \leq L$.

Proposition

If $L \leq OPT$, then any f-approximation algorithm with isolated days is a $[(2 + L/OPT) \cdot f]$-approximation algorithm for Observation Scheduling.

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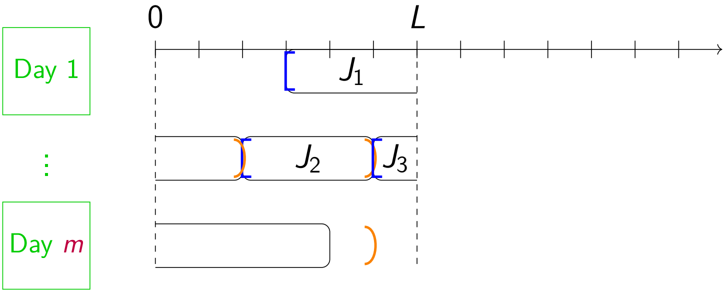

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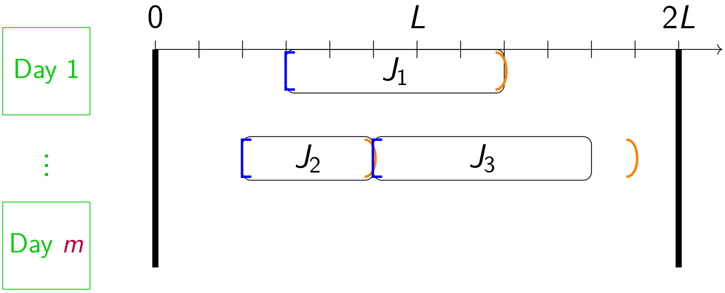

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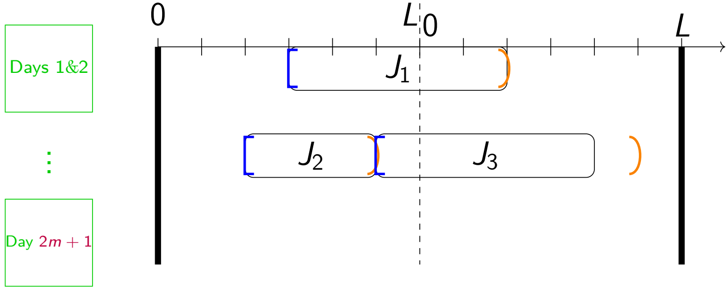

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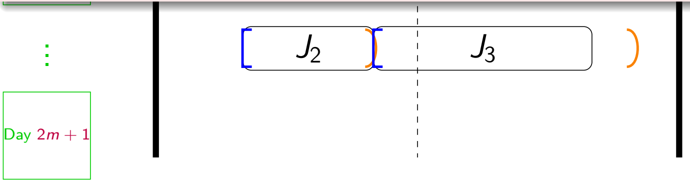
Constant approximation algorithm?



Outline

- Problem $\mathcal{P}$, instance $\mathcal{I}$.
- P: deterministic time $poly(|\mathcal{I}|)$.
- NP: nondeterministic time $poly(|\mathcal{I}|)$.

Going beyond NP-hardness

- Problem $\mathcal{P}$, instance $\mathcal{I}$.
  - Parameter $\bar{k}$, instance $\mathcal{I}$ with parameter value $\leq k$.
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  - FPT: deterministic $f(k) \cdot poly(|\mathcal{I}|)$ time.
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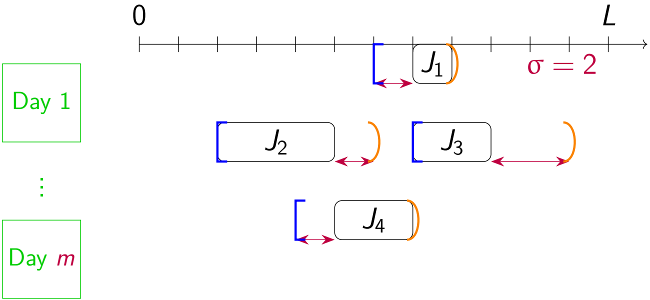
  - para-NP-complete: NP-complete for some fixed k.

Scheduling with tight windows



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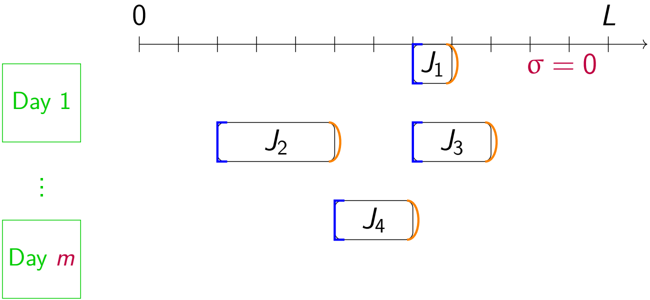

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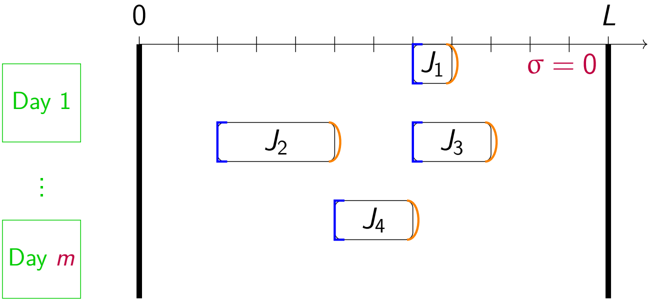

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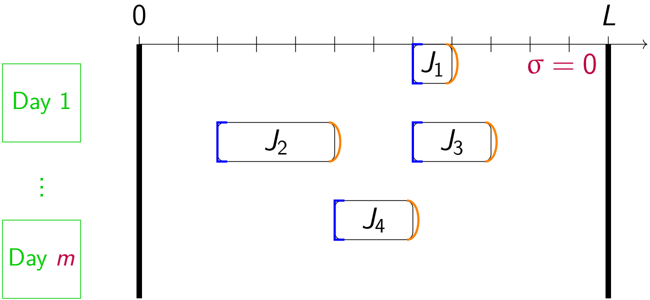
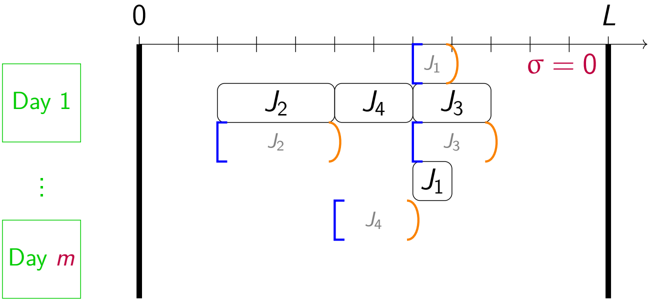


Interval graph coloring: $\mathcal{O}(n \cdot \log(n))$ time.



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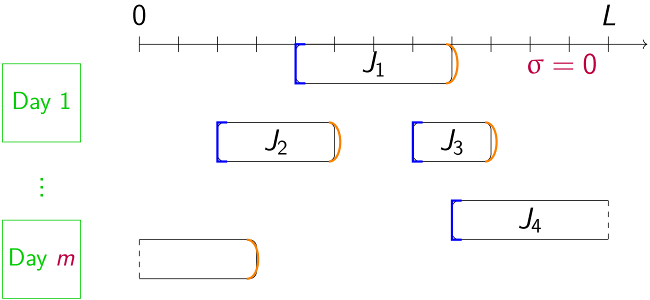

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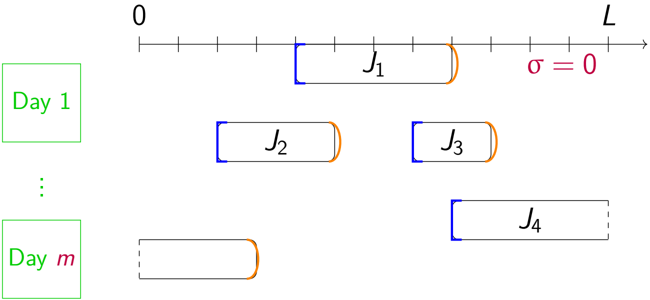

Circular-arc graph coloring: NP-complete [Garey et al., 1980].

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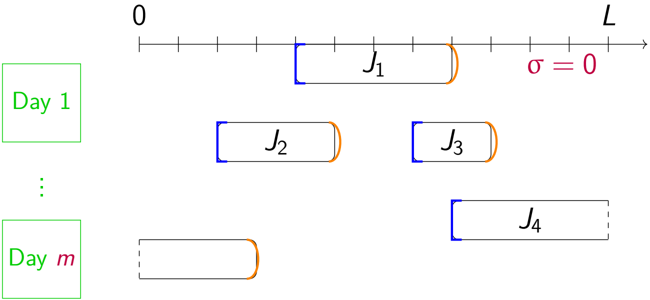

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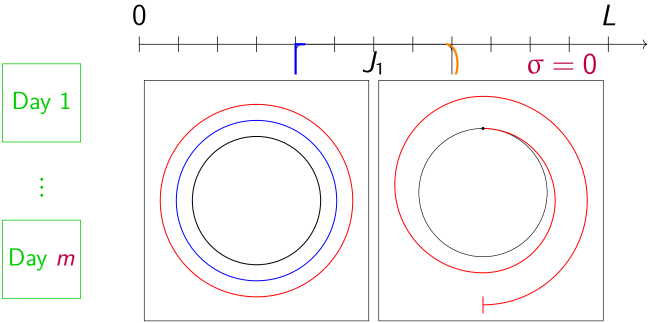

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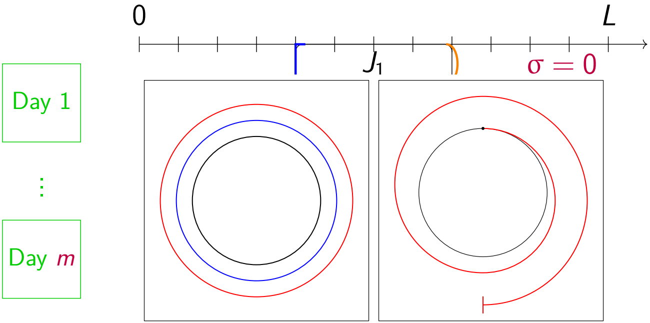

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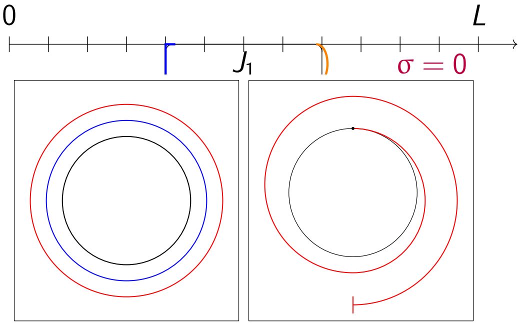
Multi-slot Just-in-time: $\mathcal{O}(n\log(n)^2)$ time [Dereniowski and Kubiak, 2010].

Scheduling with tight windows

Day 1

⋮

Day m



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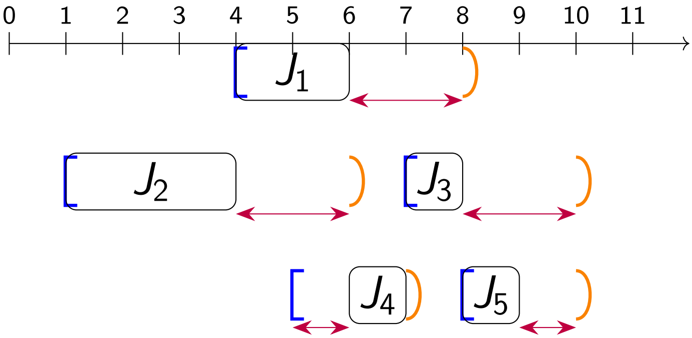
→ TSP with a Gilmore-Gomory distance matrix.

[Gilmore and Gomory, 1964, Vairaktarakis, 2003]

Width μ



$\sigma = 2$: NP-complete [Cieliebak et al., 2004].

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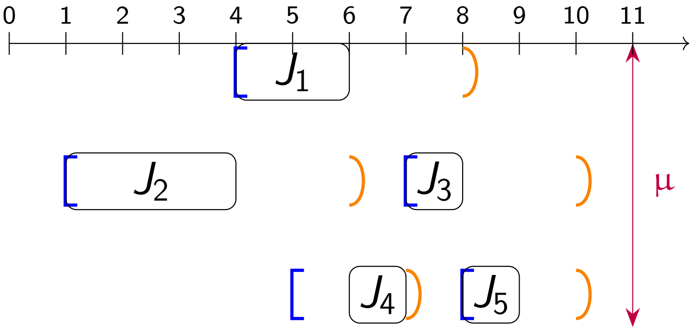

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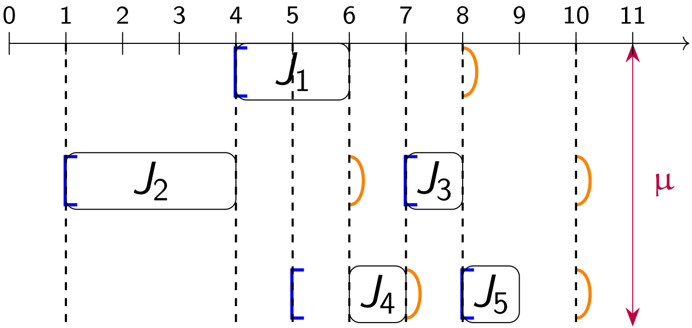

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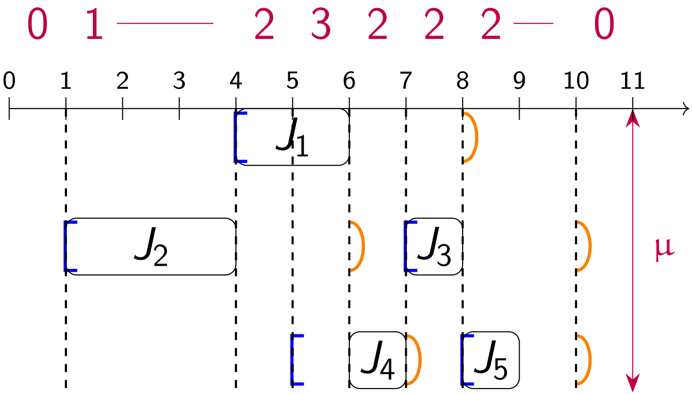

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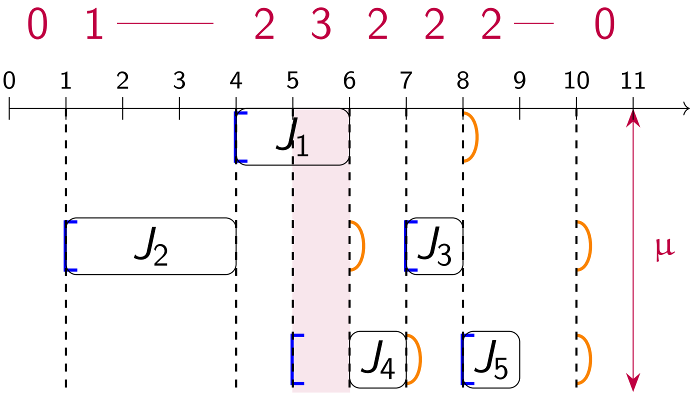

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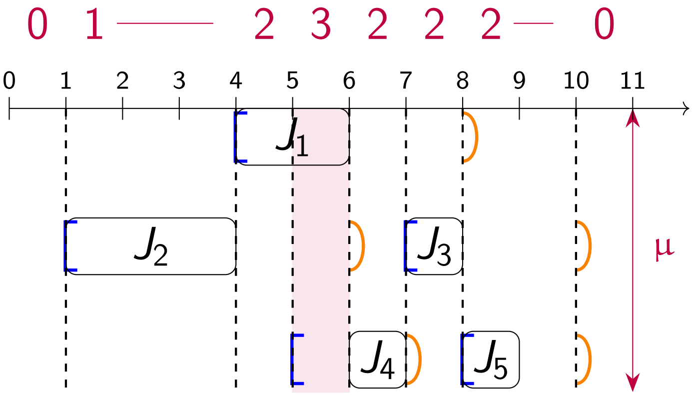

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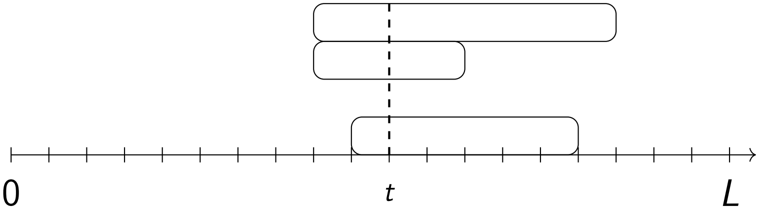
$\mu = 4$: NP-complete [Hanen and Munier Kordon, 2023].

FPT membership with $(\mu + \sigma)$: border schedules



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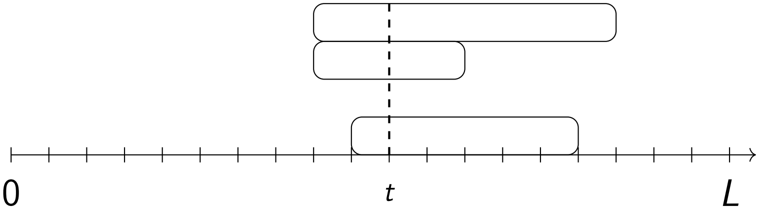

- $\mathcal{O}([\mu(\sigma+1)+1]^m)$ border schedules associated to t mod L.

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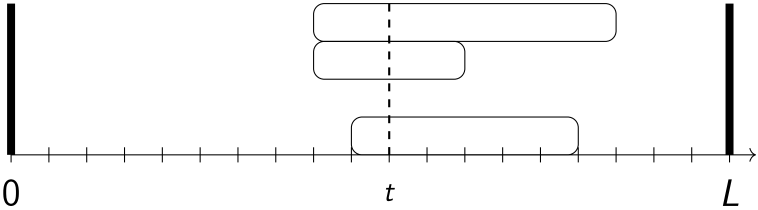

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- Isolated days: in FPT$(\mu + \sigma)$.

  [Cieliebak et al., 2004, Hanen and Munier Kordon, 2023]

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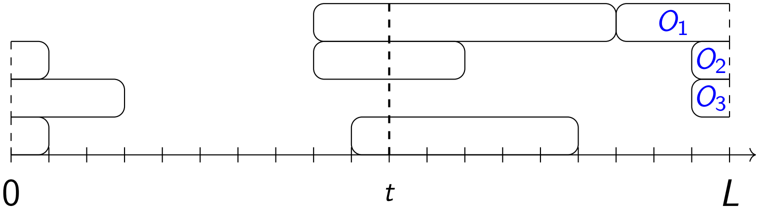

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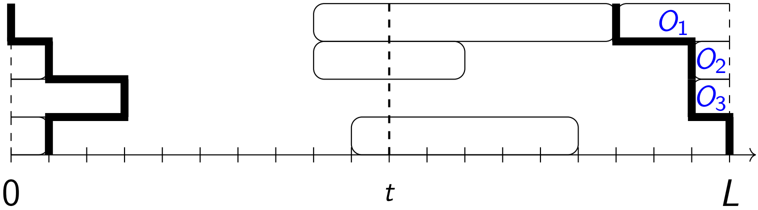

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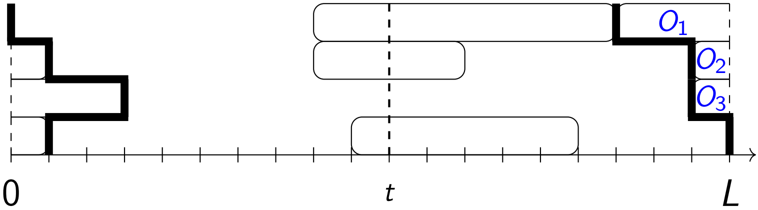

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Idea

For each of the $\mathcal{O}([\mu(\sigma + 1) + 1]^{m-1})$ border schedules associated to 0 mod L, solve an instance of "$P|r_j, d_j|\min(m)$ with machine availabilities".

Outline

Results: same as with isolated days

- $\mathcal{O}(\log(n))$-approximation algorithm.
- No $(2-\varepsilon)$-approximation algorithm (unless P = NP).
- Fixed-parameter tractability: σ, μ and $(m+\sigma)$.

Conclusion

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Future work

- Constant approximation algorithm?
- Improve the fixed-parameter algorithms.
  - $\mathcal{O}([\mu(\sigma+1)+1]^{3m-1} \cdot 2^{3\mu} \cdot m^{\mu} \cdot \mu \cdot n + n \cdot \log(n))$ time.
- “Earliest Start Time” performance guarantee?
- Refine the model: unary/fixed L; limited preemption.

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- "Earliest Start Time" performance guarantee?
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- **Other applications?**

References I

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