Learning-Augmented Bidding in Stochastic Settings

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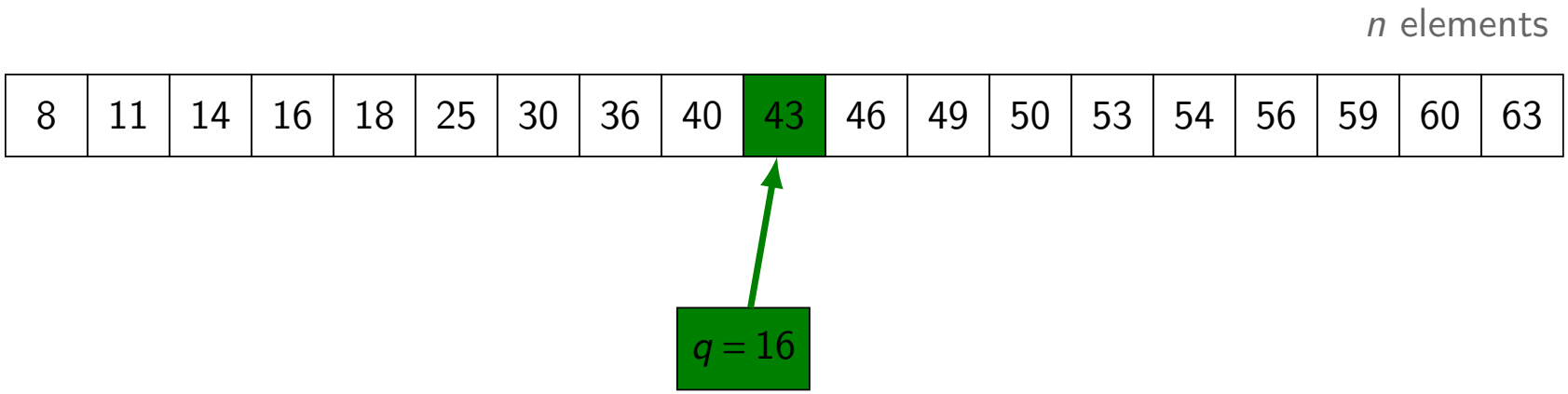
Algorithm with Predictions example : binary search

n elements

8	11	14	16	18	25	30	36	40	43	46	49	50	53	54	56	59	60	63

$q = 16$

Algorithm with Predictions example : binary search



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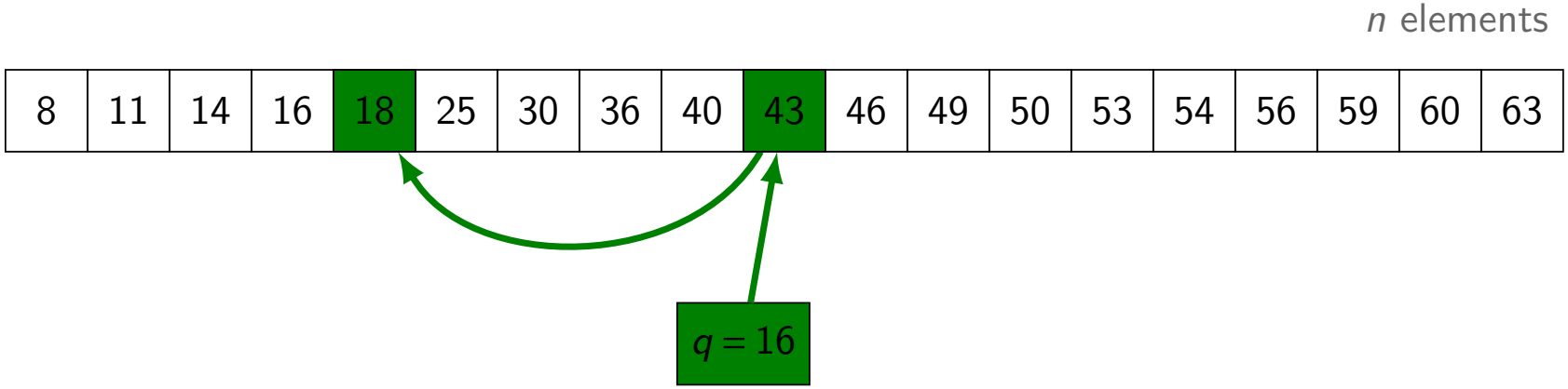

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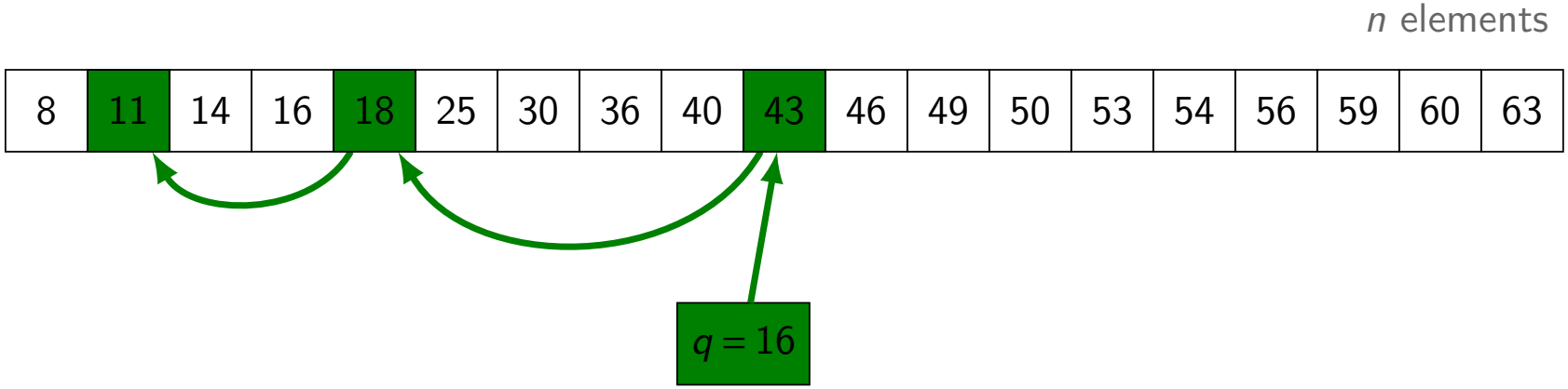

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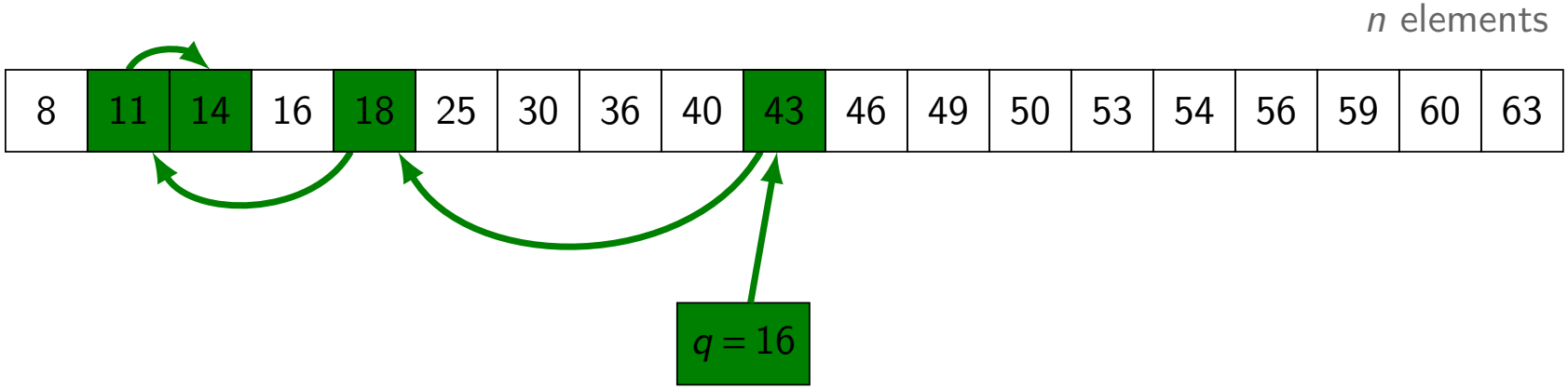

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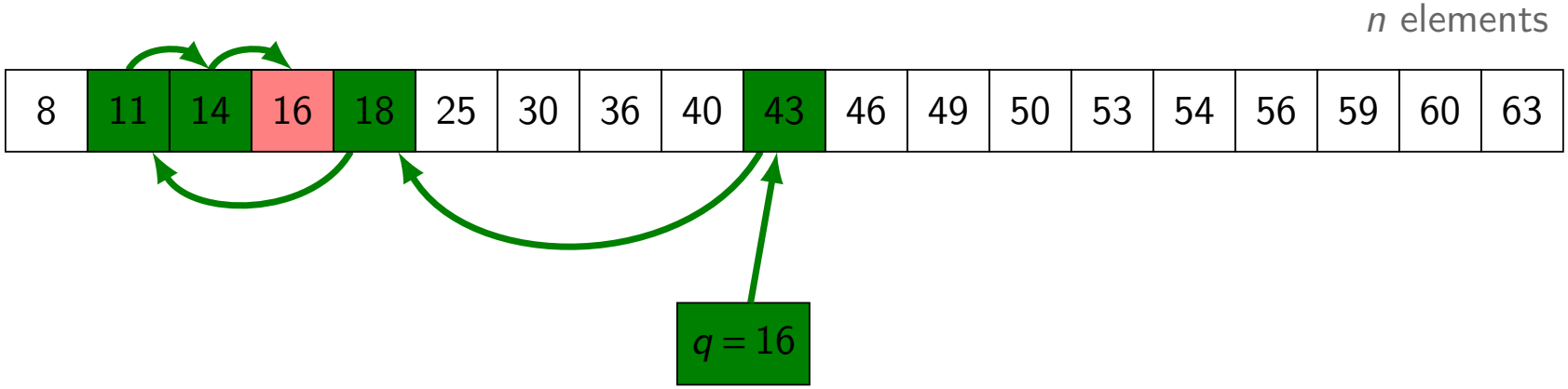

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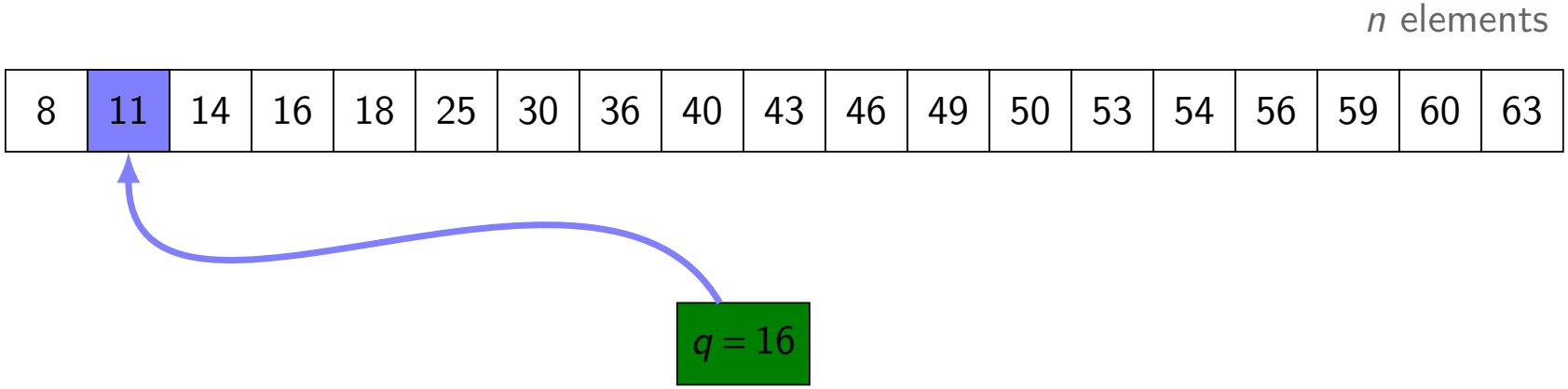

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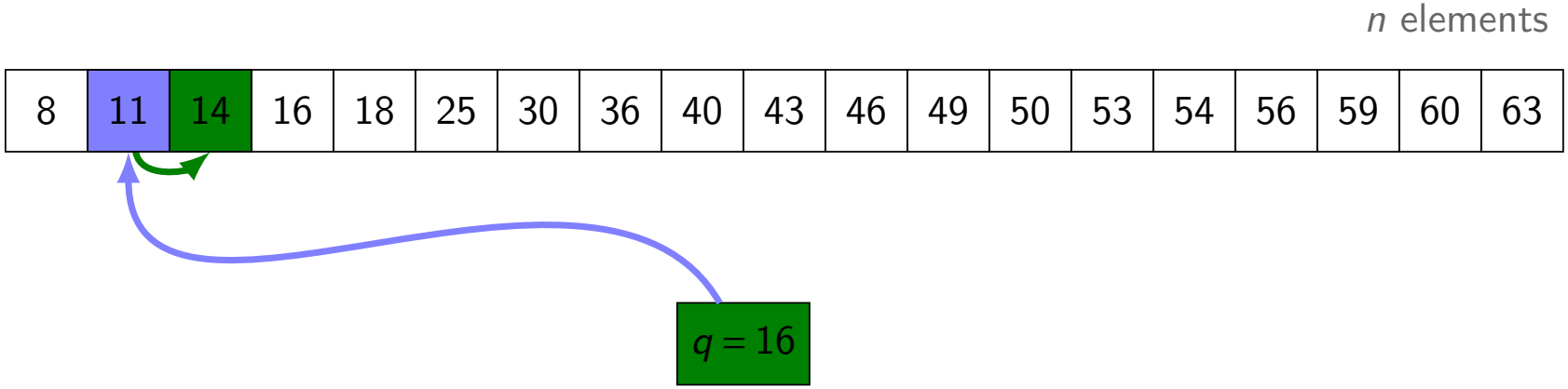

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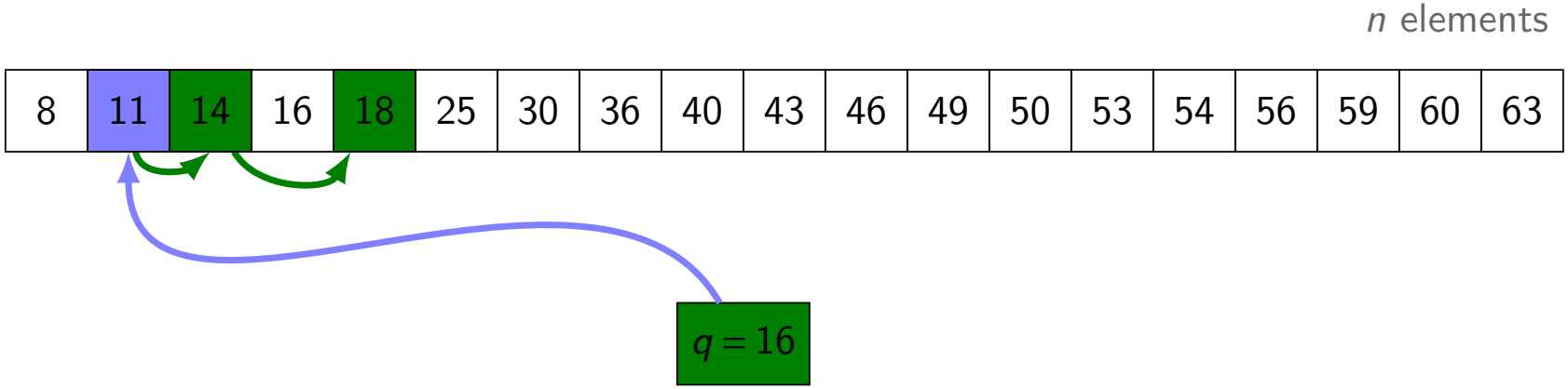

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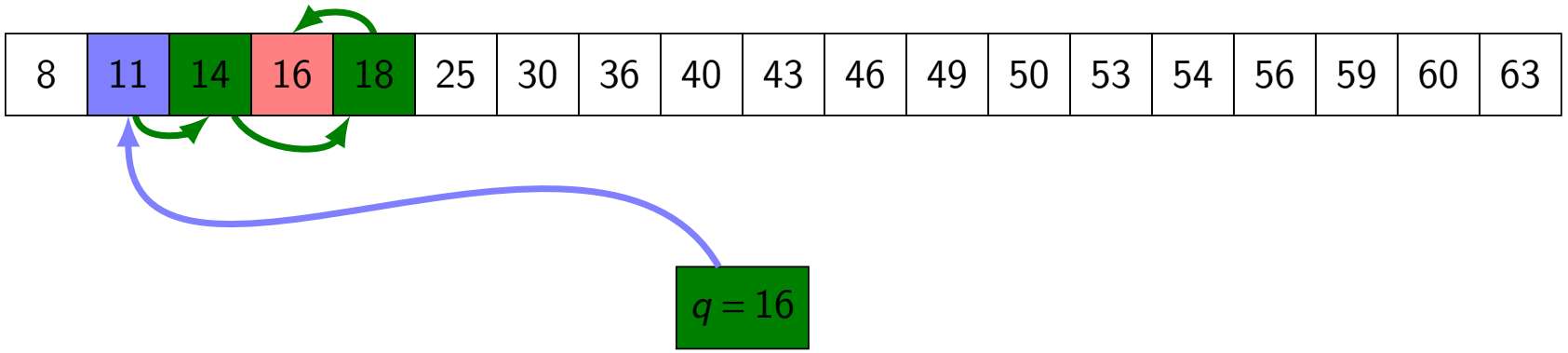

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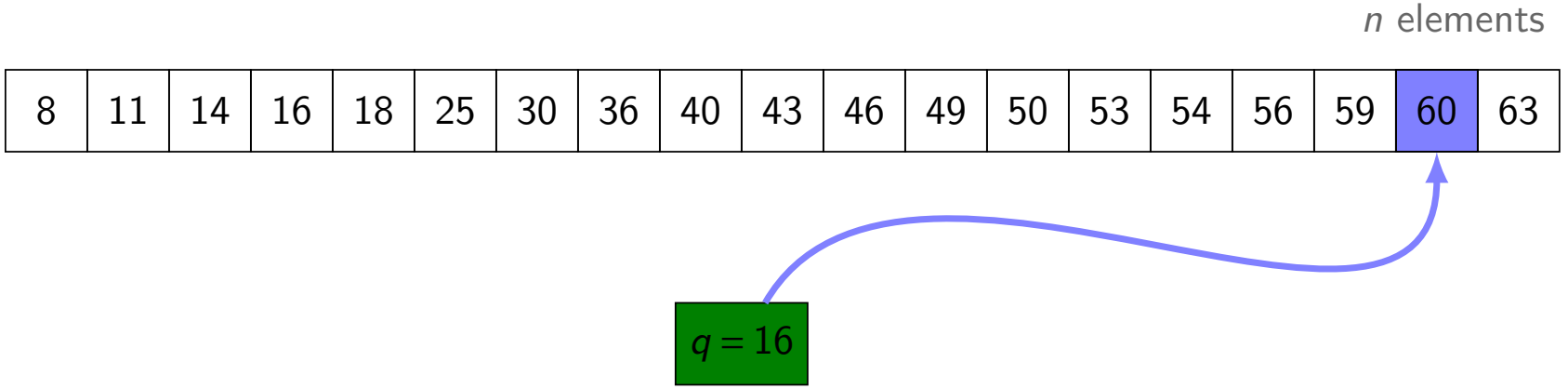

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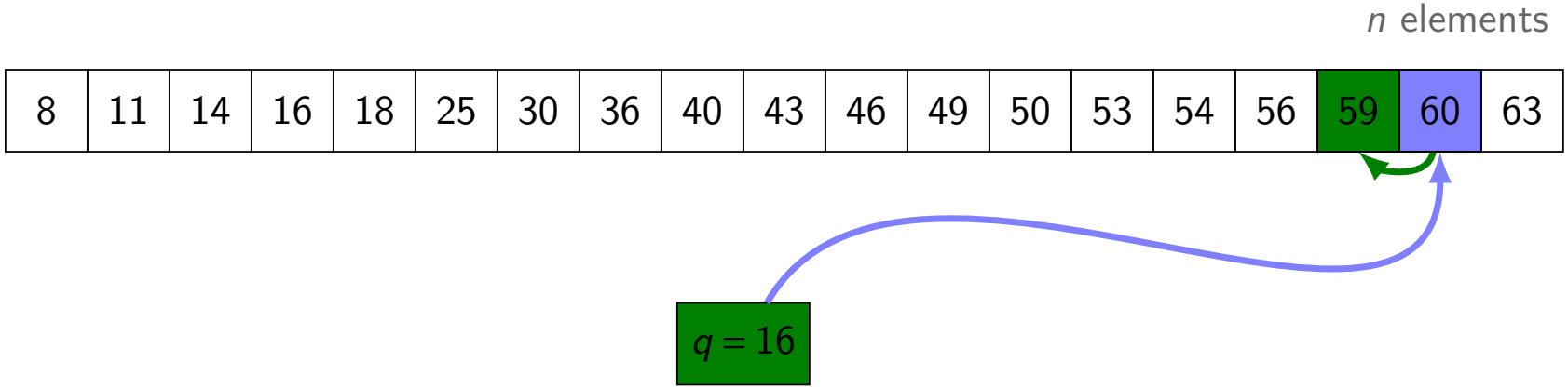

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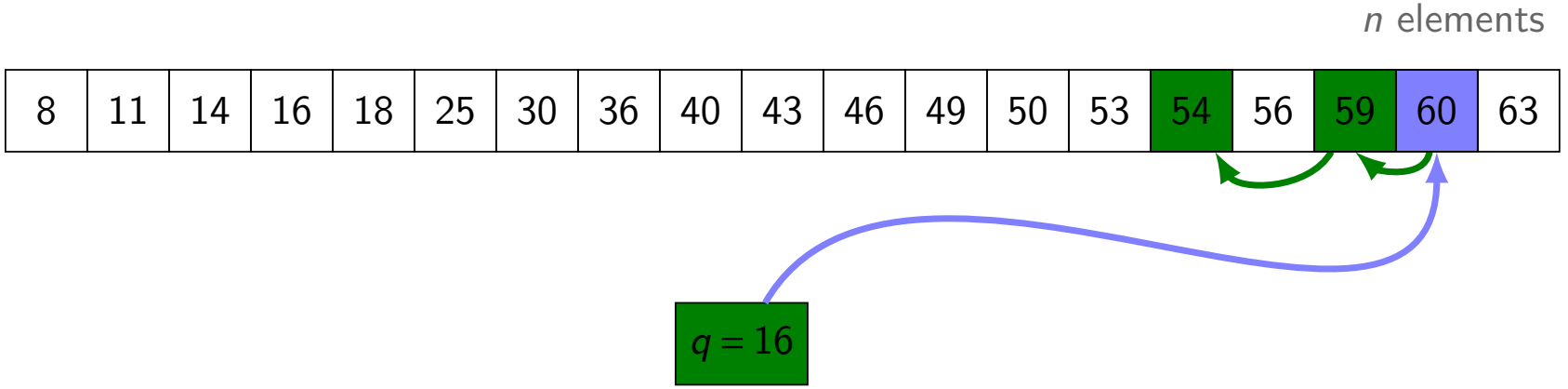

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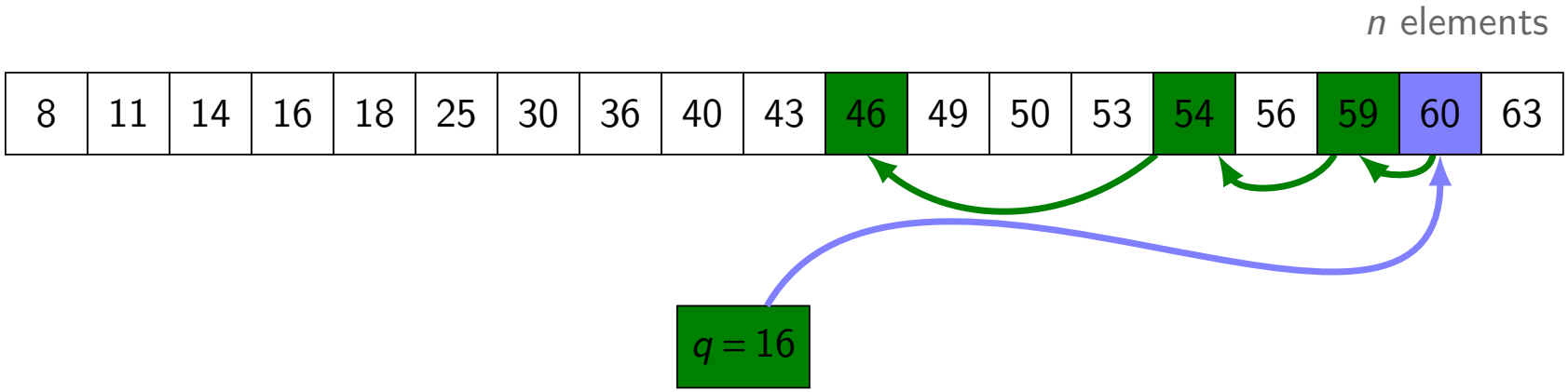

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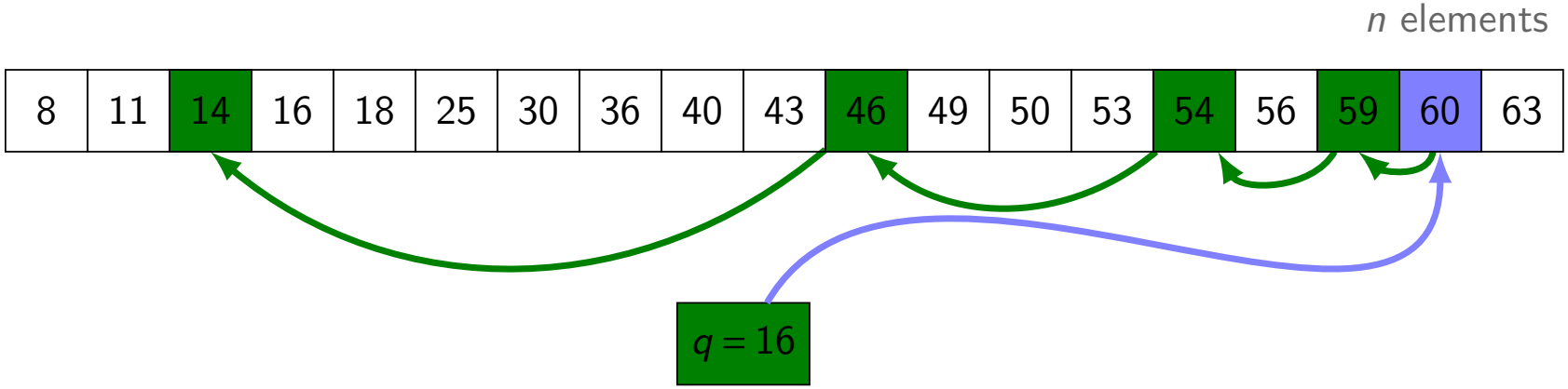

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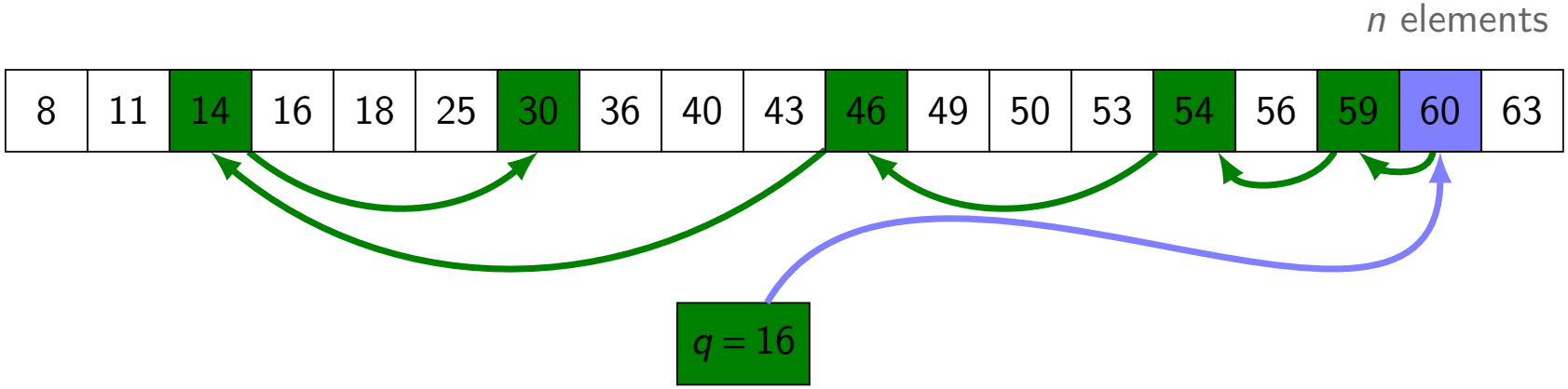

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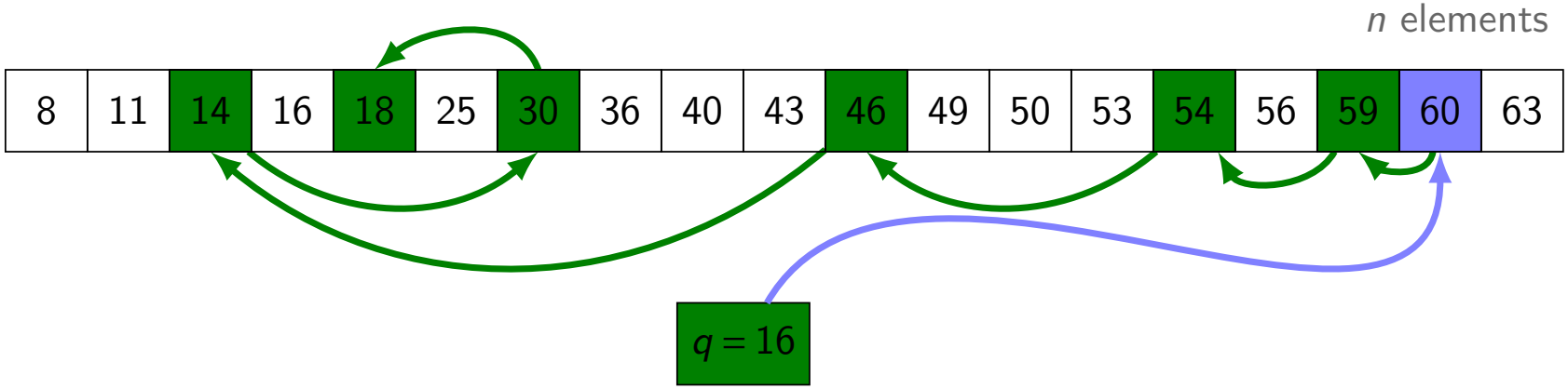

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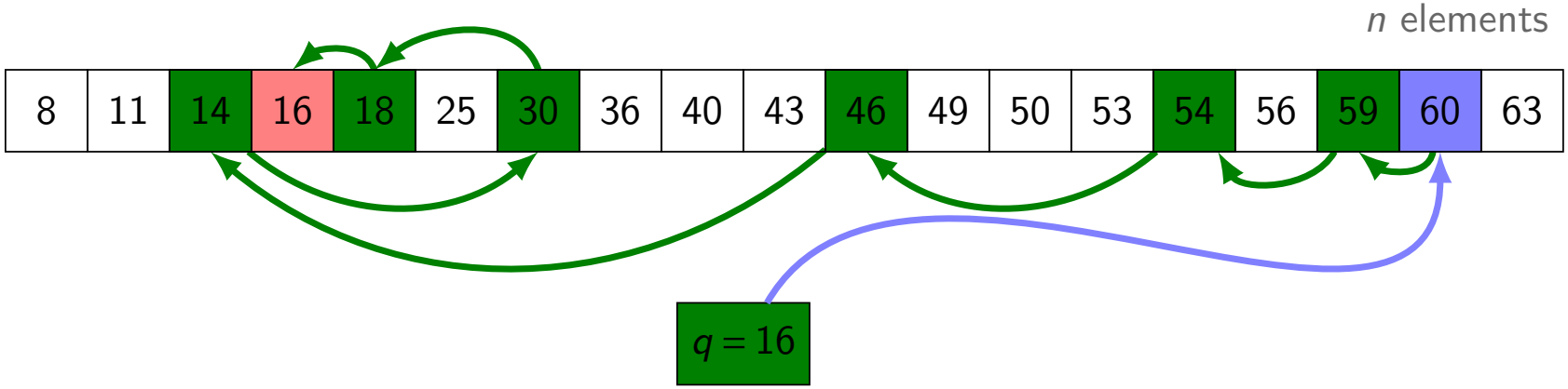

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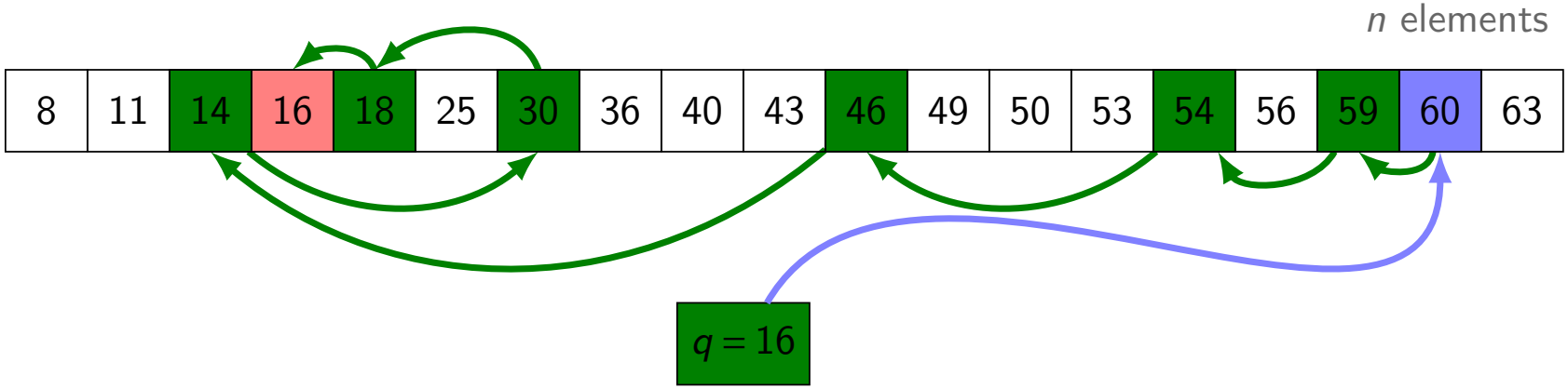

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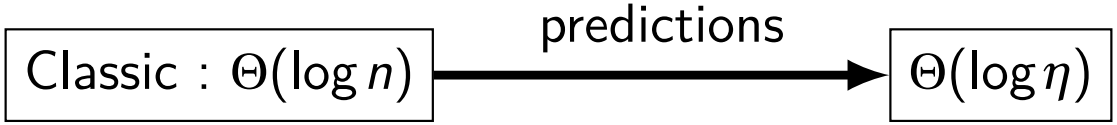

Practical applications [KraskaBCDP'18] – 400+ papers since 2020

Our work in a nutshell [Angelopoulos, Simon, NeurIPS 25]

- Learning-augmented algorithms for a classic online resource allocation problem
- **Stochastic** settings
    - Distributional prediction oracles
    - Randomized online algorithms
- **Upper** and **lower** bounds on performance metrics
- Theoretical / experimental evaluation

Online bidding

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Online bidding

u

Online bidding

x_0

u

Online bidding

x_0

x_1

u

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u

x_0

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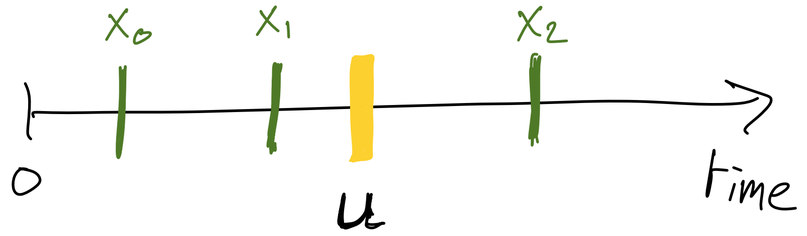

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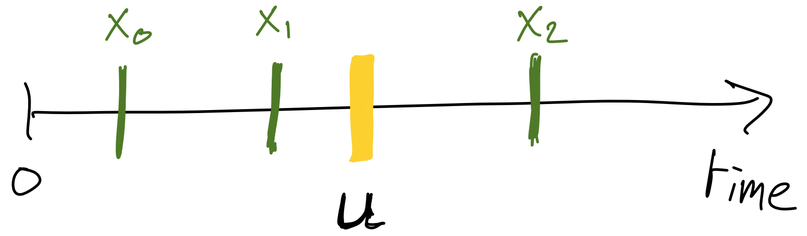

Competitive ratio of $X : (x_0, x_1, \ldots)$ $= \sup_u \frac{\text{cost}(X, u)}{u} = \sup_u \frac{x_0 + x_1 + \ldots x_j}{u}$

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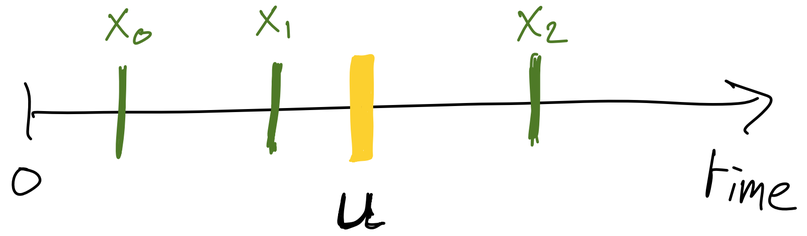

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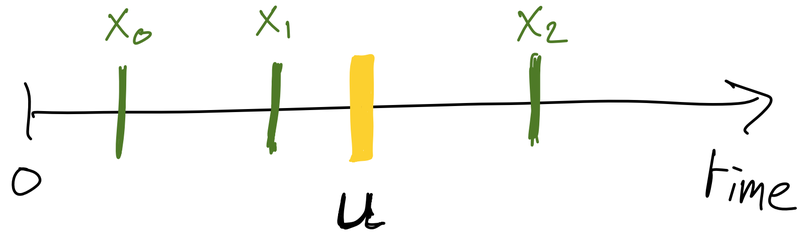

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$$= \sup_{j \geq 1} \frac{\sum_{i=0}^{j} x_i}{x_{j-1}}$$

Previous work : the standard setting

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- Tight **deterministic** competitive ratio = **4**, using the strategy $(2^i)_{i\geq 0}$

- Tight **randomized** competitive ratio = e, using the strategy $X = (e^{i+s})_{i\geq 0}$, $s \sim U[0,1]$ [Chrobak, Mathieu, Noga, and Young 2008]

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- Applications in many online and offline problems [Chrobak and Mathieu 2008]

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- Point prediction $\hat{u}$ on the real target u, hence error $\eta = |\hat{u} - u|$

$$\textbf{Consistency} = \sup_{\hat{u}} \frac{\text{cost}(X_{\hat{u}}, \hat{u})}{\hat{u}} \quad \text{whereas} \quad \textbf{robustness} = \sup_{\hat{u},u} \frac{\text{cost}(X_{\hat{u}}, u)}{u}$$

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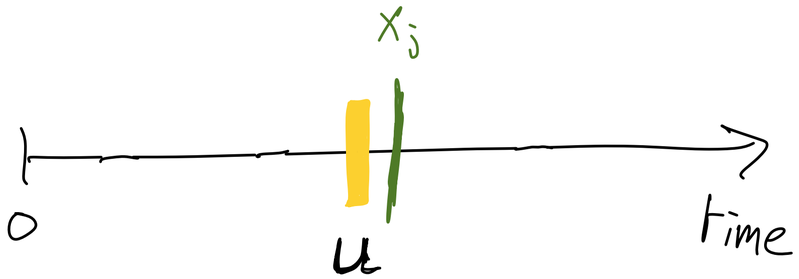
- Deterministic Pareto-optimal algorithms: $\frac{b}{b-1}$ - consistency and $\frac{b^2}{b-1}$ robustness, $b > 1$ [Angelopoulos, Durr, Jin, Kamali, Renault 2020]



- Pareto-optimal algorithms are **brittle** [Elenter, Angelopoulos, Dürr, and Lefki 2024], tradeoffs between consistency, robustness and smoothness, also in [Benomar, Perchet 2025]

First stochastic setting: Distributional predictions

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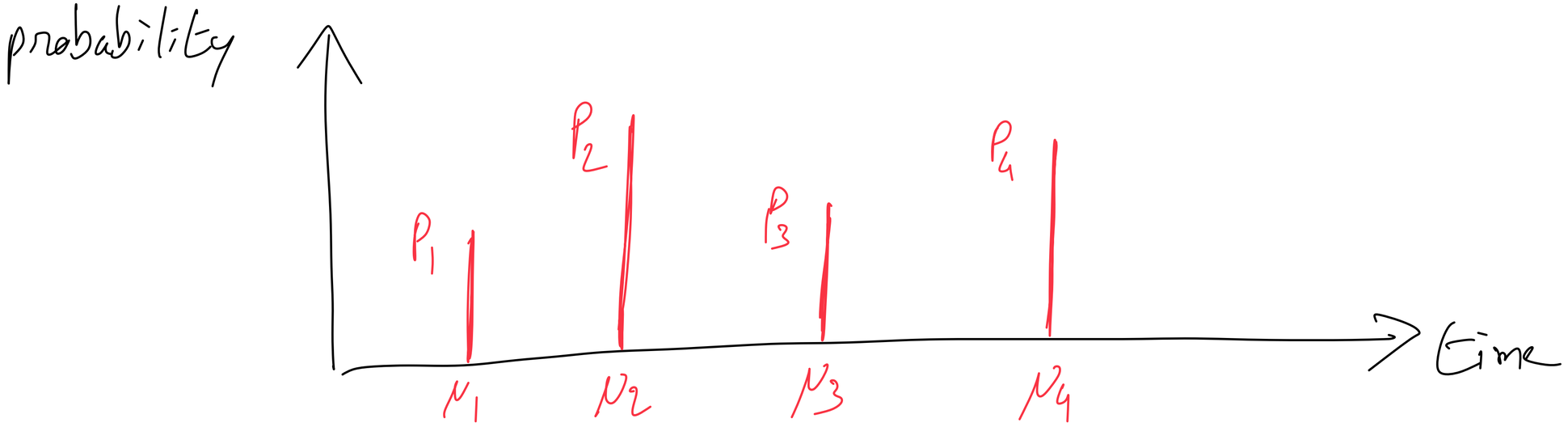

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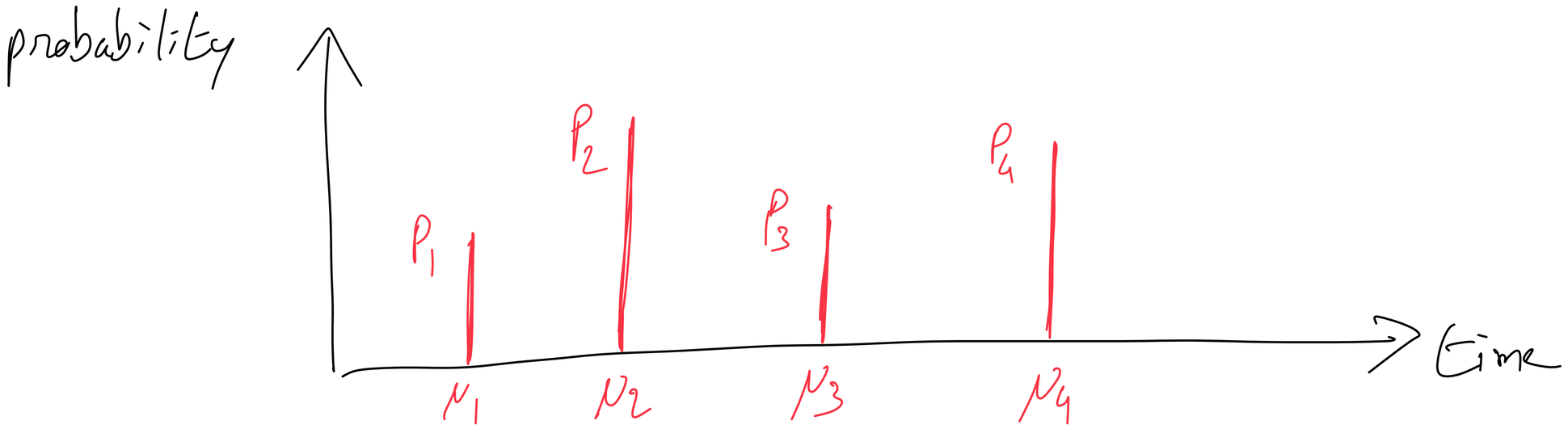

Objective : Given $r \geq 4$ find a deterministic $r-$robust algorithm X that minimizes the consistency wrt μ

$$\mathrm{cons}(X, \mu) = \frac{\mathbb{E}_{z\sim\mu}[\mathrm{cost}(X, z)]}{\mathbb{E}_{z\sim\mu}[z]}$$

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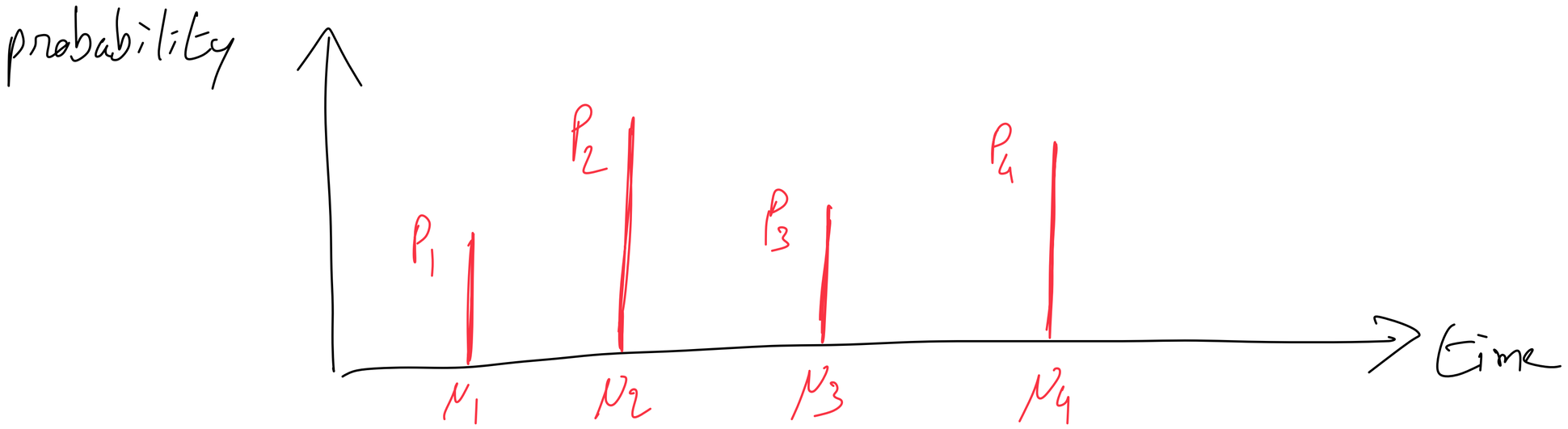

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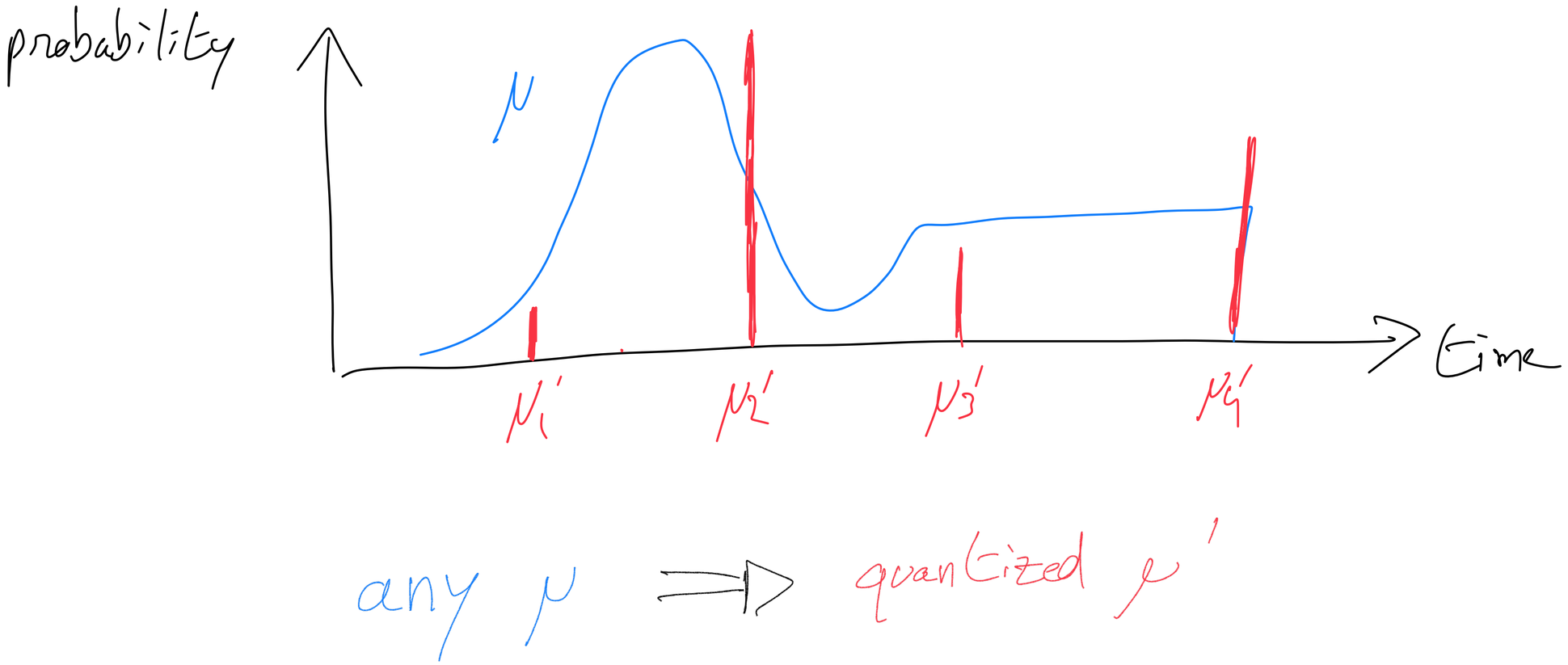
Main result: Pareto-optimal algorithm for this setting

Handling general distributions via quantization



any μ ⇒ quantized μ'

Outline of the approach

Challenge: Pareto-optimal strategies may have a non-obvious structure

Theorem: Even if $k = 2$, every geometric strategy $(\lambda \cdot b_r^i)_{i \geq 0}$ has *unbounded consistency* as $r \to \infty$

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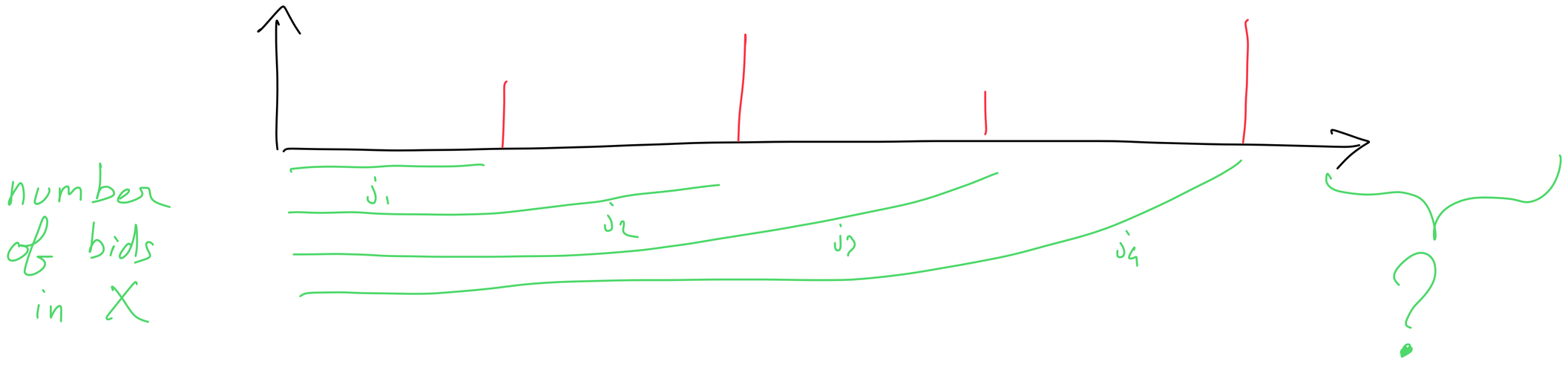

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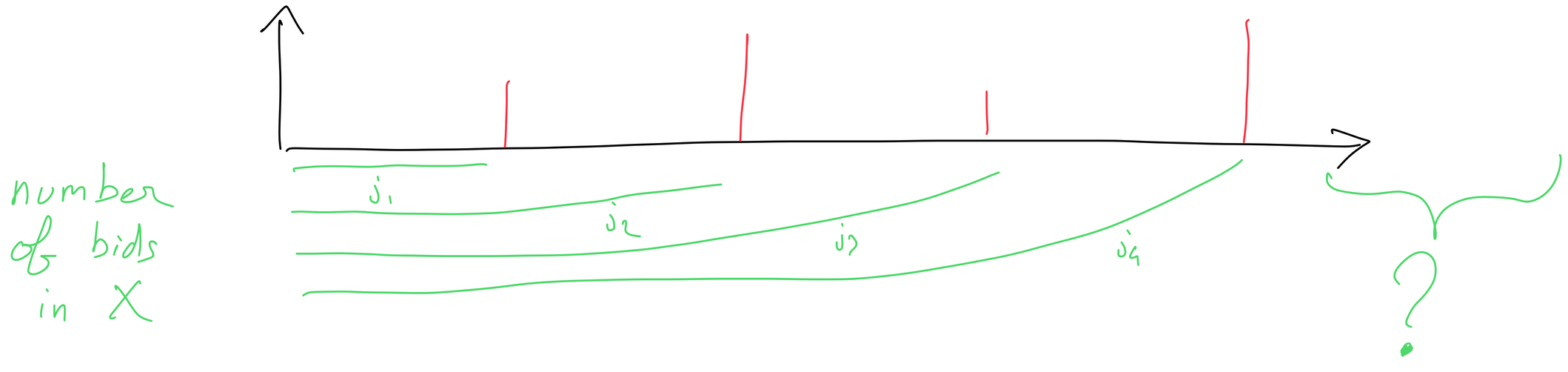
- We can write the objective as: $\mathbb{E}_{u \sim \mu} \text{cost}(X, u) = \sum_{i=1}^{k} p_i \sum_{q=0}^{j_i+1} x_q$



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$$\text{subj. to} \quad x_i \leq r x_{i-1} - \sum_{j=0}^{i-1} x_j, i \in [1, j_k + 1]$$

robustness “up to the prediction”

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Theorem: A partial $r-$robust strategy can be extended to an infinite one if subsequent robustness constraints are saturated

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$$\sum_{i=0}^{j_k+1} x_i \leq \frac{r + \sqrt{r(r-4)}}{2} x_{j_k+1}$$

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Complexity, brittleness, and error

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Complexity: $O(c^c)$ LPs, $c = O(\log^k \mu_k)$

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- EMD-based analysis is likely possible using techniques from contract scheduling [Angelopoulos, Bienkowski, Dürr, and Simon 2024]

  (but some assumptions on the distribution will be required)

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Recall: $X = (e^{i+s})_{i \geq 0}$ with $s \sim U[0,1]$ has optimal randomized competitive ratio = e

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How much improvement relative to deterministic algorithms ?

Robustness	Randomized consistency	Deterministic consistency
$r = 4$	1.724 ($\delta^* \approx 0.9$)	2
$r \geq 5$	**very similar values** $\delta^* \approx 0.99$	

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Main idea: Apply **Yao's principle** on the **scalarized** objective **Cons + $\lambda \cdot$ Rob,** **for all** $\lambda \in [0,1]$

(inspired by two-person zero-sum games [Angelopoulos, Lidbetter, Panagiotou 2024])

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- Adversarial distribution $\mathscr{D}$ with a hyperbolic pdf (also recovers the $e-$competitive ratio)
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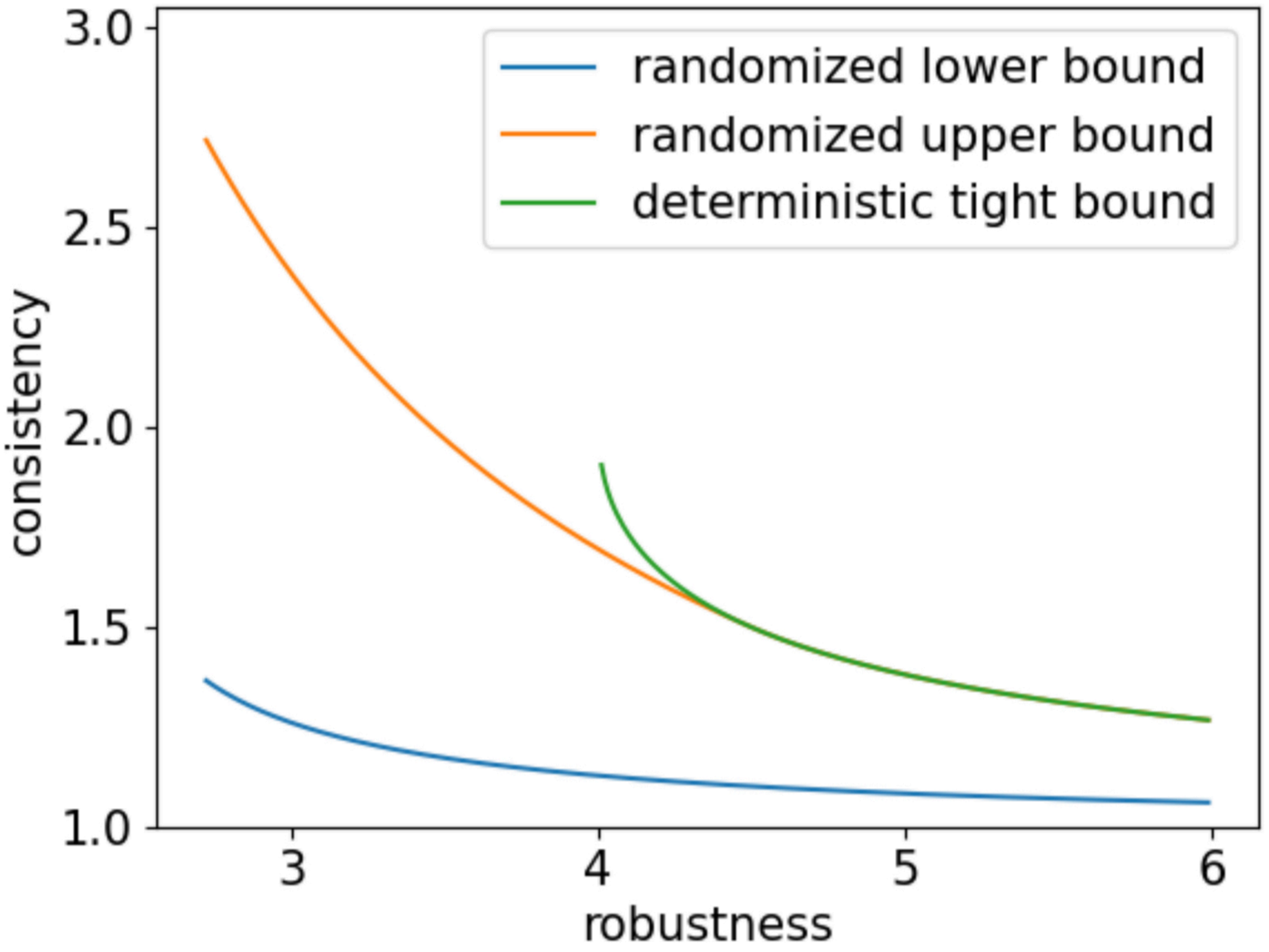
Comparison between the bounds



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Searching on the infinite line

- Pareto-optimal deterministic algorithms for distributional, **positional** advice

  (robustifies the average-case analysis of [Gal 1980])

- Upper/lower bounds for randomized algorithms with single-point **positional** prediction

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Bidding with dynamic predictions

Setting: Bidding with a sequence of predictions $\hat{u}_1, \hat{u}_2, \ldots$

Objective: Lexicographically Pareto-optimal deterministic algorithm

Result: LP-based polynomial-time solution

Future work

- Improve the time complexity of the LP-based algorithm: is there an FPTAS? Lower bounds?
- Tight(er) upper / lower bounds for randomized algorithms
- Combined settings: Distributional oracles + randomized algorithms (hard)
- Other tools for lower bounds beyond scalarized Yao's principle?

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Thank you!