Scheduling recursive tasks on homogeneous and heterogeneous systems

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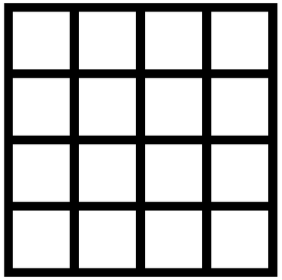
INRIA Bordeaux - Sud-Ouest -- STORM Team

Expressing a task graph

Implicit task dependencies

- Right-Looking Cholesky decomposition (from PLASMA)



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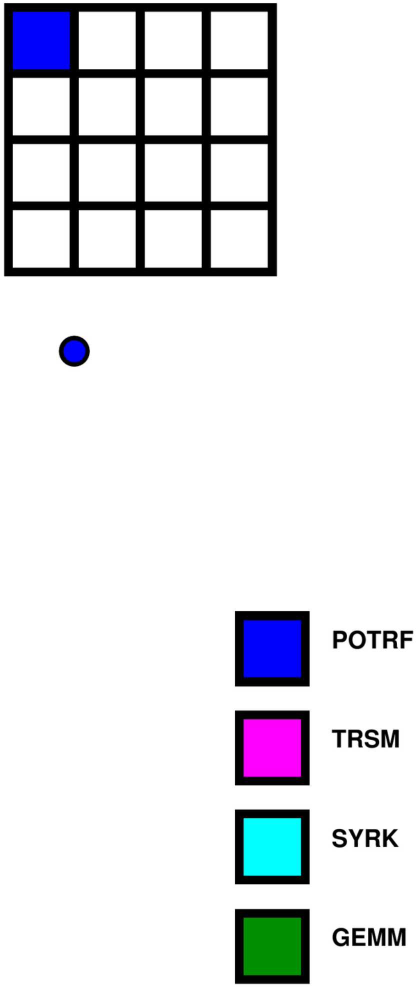

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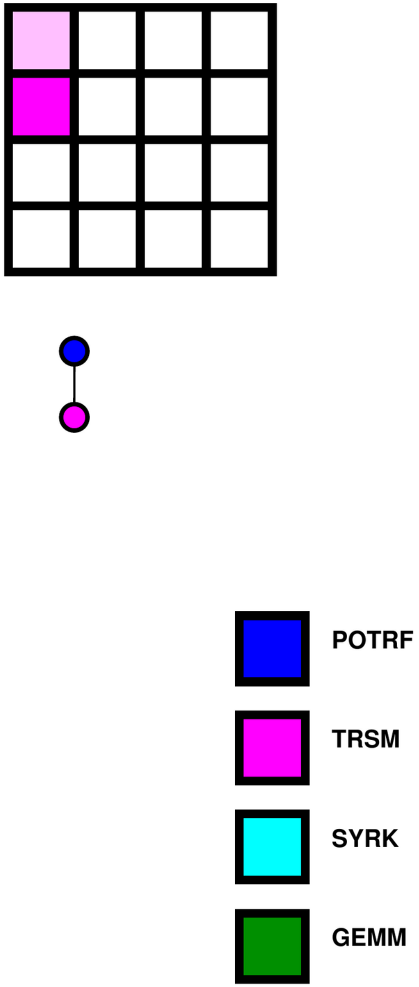

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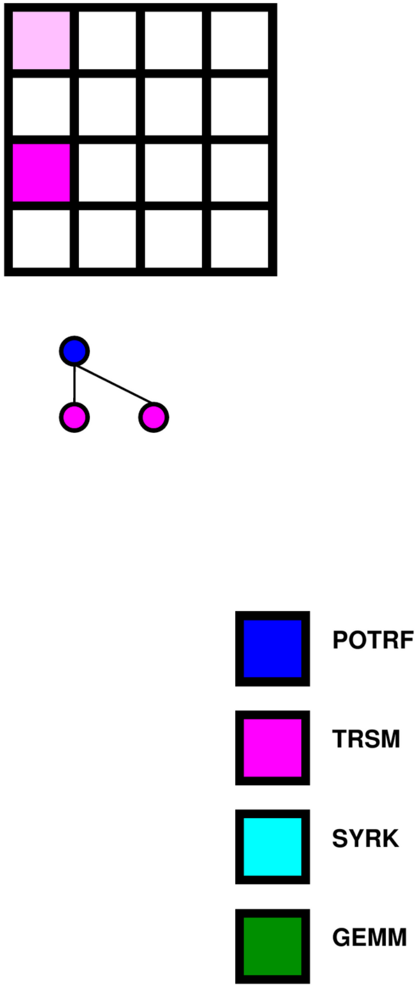

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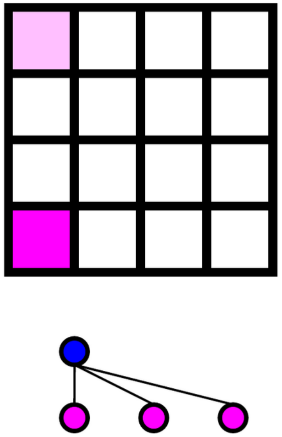

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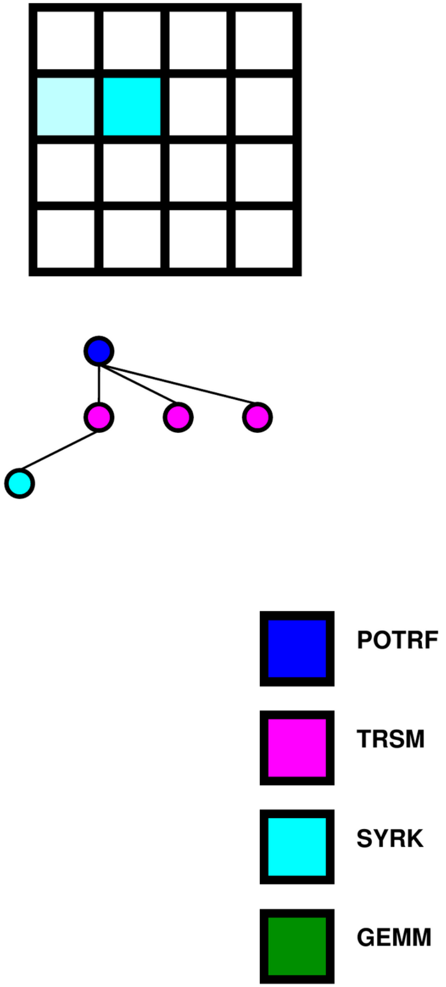

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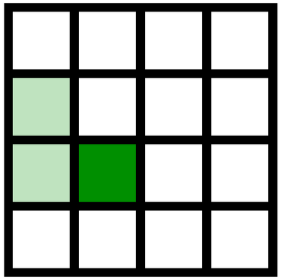

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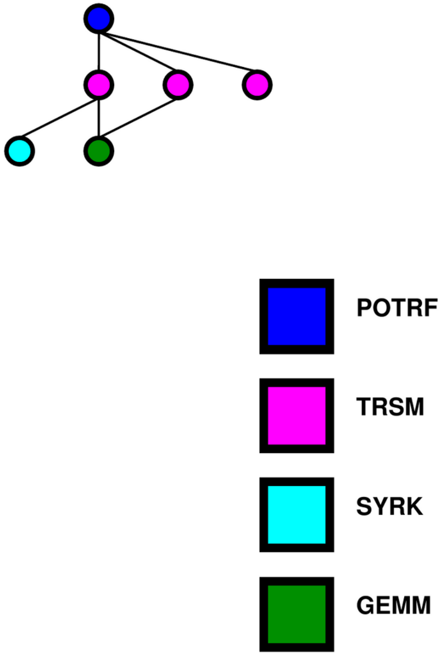

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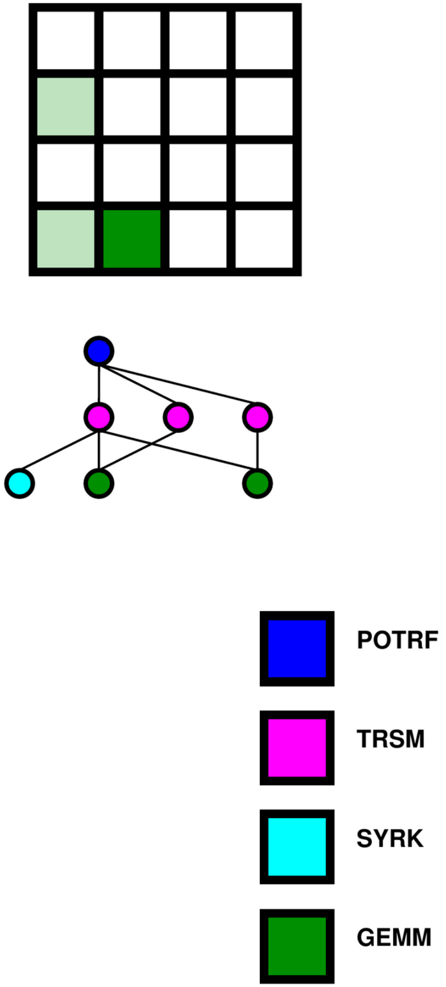

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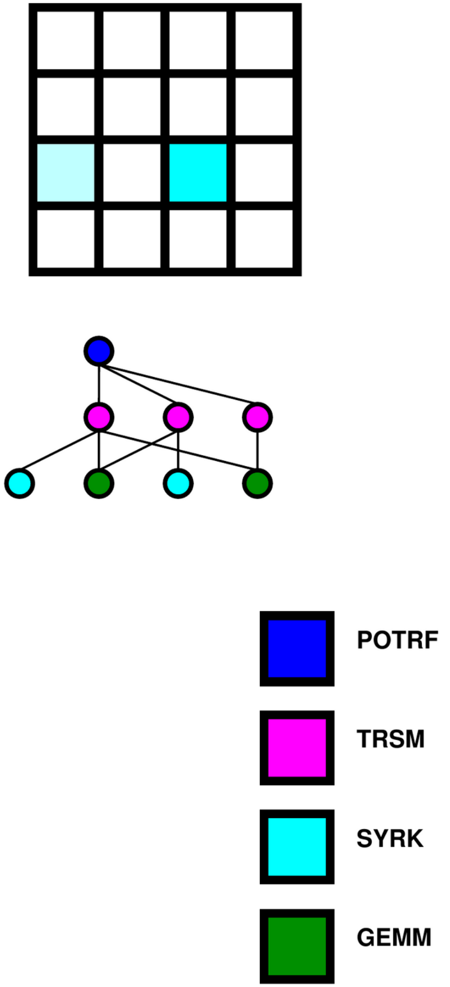

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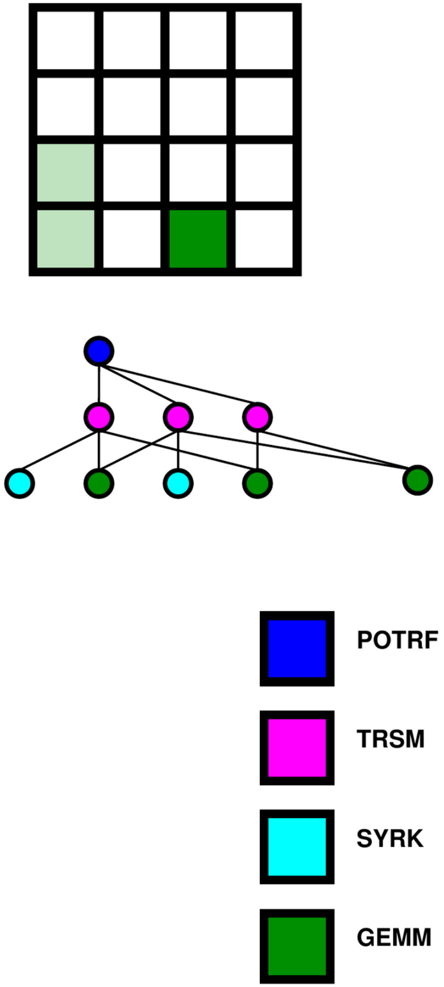

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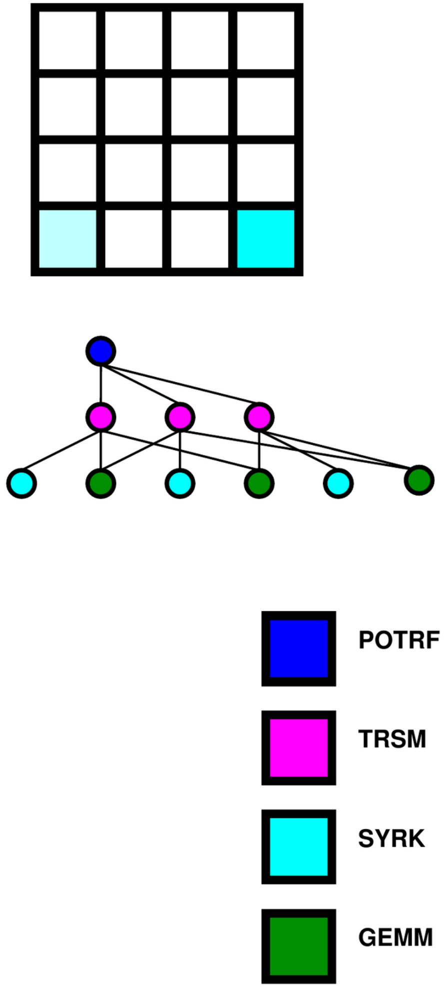

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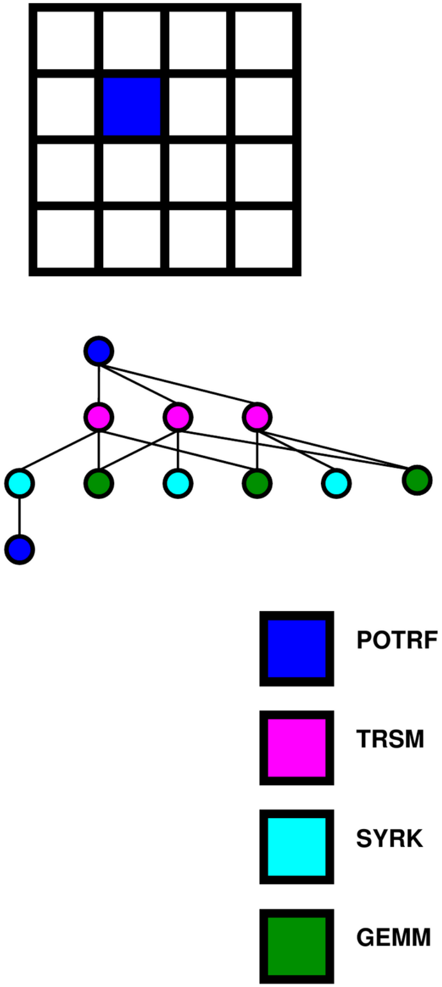

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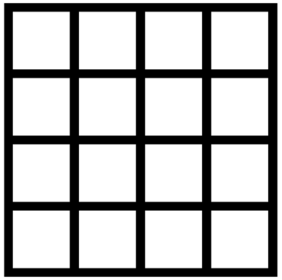

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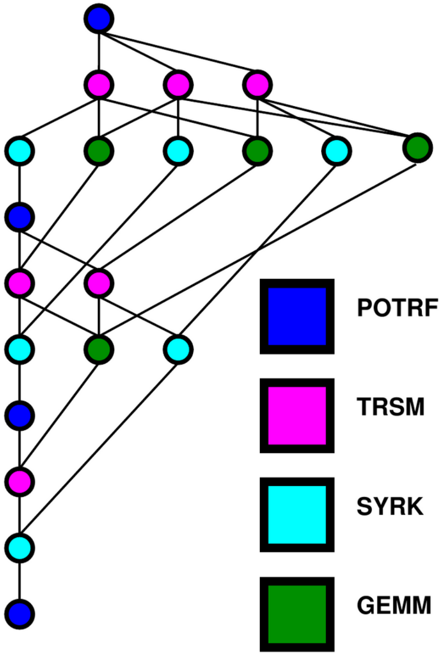

How big should a task be?

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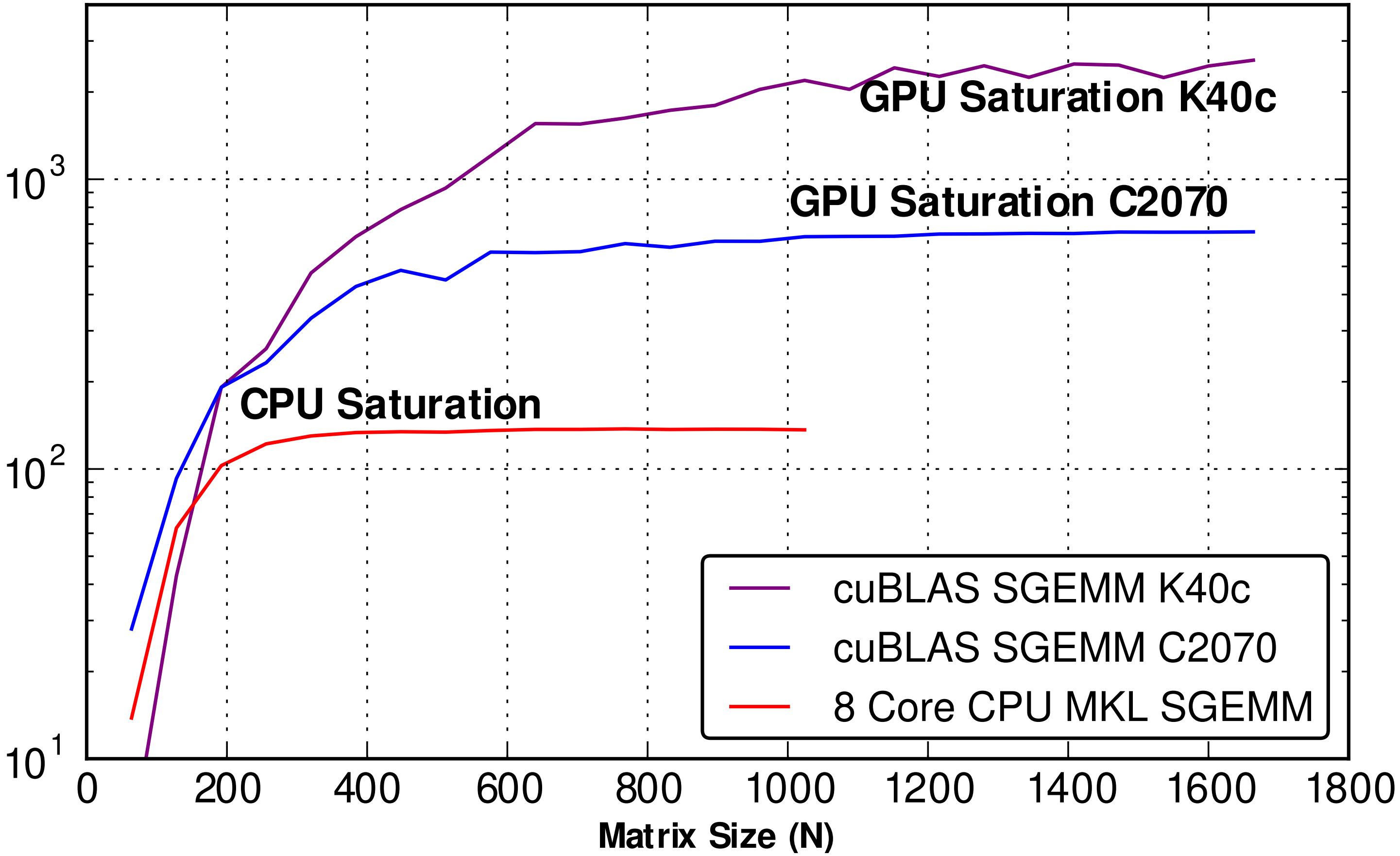
- Small enough to get parallelism to **feed** all processing units
- Large enough to **efficiently** use the processing units

How big should a task be?



From PARSEC : « Hierarchical DAG Scheduling for Hybrid Distributed Systems », Wu, Bouteiller, Bosilca, Faverge, Dongarra

How big should a task be?

GPUs

- Have **thousands** of cores to feed
- Newer generations require yet larger sizes
- Can run several kernels at the same time
  - Still limited

How big should a task be?

CPUs

- Have many independent cores

  → Need many tasks

- Can use parallel implementations (e.g. from MKL)
  - But better have subtasks to interleave them

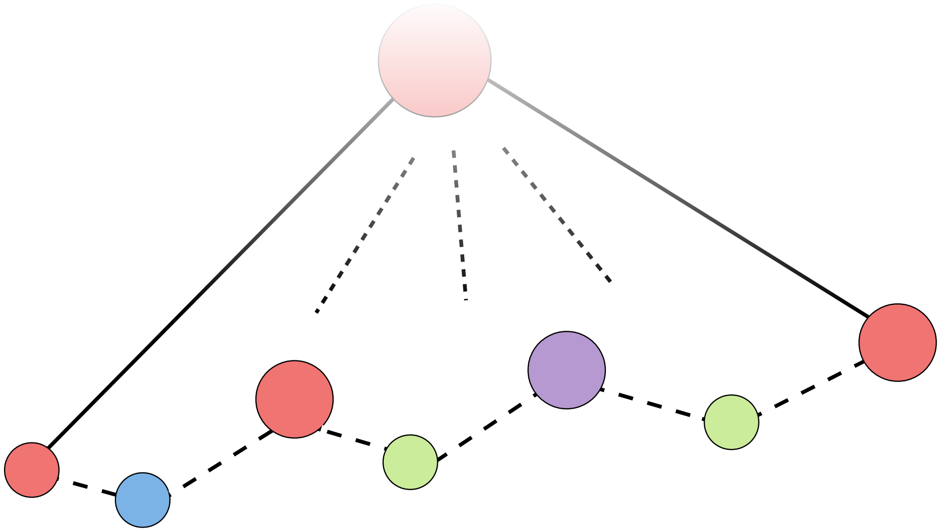
How big should a task be?

Multiple answers

- Depends on available platform parallelism
- Depends on available application parallelism
- Depends on application phases

Automatically adapt?

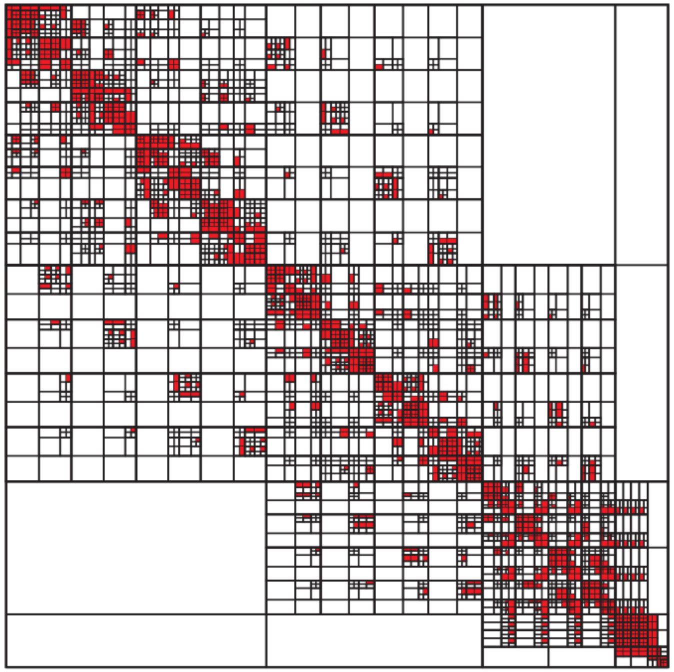
Recursive task graphs



Recursive task graphs

Applications themselves are recursive

- e.g. h-matrices



From Airbus Group

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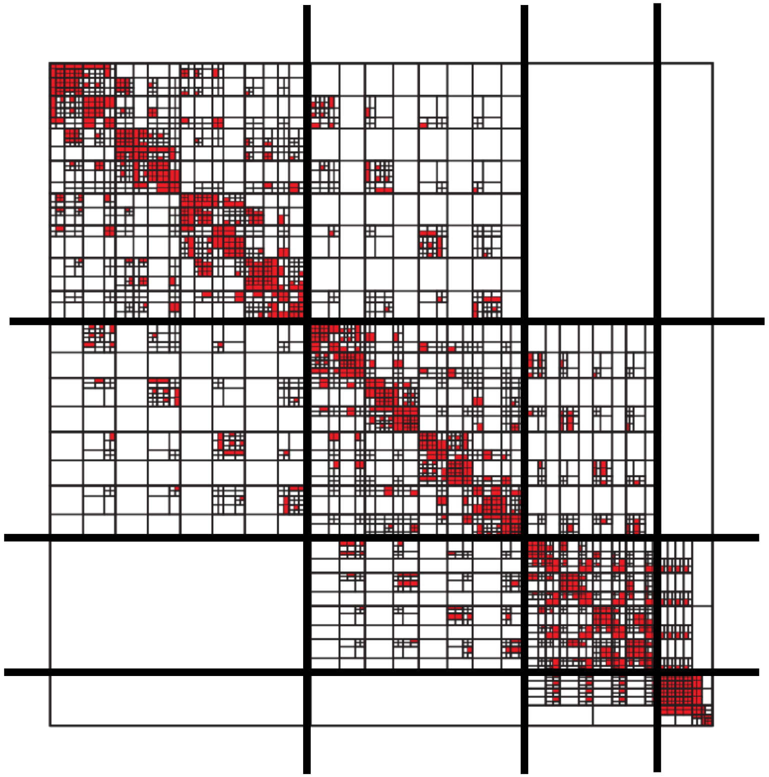

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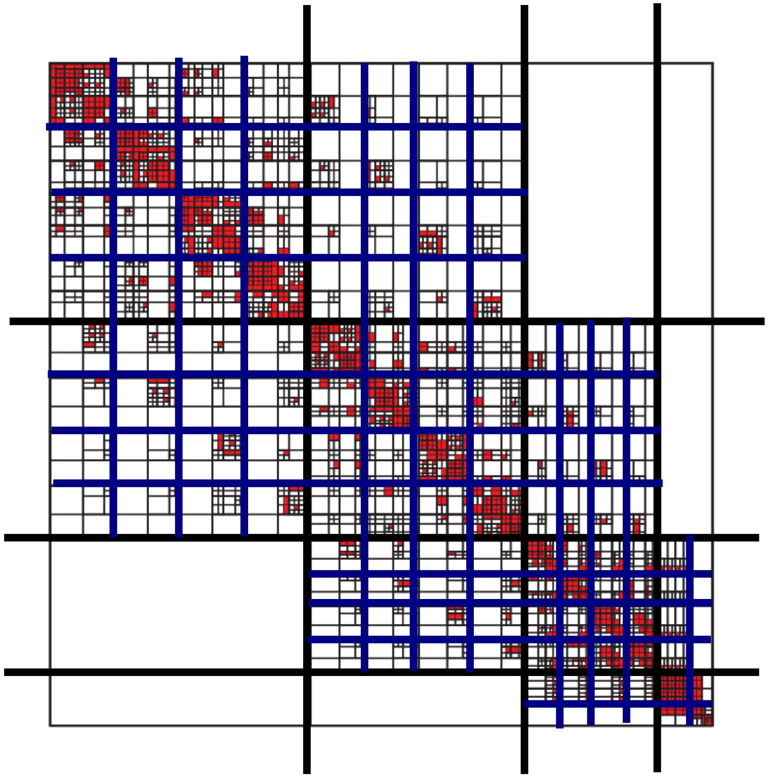

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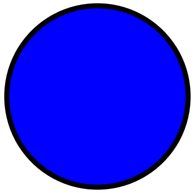
Recursive task graphs

Ideally, should just start with one huge task, and split



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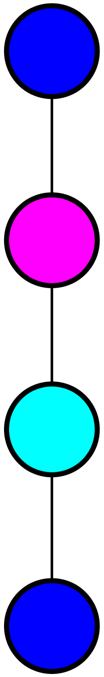

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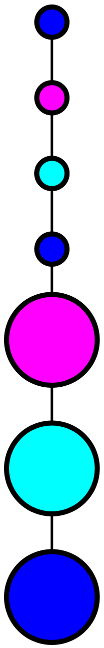

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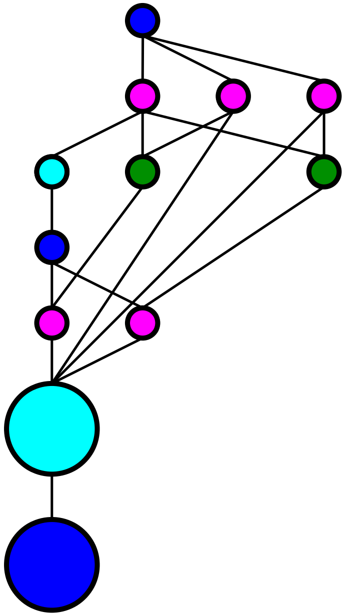

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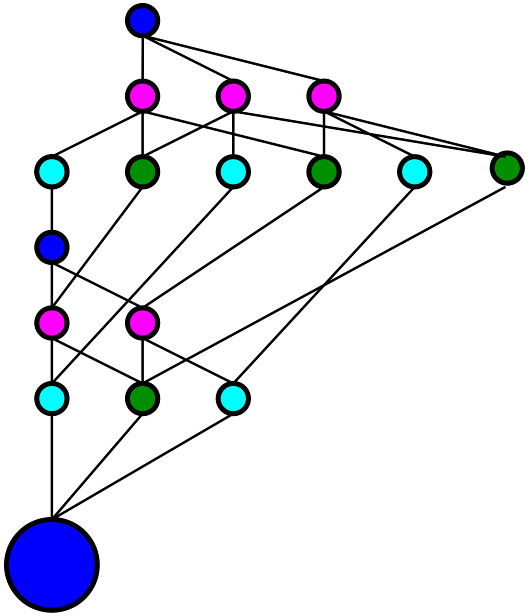

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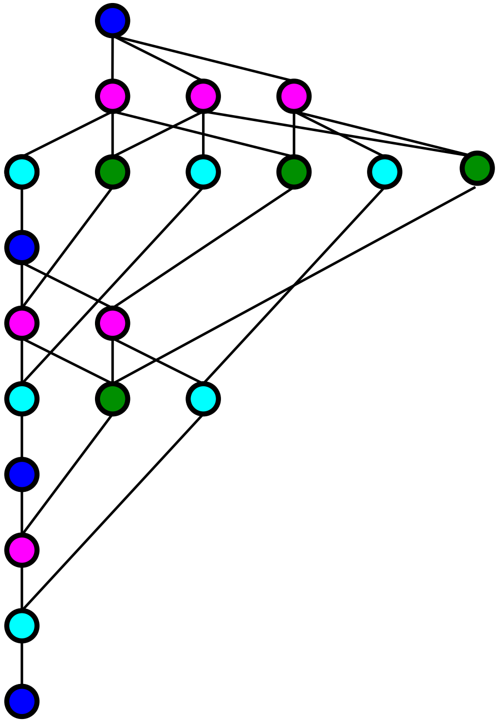

Dividing tasks

No extra synchronization

→ Can consider task graph subdivision as a tree

POTRF
Recursion

Dividing tasks

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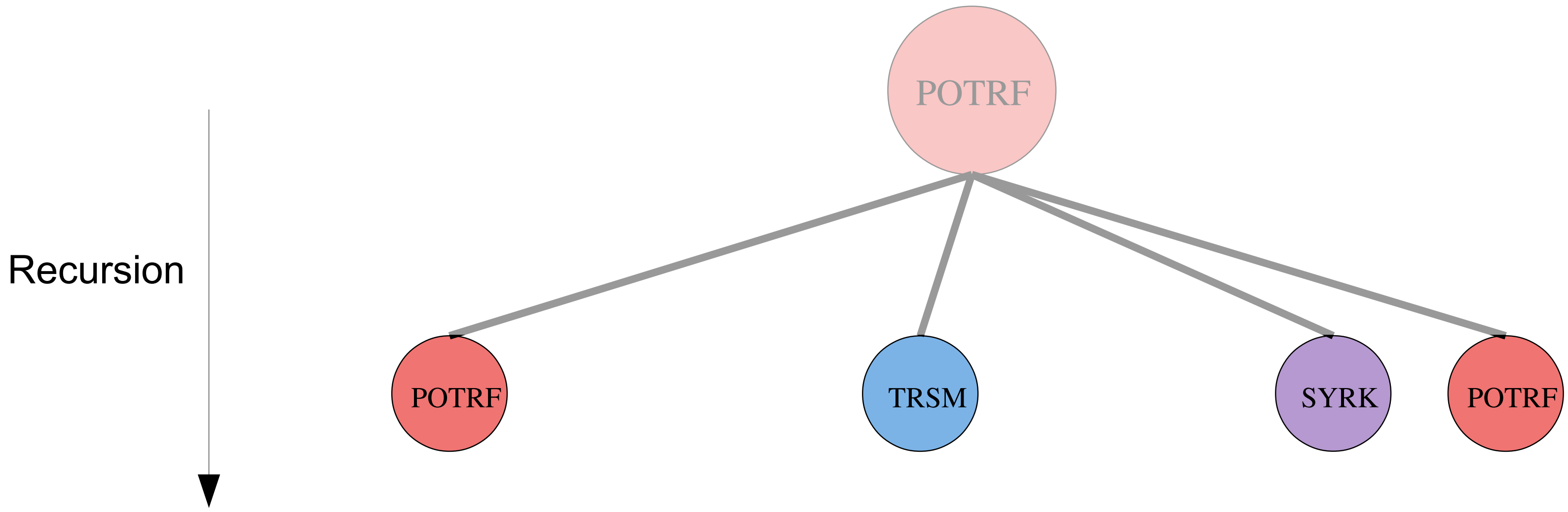

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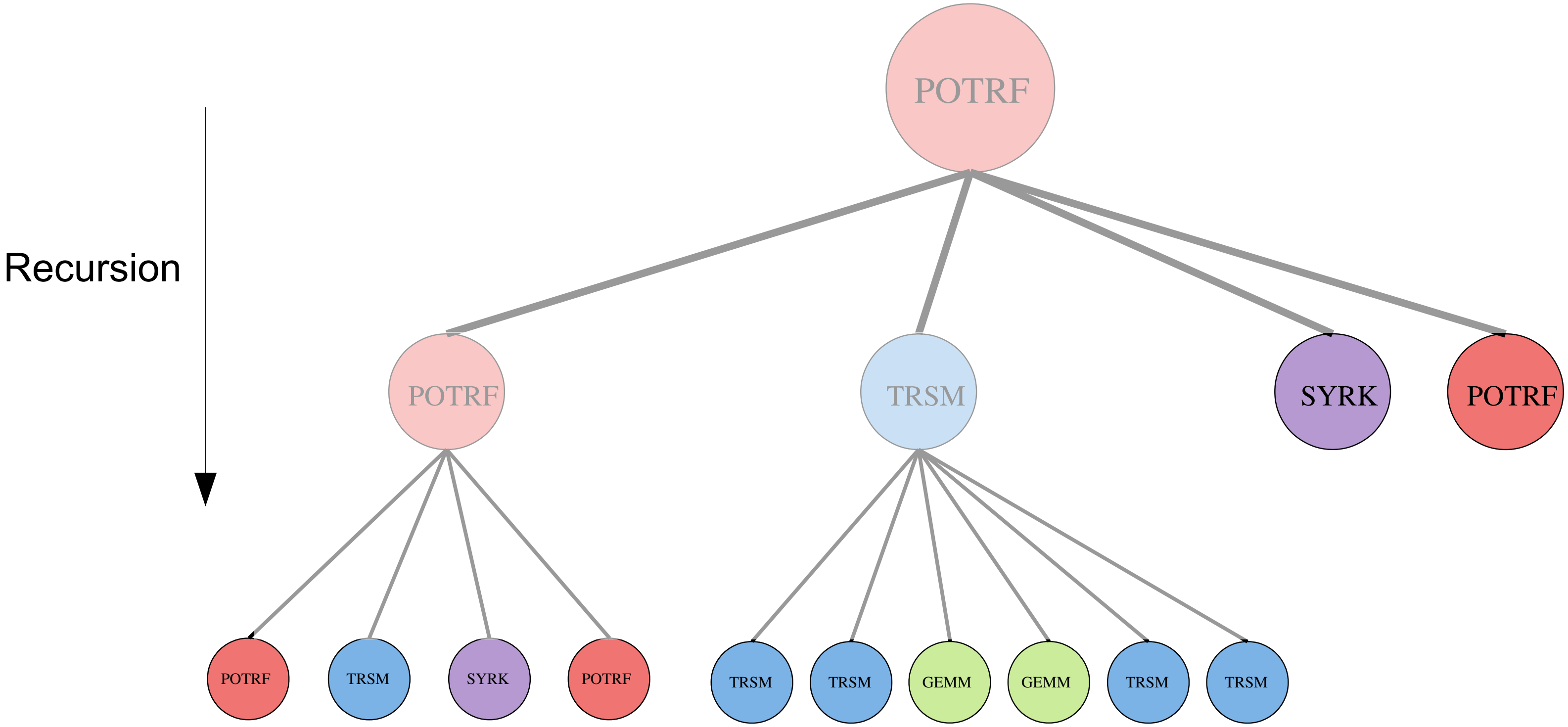
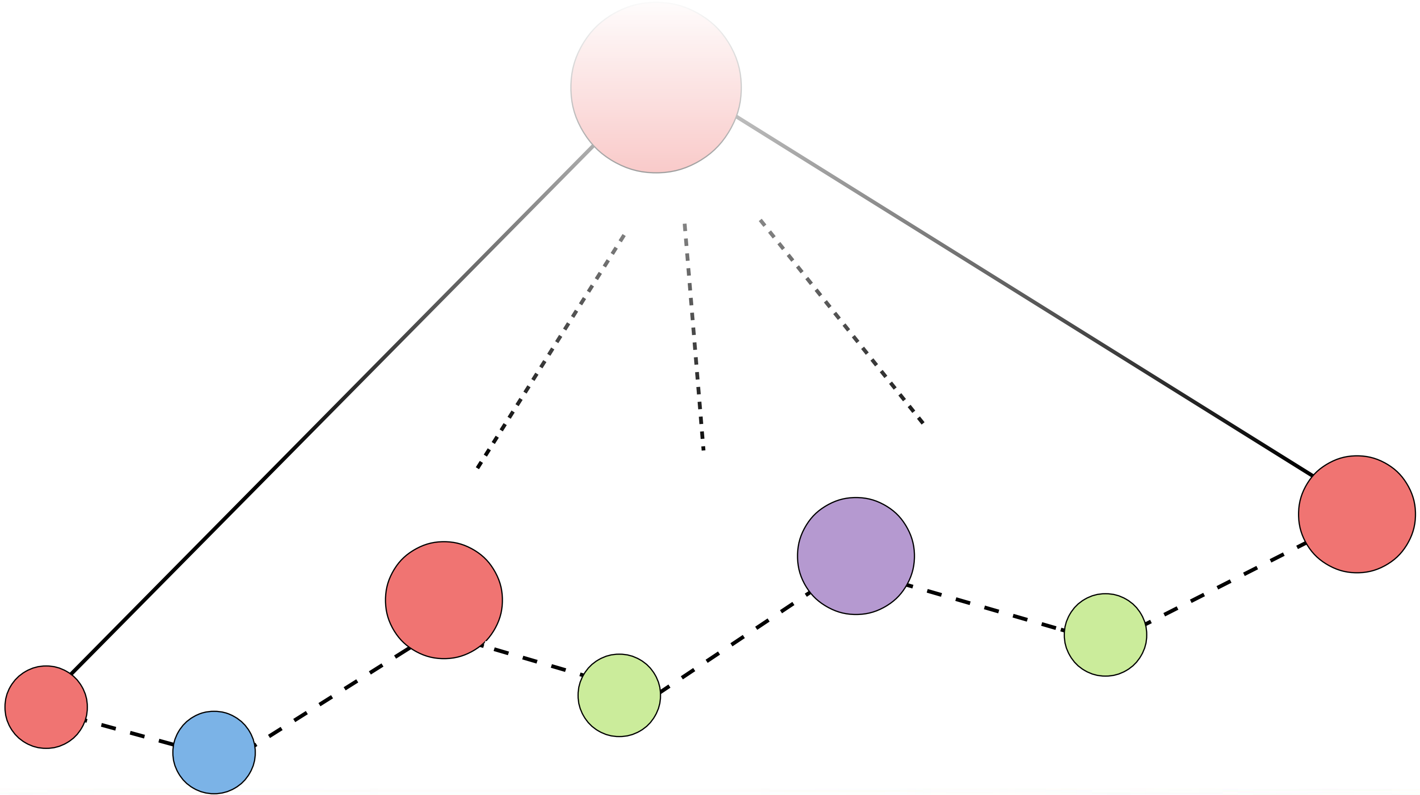


Dividing tasks

No extra synchronization

→ Can consider task graph subdivision as a tree

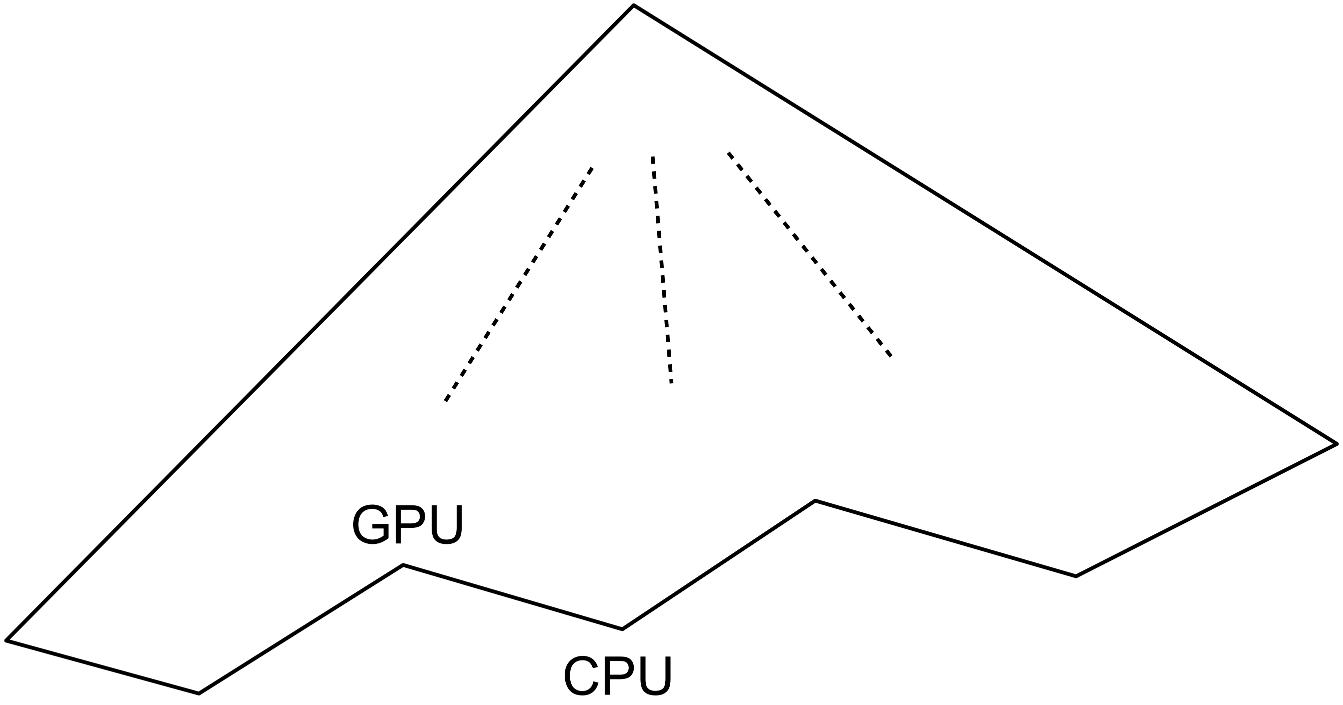
→ Decide at will where and when to stop recursing



Opportunities

GPU / CPU efficiency management

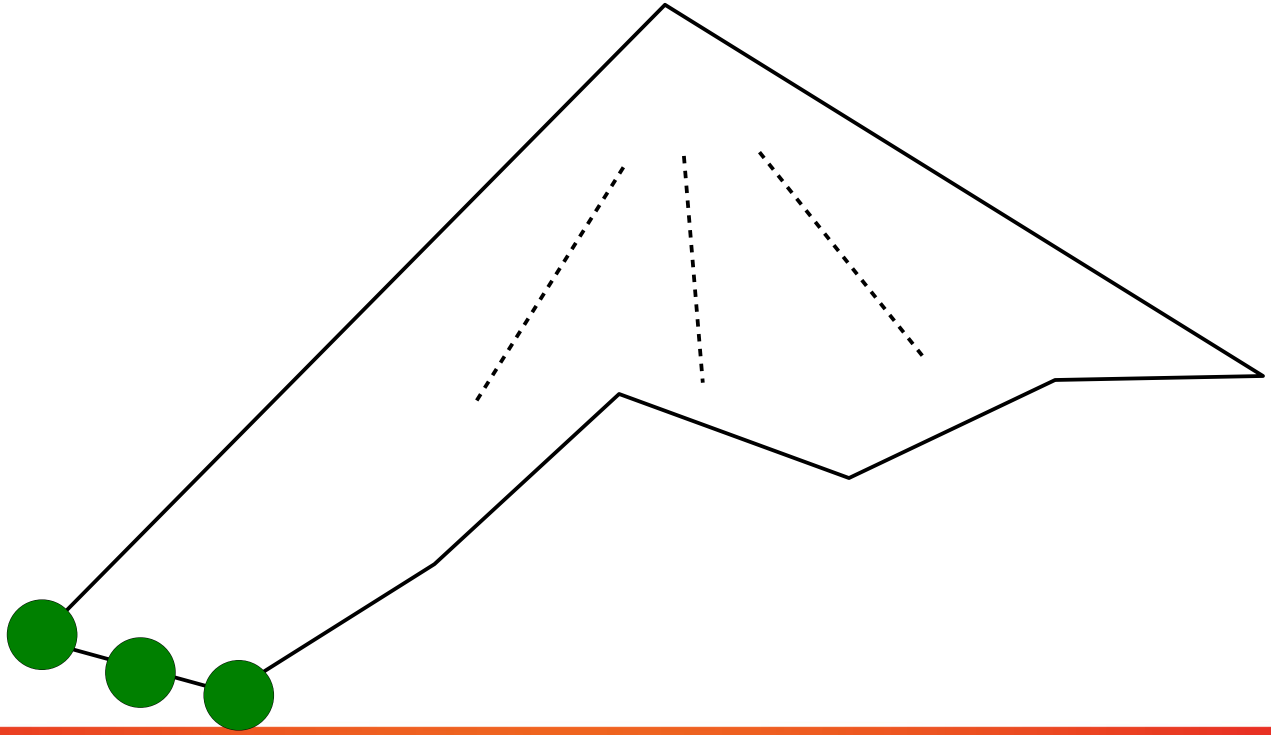
- Stop recursing at desired tile size
  - Keep large tasks for GPUs, small tasks for CPUs
- Care for latency of the task-graph critical path



Opportunities

Delay unrolling

- Observe behavior before unrolling the rest accordingly
- Decorrelate submission and execution



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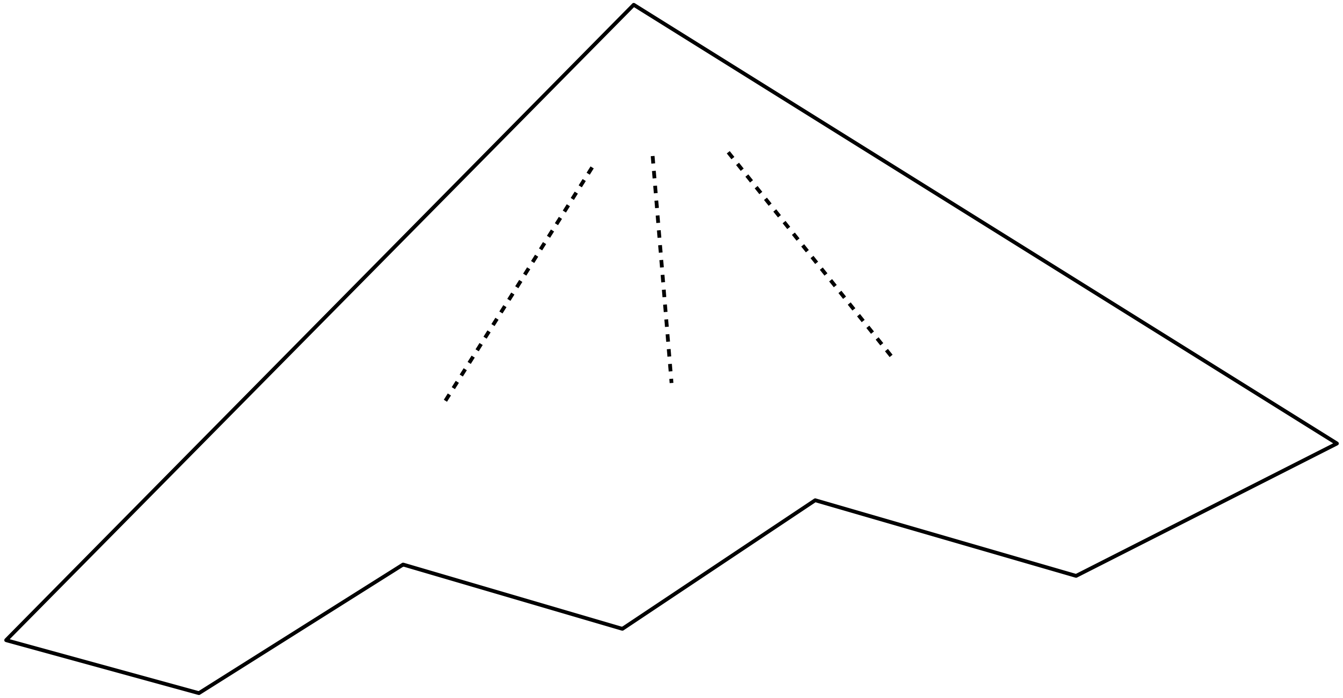


Recursive task graphs

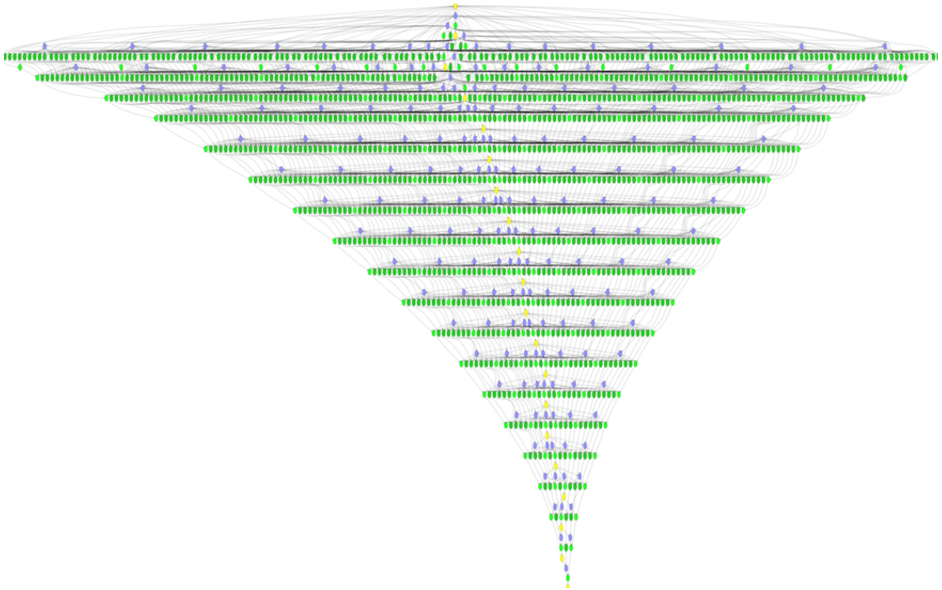
[Lucas'23]

- Made Recursive task graphs work
  - With STF etc.
  - With no spurious dependency
- Ideally, Cholesky boils down to
  - `int main(void) { potrf(A); }`
  - And the runtime splits into tasks recursively
- Now : When? How many? How? Which?

Recursive task graphs

When?

- Better wait a bit to see how things behave?
- In Cholesky case
  - Beginning: better not split too much
    → Efficiency
  - End: better split more
    → Parallelism



Recursive task graphs

How many?

- GPUs? Better not split too much
- CPUs? Better split
- Priority? Better split
- ...

Recursive task graphs

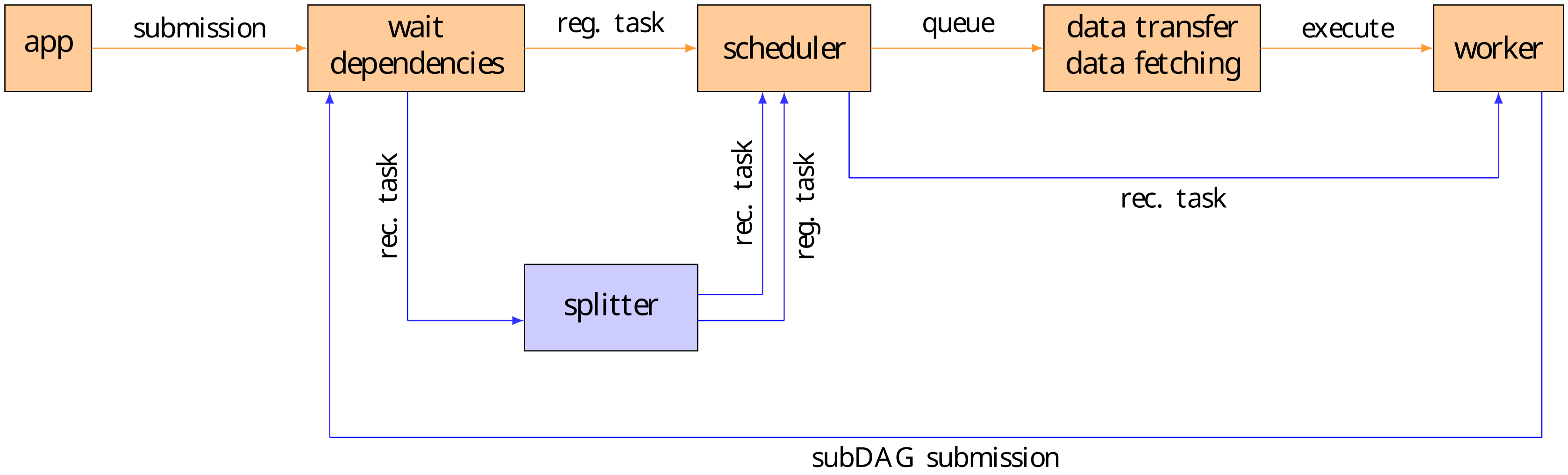
Which?

- Depends on split efficiency
  - Splitting may lower GFlop/s achieved by tasks
  - But also exposes slack
    - E.g. mix POTRF subtasks with other subtasks
- Depends on priority
  - Better split tasks on the critical path

    → Reduce latency

Recursive task graphs

When: at which scheduling stage?



- Runtime information
- Still parallel submission

Complex problem

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I’m a system guy

Complex problem

I’m a system guy

- Let’s try simple solutions, at runtime

Complex problem

I’m a system guy

- Let’s try simple solutions, at runtime
- Hoping to inspire you into complex solutions

3 simple solutions presented here

First results : heterogeneous

[Morin'25]

Consider acceleration and parallelism availability

- Big tiles for GPUs
- Small tiles for CPUs
- Find a balance between the two

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[Morin'25]

Consider acceleration and parallelism availability

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Run a linear program every 50 tasks (takes < 1ms)

- Optimizes splitting ratios per task type and recursion level
- Ratios serve as splitting guide

First results : heterogeneous

Parameters

$\mathcal{T}, N_{t,l}^{tot}$	$t \in \mathcal{T}$	Set of task types, Number of task not split of type t at level l.
$\mathcal{R}, R^u$	$u \in \mathcal{R}$	Set of processing unit types, Number of PU of type u.
$\mathcal{L}$		Maximum level of recursion.
$MinN_u$	$u \in \mathcal{R}$	Minimal wanted number of tasks on each PU of type u

Variables

exT		Total execution time.
Ns_l^t	$t \in \mathcal{T}, l < \mathcal{L}$	Number of split task of type t and level l.
$Ne_{l,u}^t$	$t \in \mathcal{T}, l \leq \mathcal{L}, u \in \mathcal{R}$	Number of task of type t and level l executed on PU u

First results : heterogeneous

Minimize exT

Subject to

Task number splitting.

$$\sum_{u\in\mathcal{R}} Ne^{t}_{l,u} + Ns^{t}_{l} - \sum_{p\ \in\ par(t)} nch^{t}_{p,l}\cdot Ns^{p}_{l-1} \geq N^{tot}_{t,l} \qquad t\in\mathcal{T},\ l\leq\mathcal{L}$$

No last−level splitting.

$$Ns^{t}_{\mathcal{L}} = 0 \qquad \forall\ t\in\mathcal{T}$$

Completion time when executing tasks.

$$\sum_{\substack{t\in\mathcal{T}\\ 0\leq l\leq\mathcal{L}}} Ne^{t}_{l,u}\cdot Ex^{u}_{t,l} - R^{u}\cdot exT \leq 0 \qquad u\in\mathcal{R}$$

Minimal number of tasks on PU type.

$$\sum_{\substack{t\in\mathcal{T}\\ 0\leq l\leq\mathcal{L}}} Ne^{t}_{l,u} - R^{u}\cdot MinN_{u} \leq 0 \qquad u\in\mathcal{R}$$

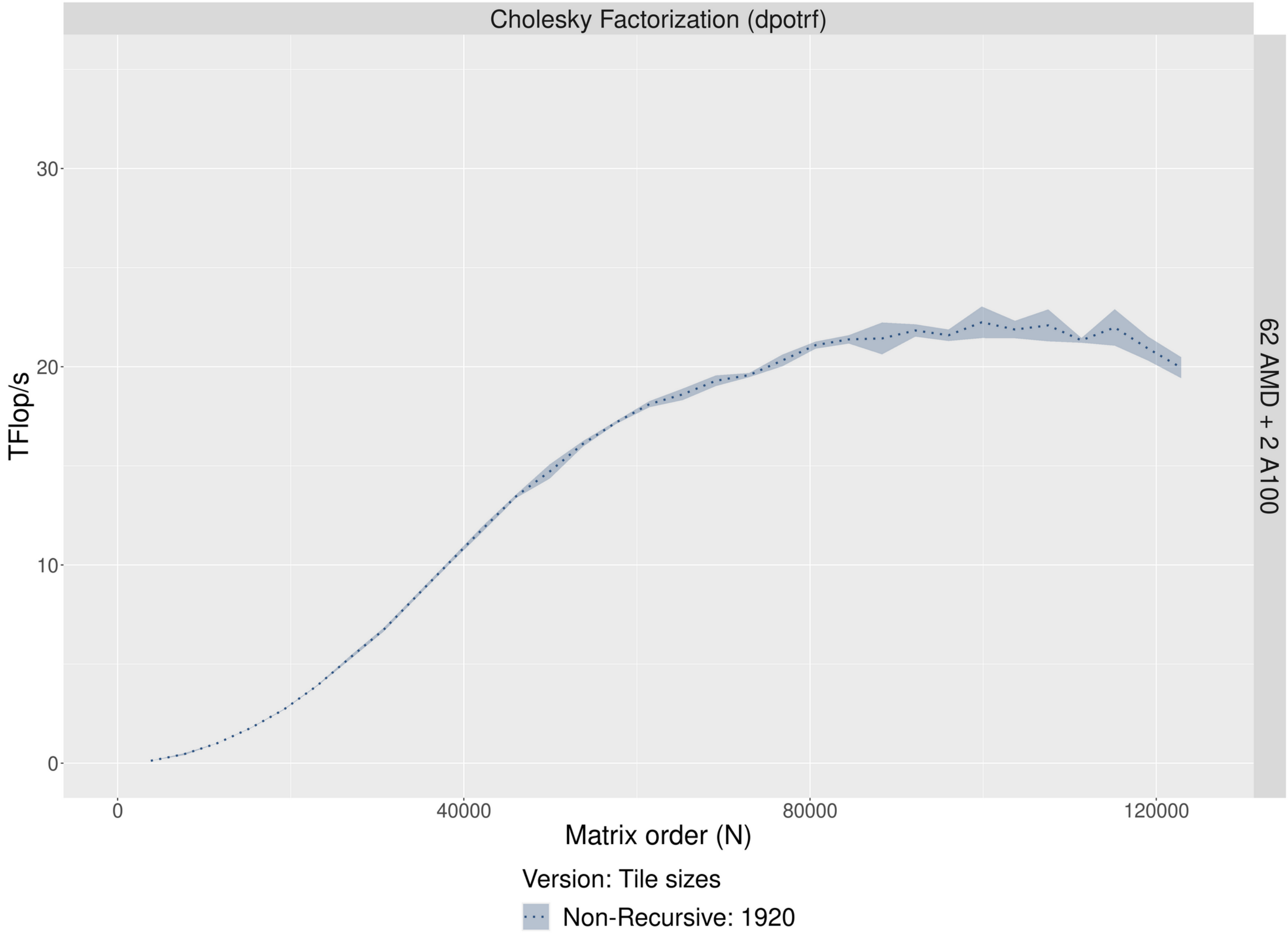
First results : heterogeneous

Cholesky factorization

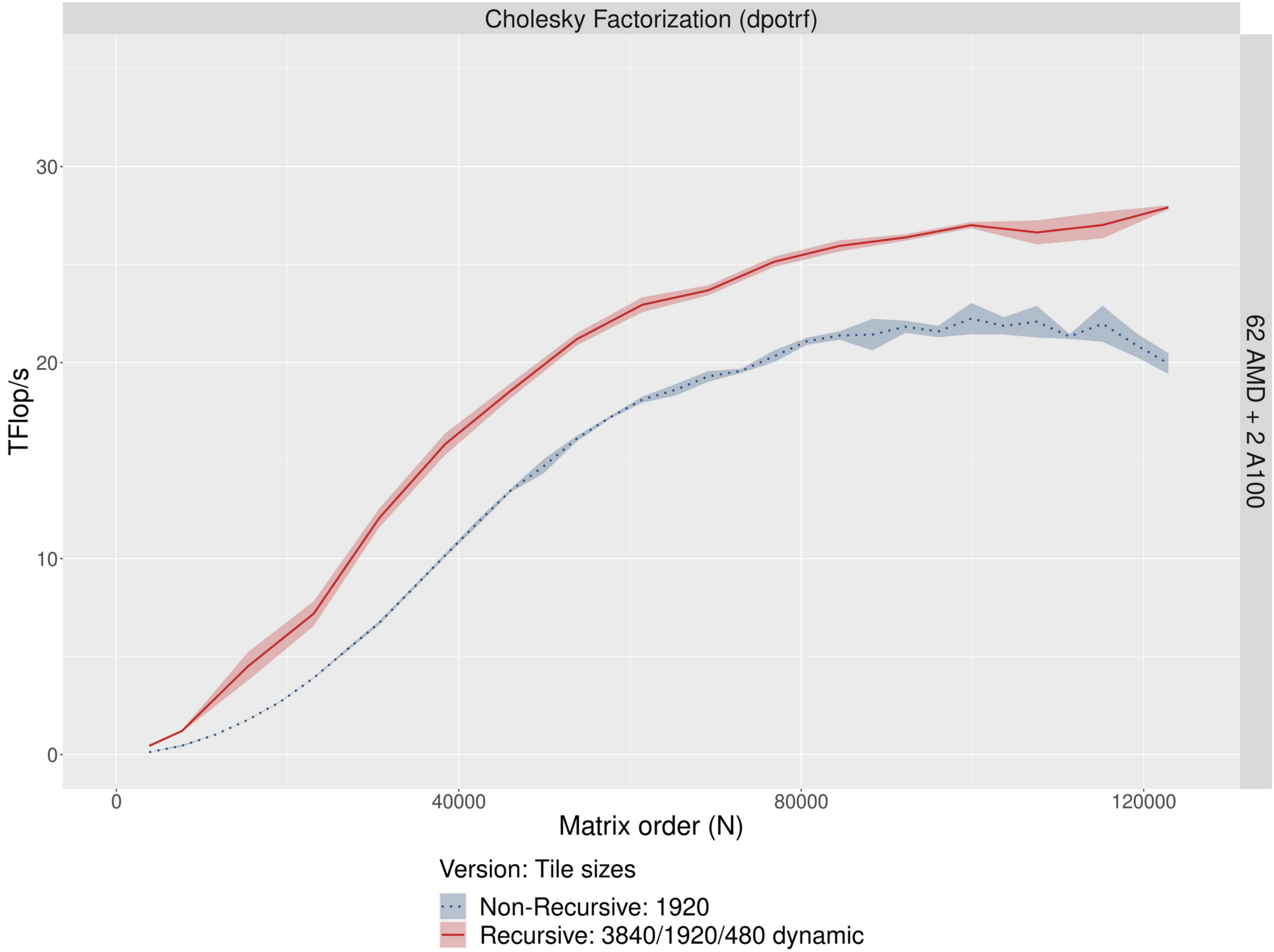
- 2x32 core AMD Zen3 EPYC 7513 @ 2.6GHz
- 2x NVIDIA A100
- Scheduler : Deque Model Data-Aware Ready (DMDAR, ≃HEFT)

- Tile sizes
- « big » : 3840, the most efficient
- « small » : 480, parallelism, for CPU cores
- « medium » : 1920, 2560, trade-off

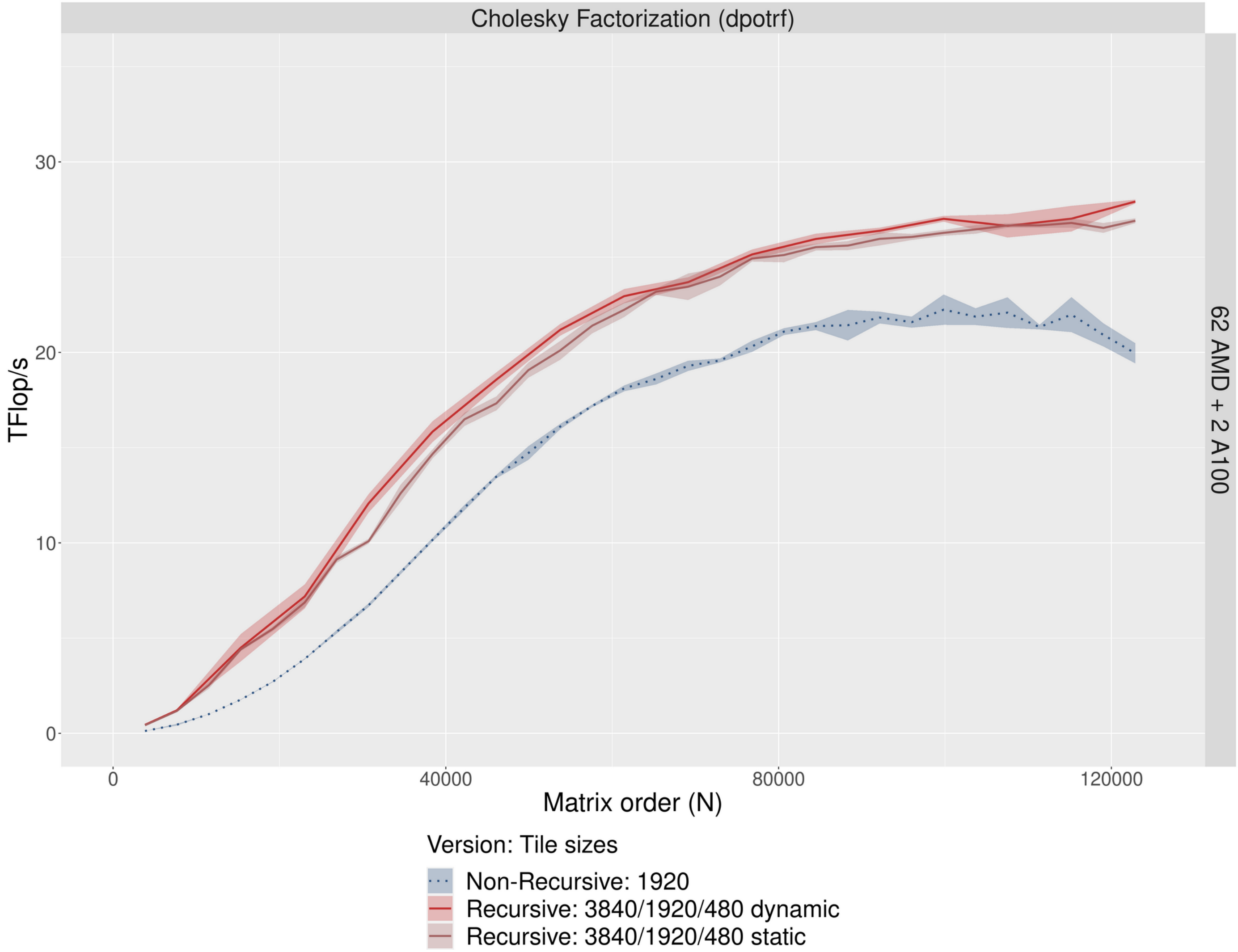
First results : heterogeneous Cholesky



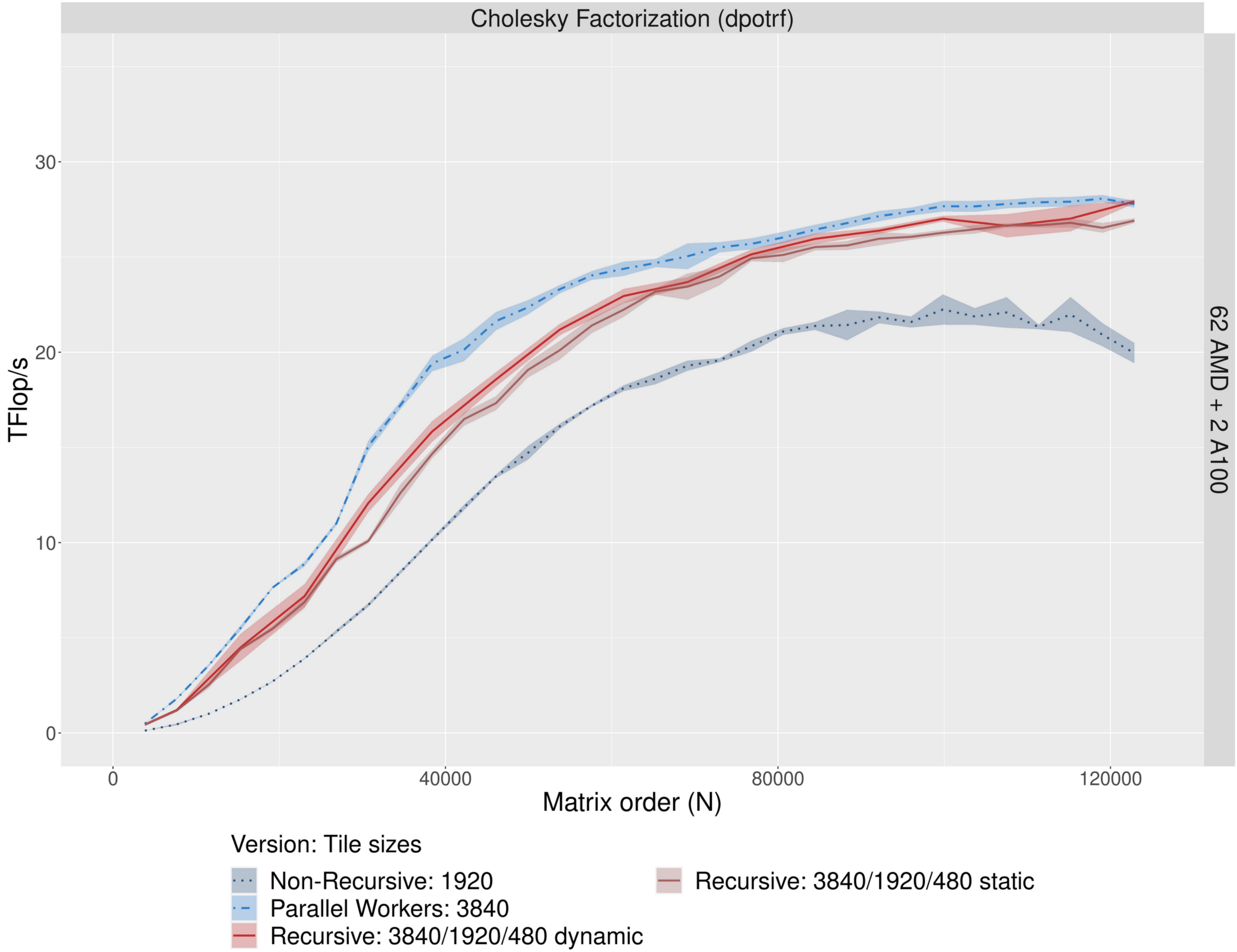
First results : heterogeneous Cholesky



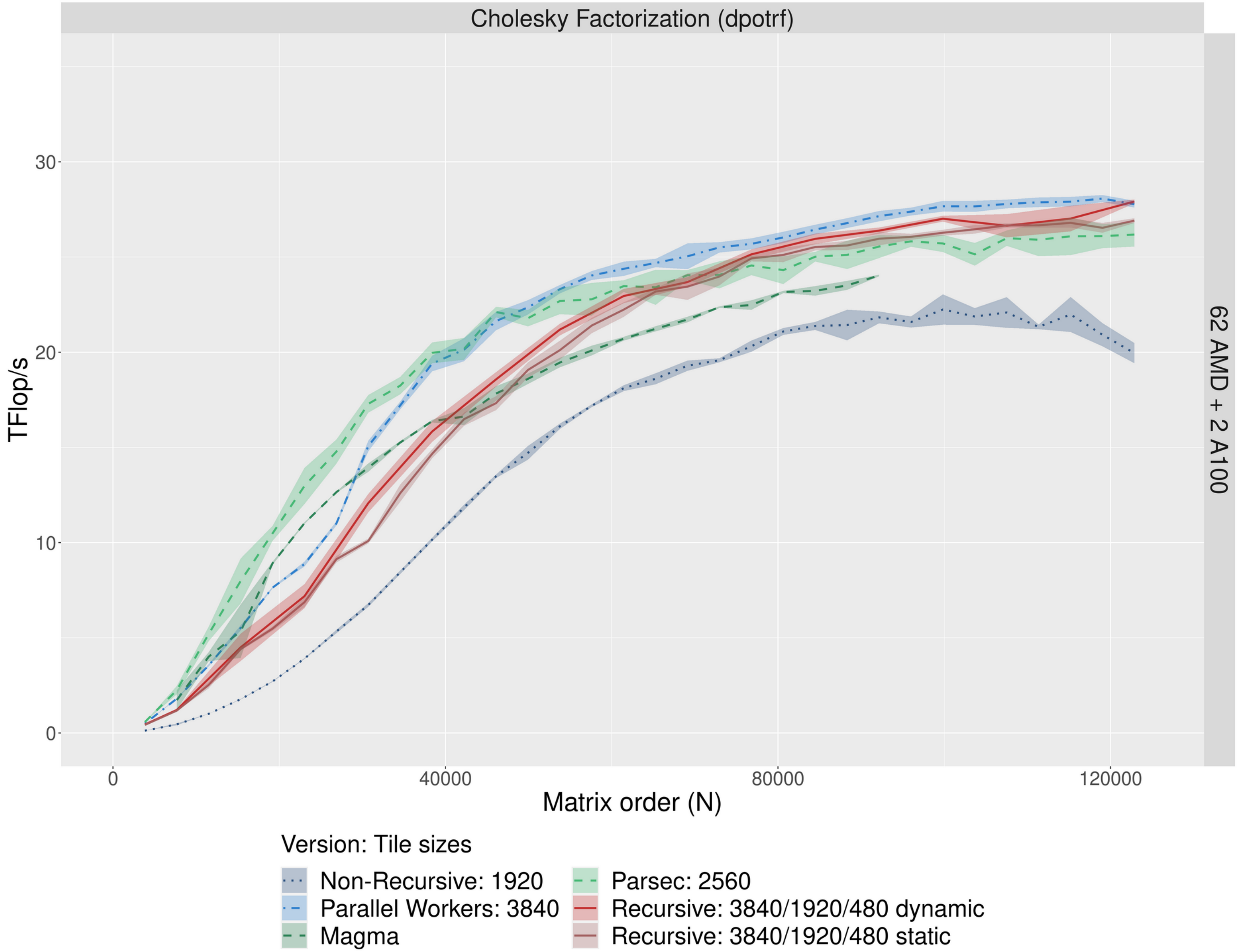
First results : heterogeneous Cholesky



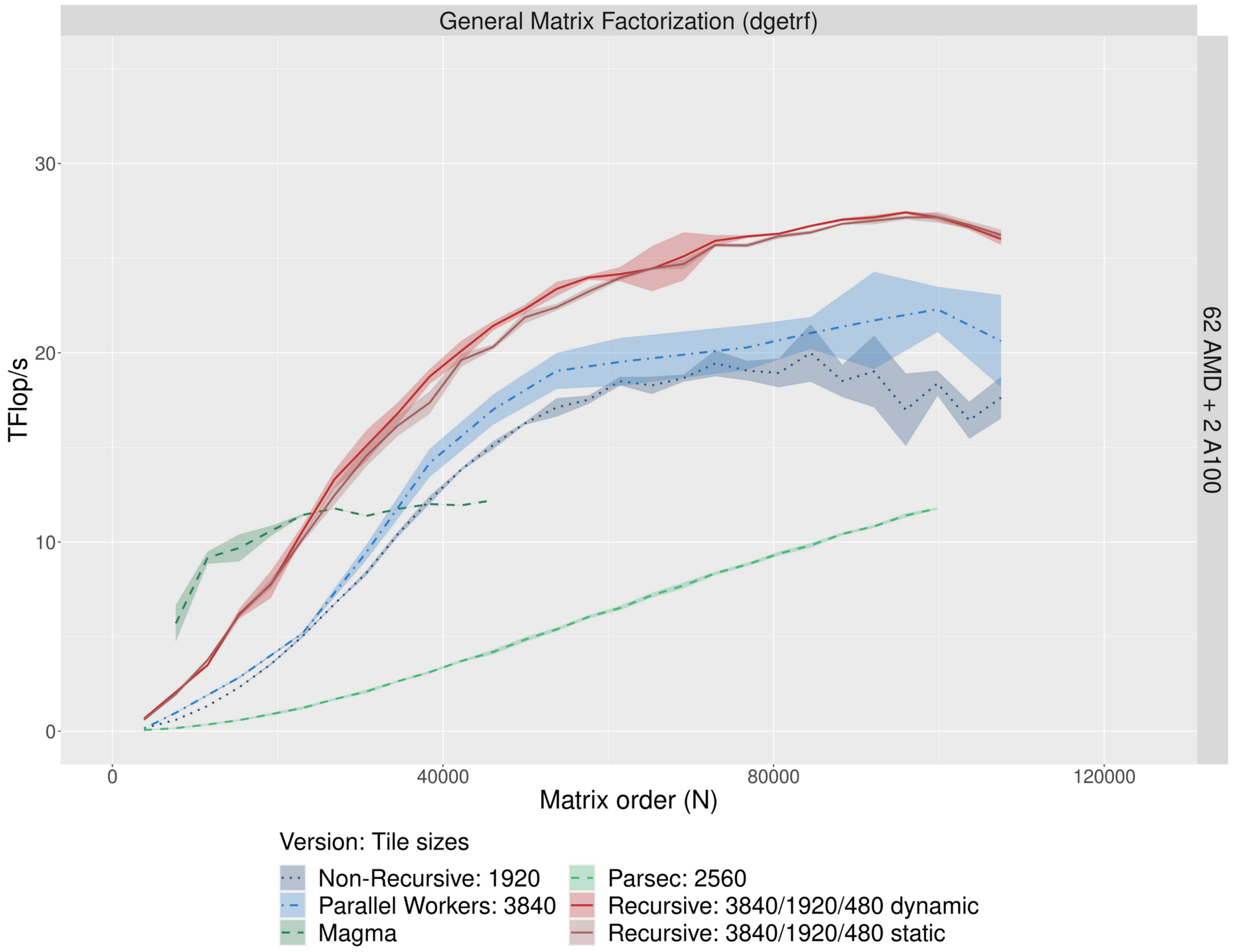
First results : heterogeneous Cholesky



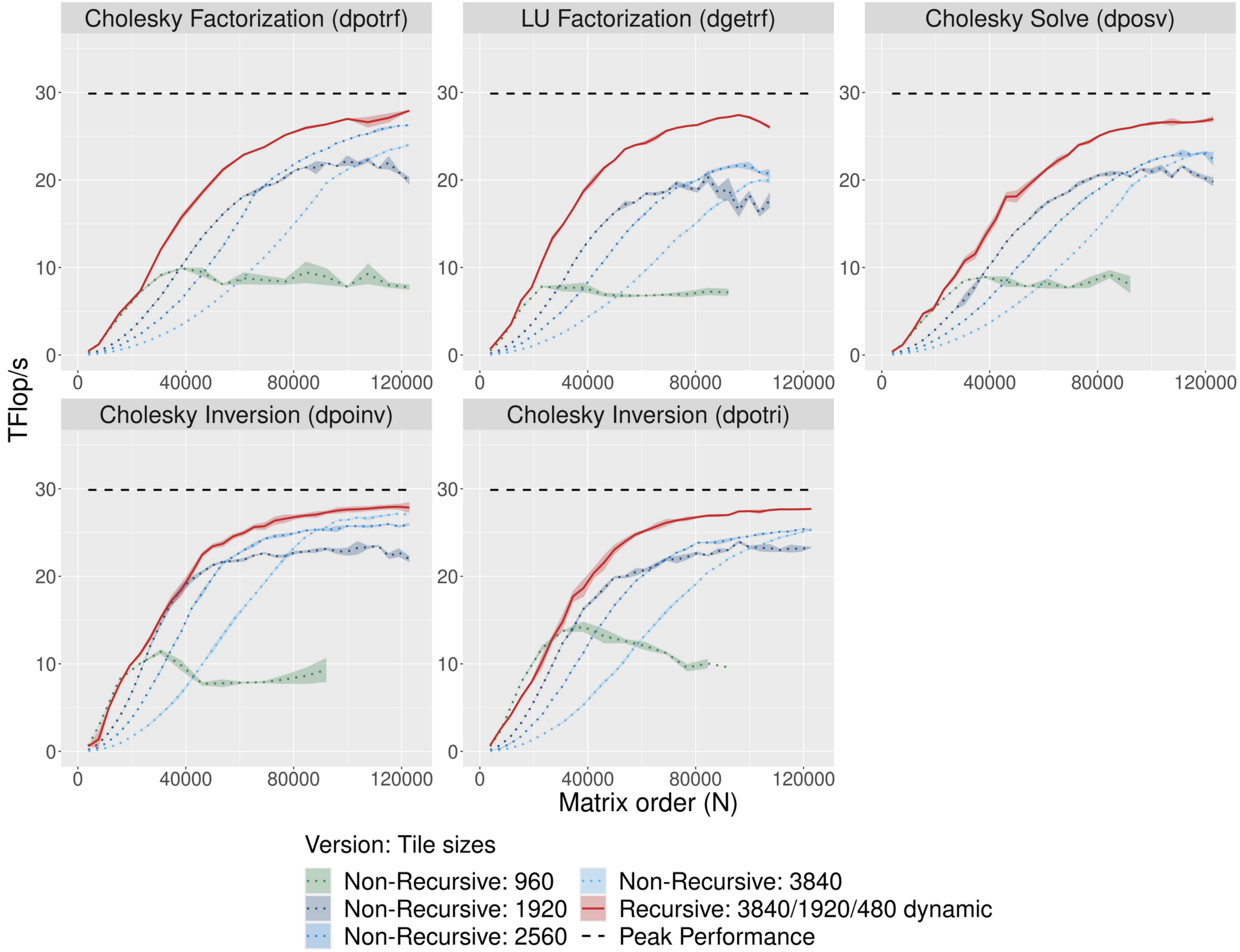
First results : heterogeneous Cholesky



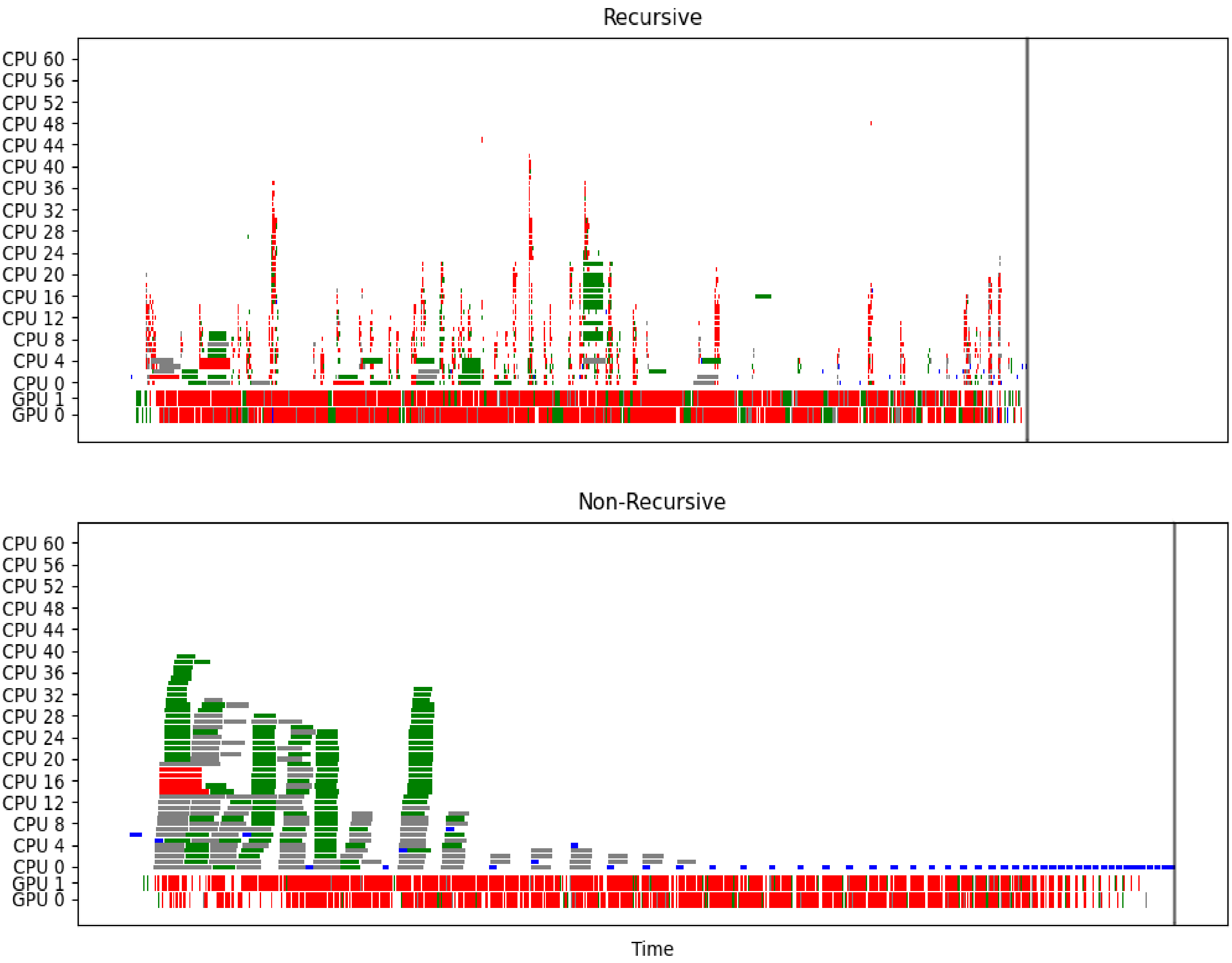
First results : heterogeneous LU



First results : heterogeneous



First results : heterogeneous



First results : heterogeneous

Linear Program

- Generic, automatic
- Requires performance models

but

- Not very reactive to new situation
- Doesn't take data transfers into account
- Global view, hard to turn into local action
  - Splitting ratios
  - Sometimes uses too much CPU time
- Not really intuitive behavior

Mid results : more greedy approach

[Morin’26]

Why splitting?

- Decrease GPU use for non-really-GPU-efficient tasks
- Occupy CPU cores

Mid results : more greedy approach

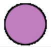
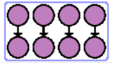
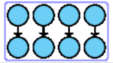
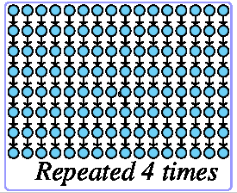
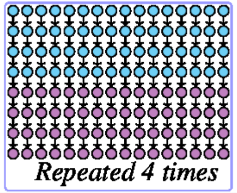
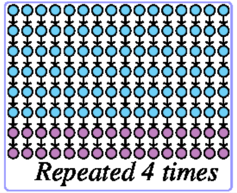
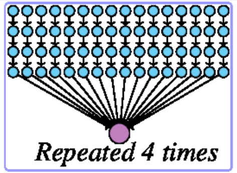
Recursive performance model : for each possible subDAG, record

- Global finish time
- Sequential GPU time
- Average # of CPU cores used

and prune symmetries

→ Splitting Profiles

Example on GEMM with 2 GPUs:

	Sub-DAG-placement (CPU / GPU)	Makespan	GPU time	Mean CPUs used (% total CPUs)
1		7 ms	7 ms	0 core (0%)
2		6 ms	12 ms	0 core (0%)
3		1088 ms	0 ms	4 cores (6%)
4		74.4 ms	0 ms	64 cores (100%)
5		43.6 ms	12.8 ms	54.7 cores (85%)
6		59 ms	6.4 ms	60.6 cores (95%)
7		40.2 ms	6 ms	59.3 cores (93%)

Mid results : more greedy approach

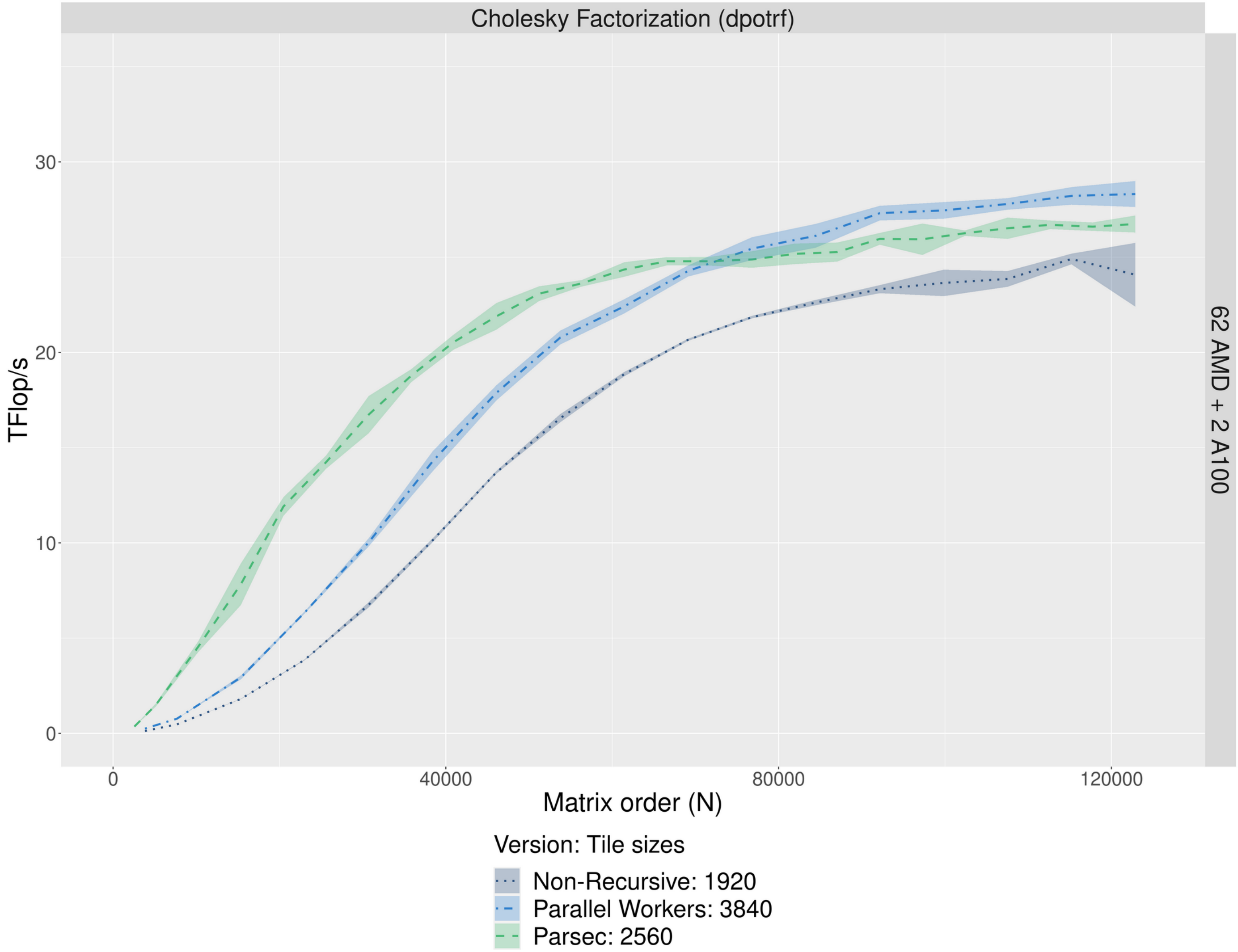
Greedy scheduler : GASPP

- Keeps ready tasks ordered by CPU/GPU acceleration
- Records how many CPUs are idle

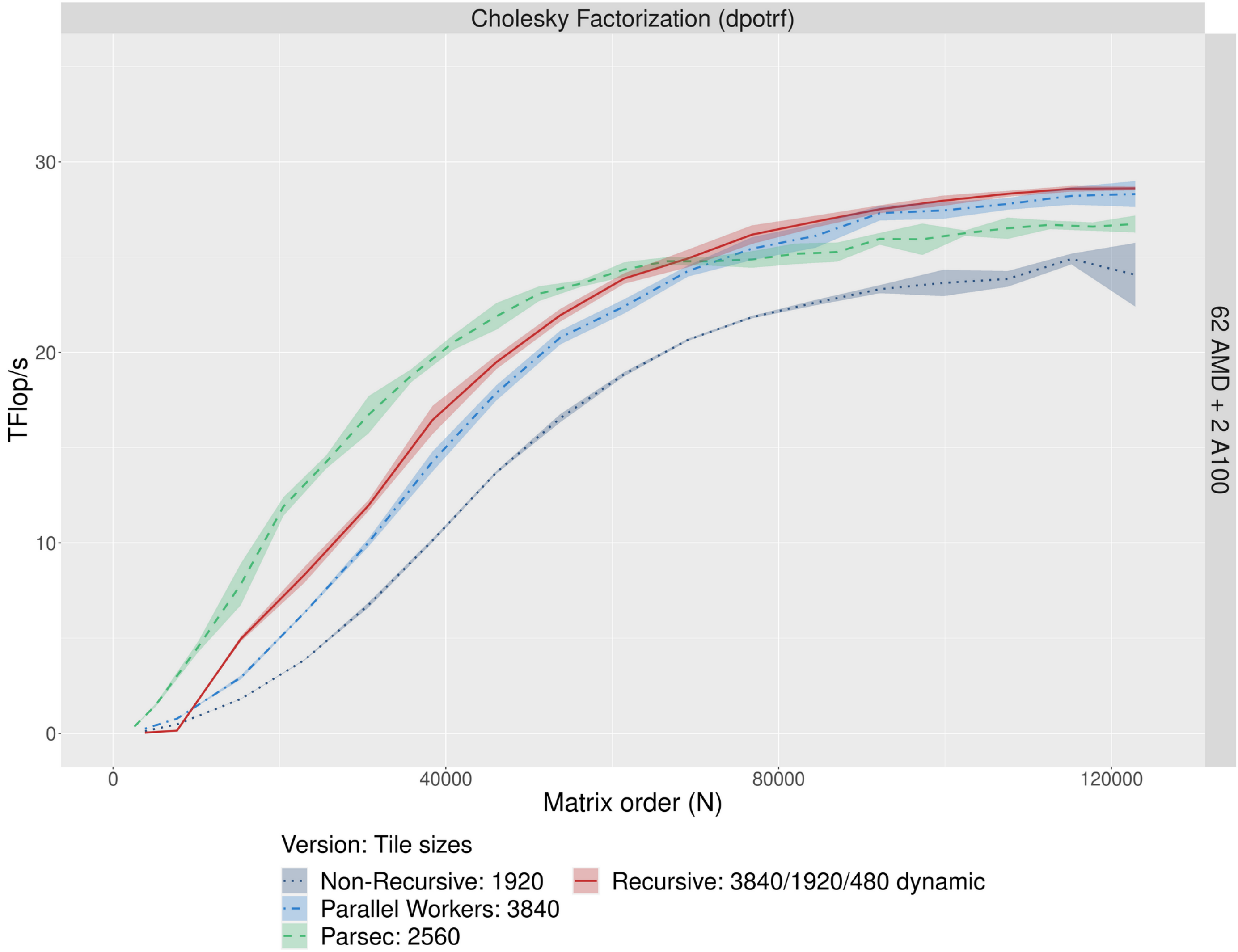
When scheduler asks for a task,

- Pick up the least-accelerated task
- Look for best splitting profile that
  - Can fit the idle CPUs
  - Allows to terminate earlier than on GPU
  - Offloads the most work from GPU
  - I.e. safe offload bet
- If none, pick up instead the most-accelerated task
  - Will go on GPU

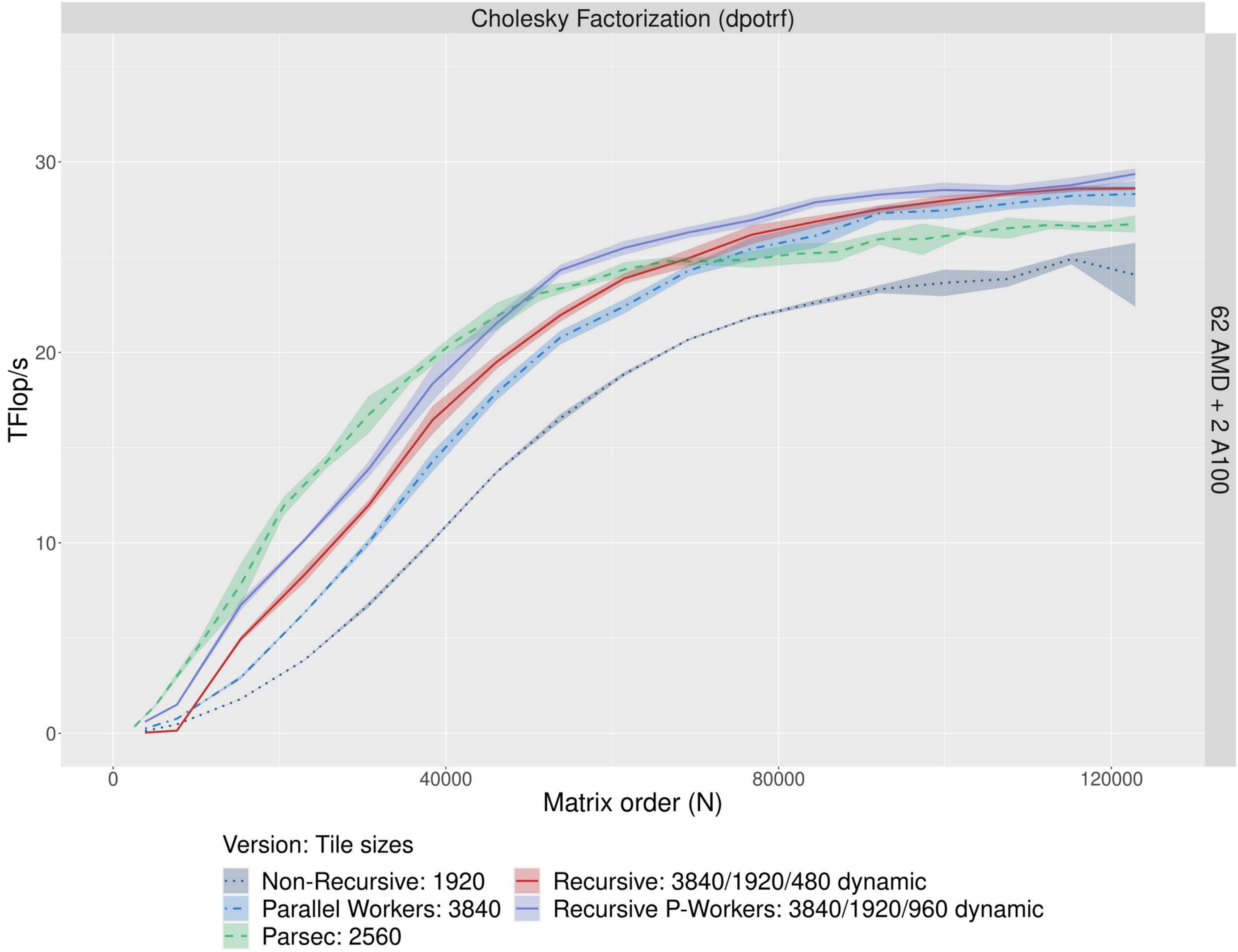
Mid results : heterogeneous Cholesky



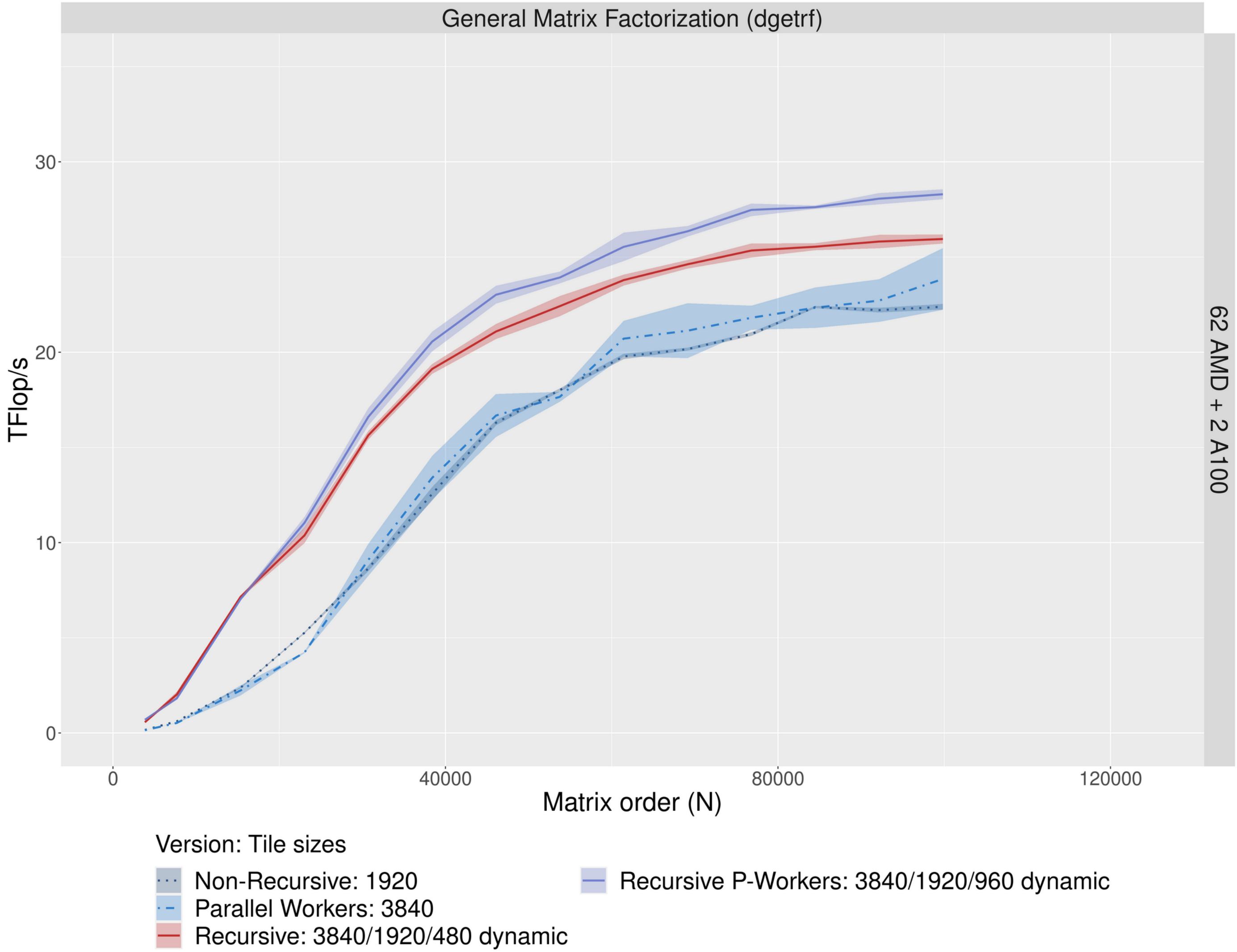
Mid results : heterogeneous Cholesky



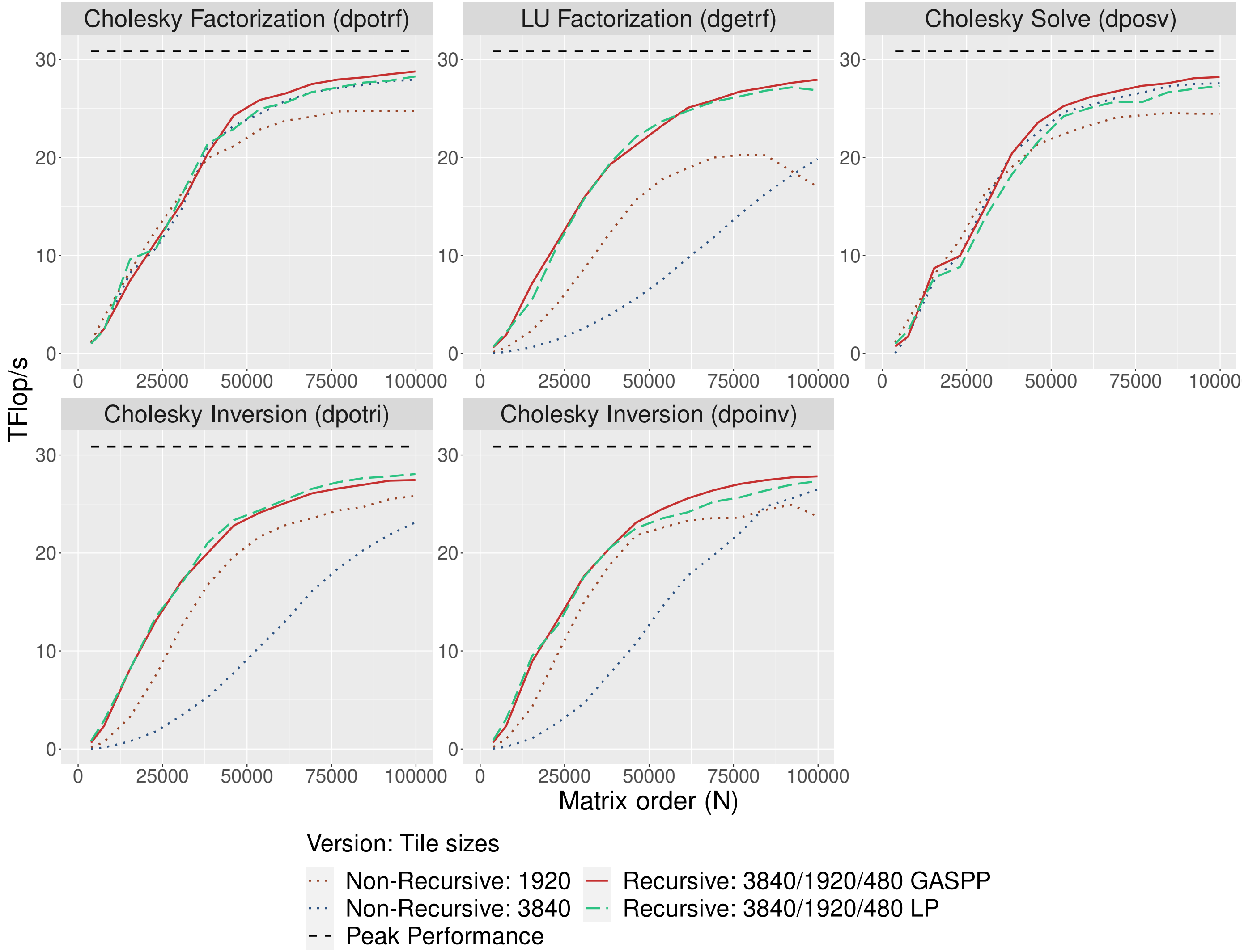
Mid results : heterogeneous Cholesky



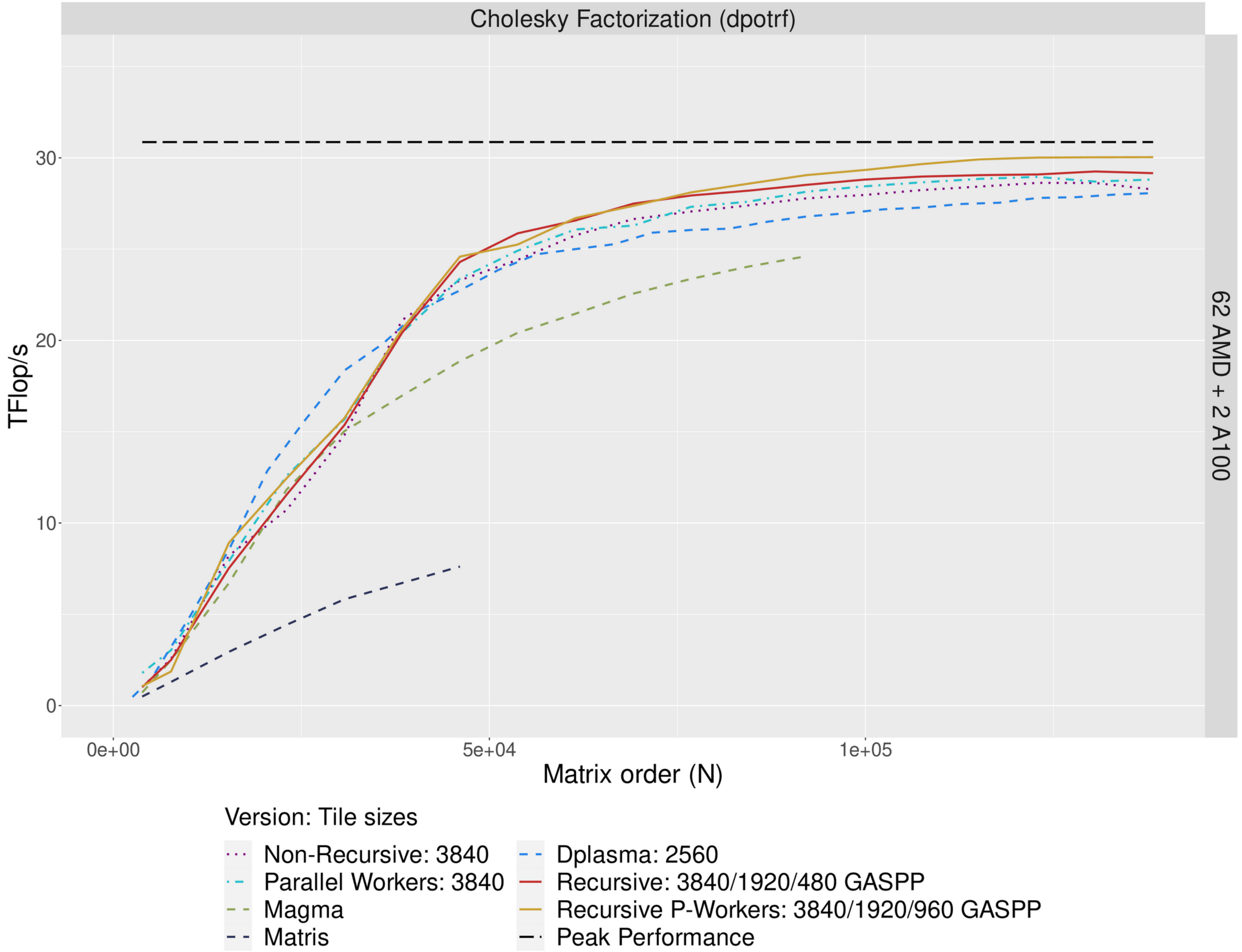
Mid results : heterogeneous LU



Mid results : heterogeneous



Mid results : heterogeneous



Current results : local/global view?

Good results asymptotically

Could be better for smaller matrices

- Not enough parallelism to occupy GPUs
- Should split a bit for GPUs too!
  - When starvation comes
  - How to evaluate starvation?

Current results : local/global view?

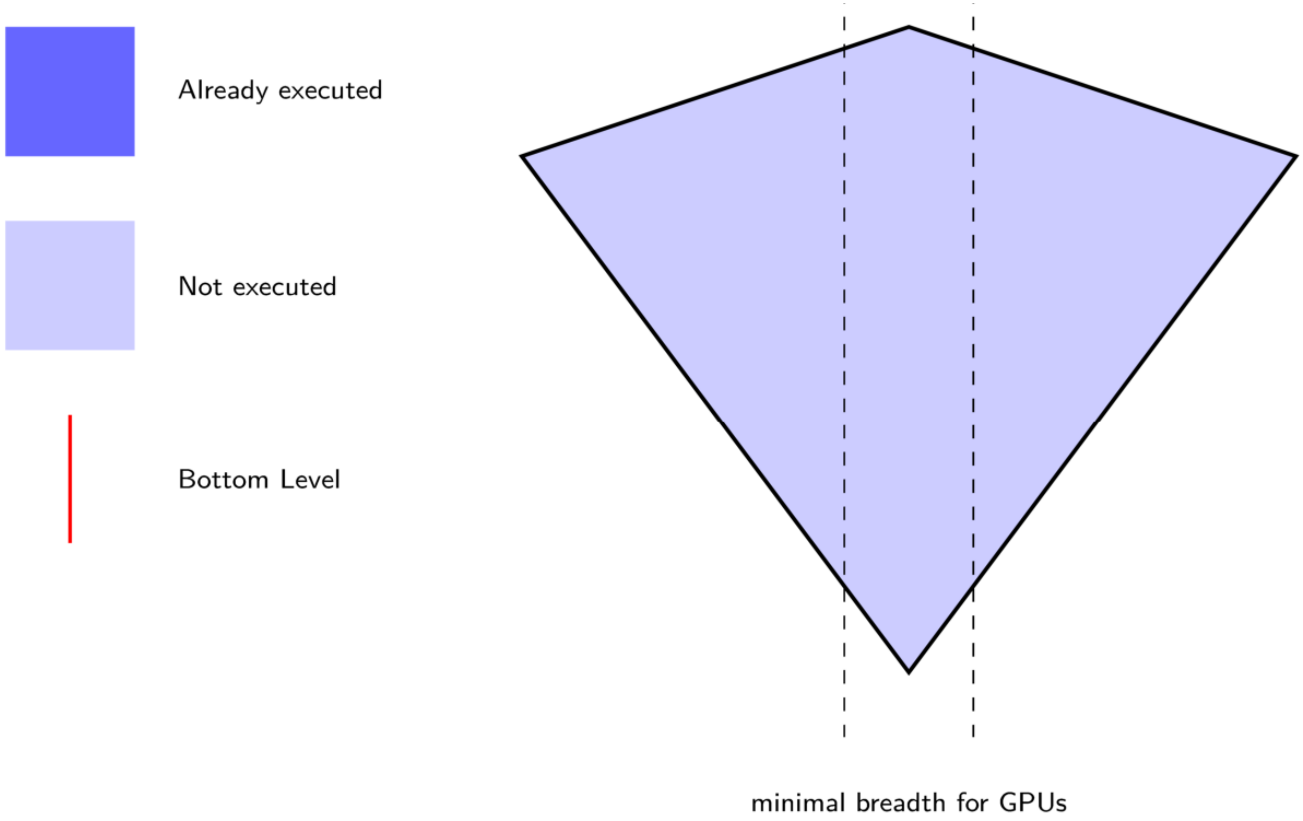
Good results asymptotically

Could be better for smaller matrices

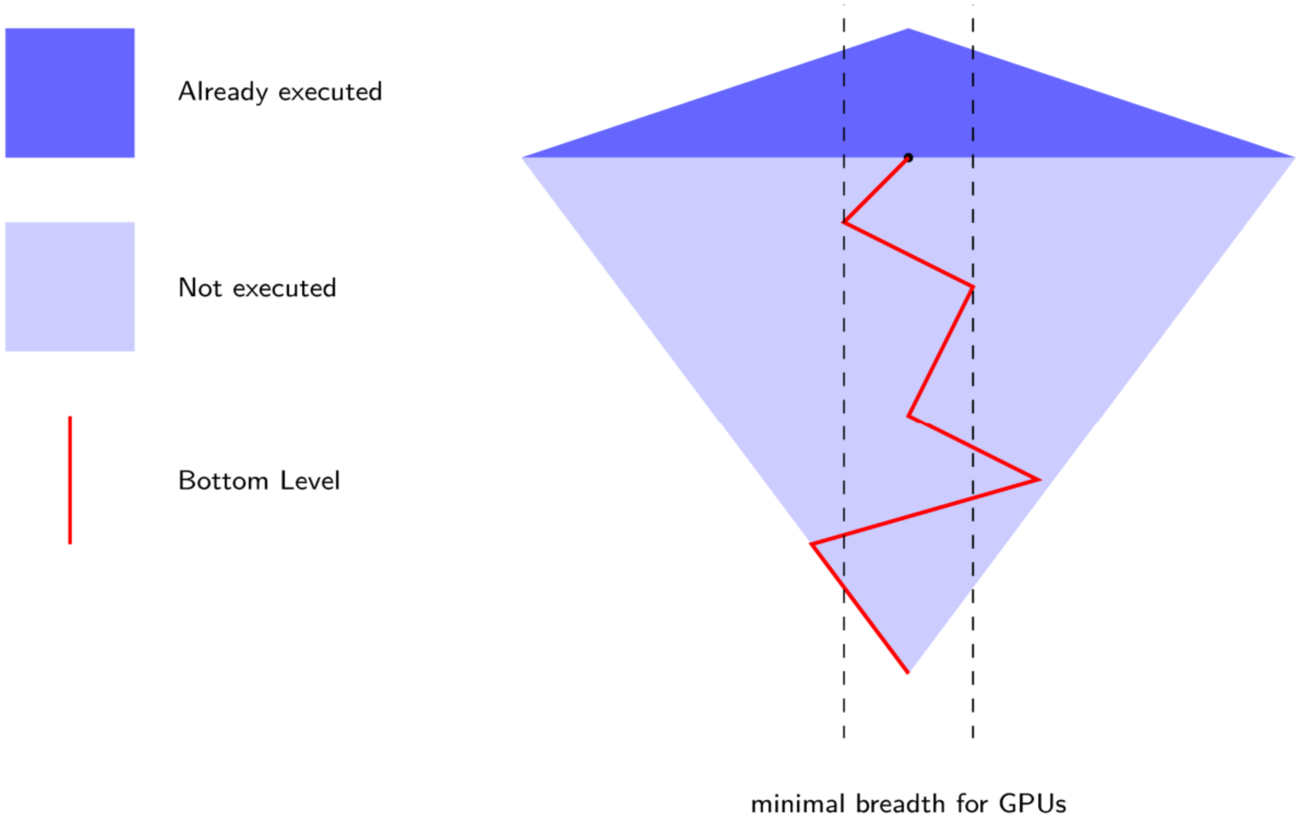
- Not enough parallelism to occupy GPUs
- Should split a bit for GPUs too!
  - When starvation comes
  - How to evaluate starvation?

- Looking at « upward time » / « bottom level »
  - Duration of the critical path from considered task
  - With affine `for` loops etc., seems computable with polyhedral model
  - If higher than the remaining work, starvation is probable

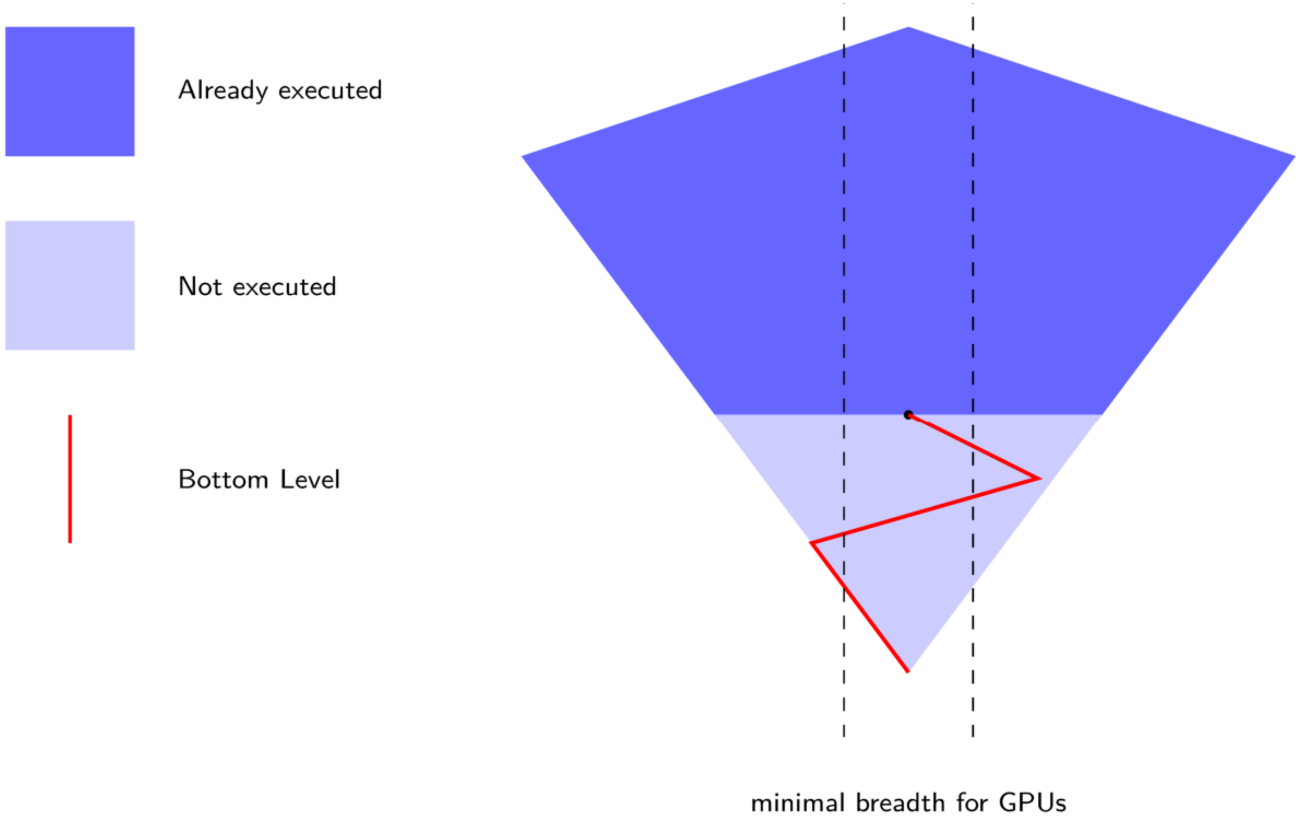
Current results : global view?



Current results : global view?



Current results : global view?



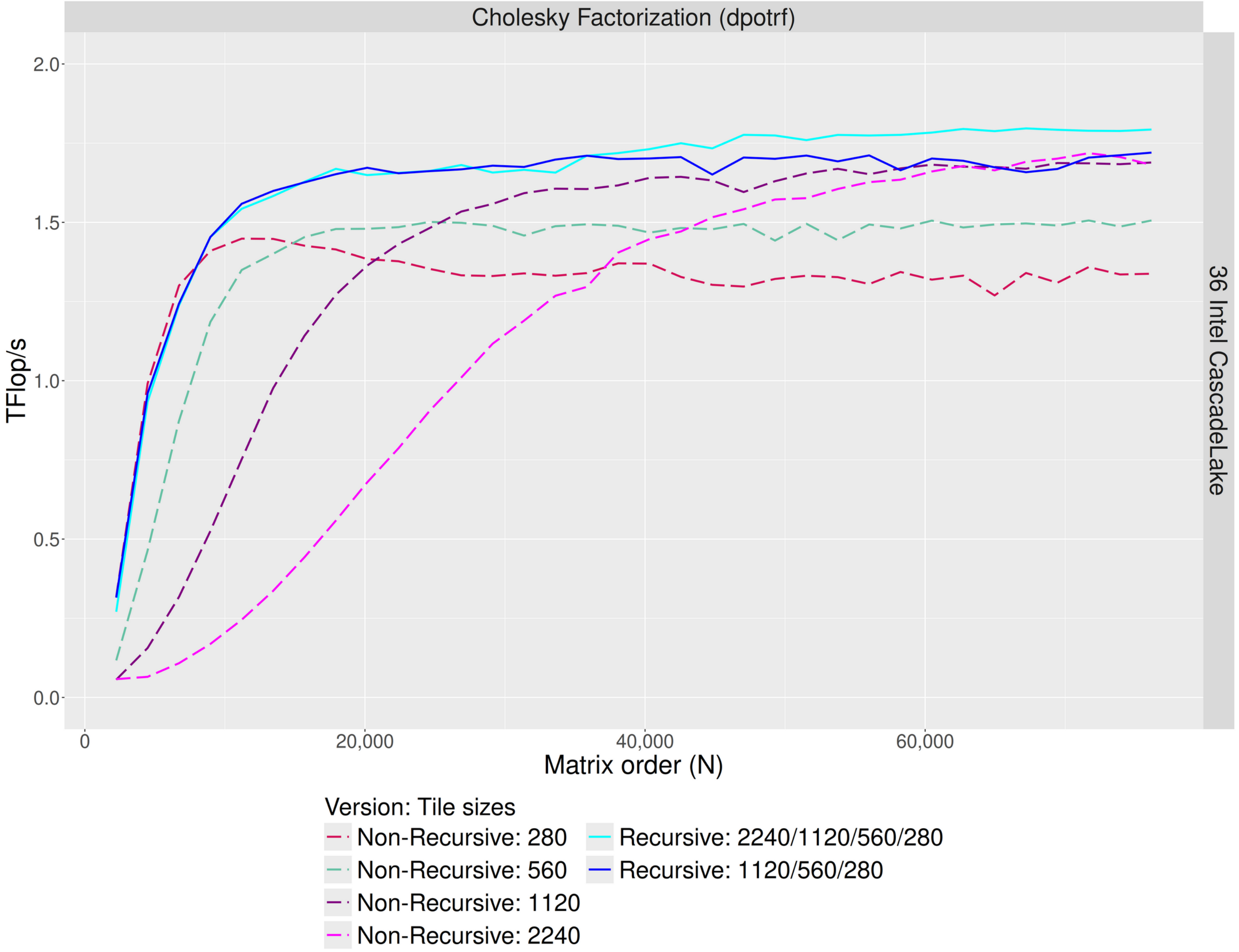
Current results : global view?

Estimating available parallelism :

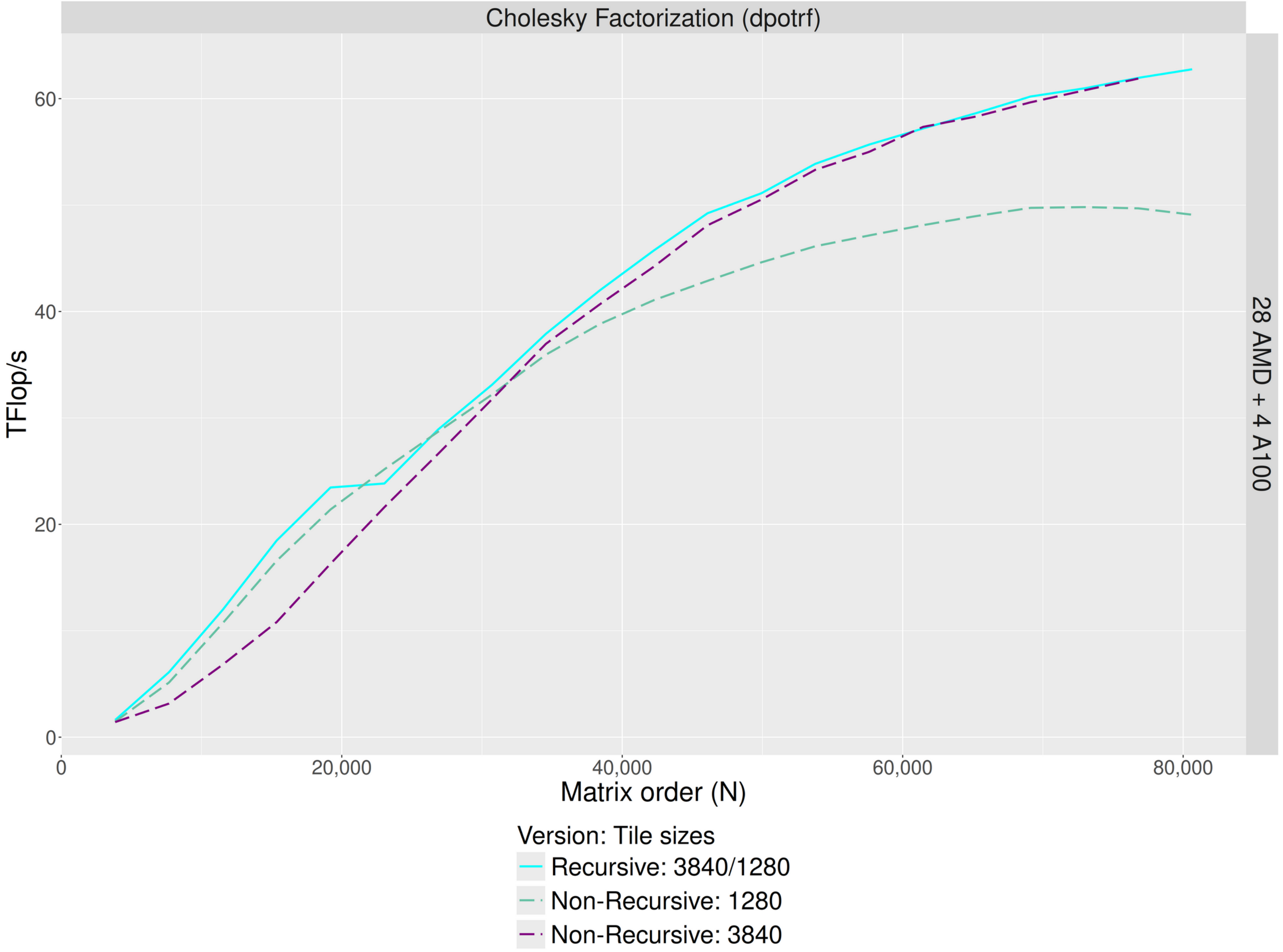
- work_time_left / critical_path_time(T)
- If less than number of GPUs, we should split T
  - To reduce critical path time
  - And create parallelism

For GPUs, can add data transfer time of T in critical path time

Current results : global view? CPUs



Current results : global view? GPUs



Conclusion

Recursive tasks

- A promising tool to steer granularity

Scheduling approaches

- Linear Program to get global view
- Greedy with local view, using Splitting profiles
- Greedy with global view using bottom level

How much global view are we willing to spend time looking at?

Perspectives

- Try with more irregular graphs
- Compute bottom levels with polyhedral model
- Scaling over MPI
  - Leverage recursivity for automatic pruning
  - Promising results with partial-pivoting LU
  - Hierarchical Load-balancing?