

Broadcast Trees for Heterogeneous Platforms

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Multi-port

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Broadcasting data

- ▶ Key collective communication operation
- ▶ Start: one processor has the data
- ▶ End: all processors own a copy
- ▶ Vast literature about broadcast, MPI_Bcast
- ▶ Standard approach: use a spanning tree
- ▶ Finding the best spanning tree: NP-Complete problem (even in the telephone model)

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Different broadcast problems

Broadcast large messages \Rightarrow **pipelining** strategies

- ▶ split the messages into slices (application level)
- ▶ route them concurrently, possibly using different spanning trees
- ▶ throughput optimization (relaxation of makespan minimization)

STA Single Tree, Atomic message
heuristics to minimize makespan: FNF...

STP Single Tree, Pipelined series of messages

MTP Multiple Tree, Pipelined series of messages

- ▶ polynomial algorithm to find optimal solution (LP formulation)
- ▶ hard to implement \Rightarrow concentrate on STP

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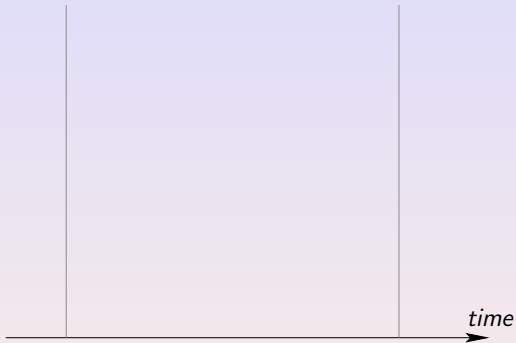
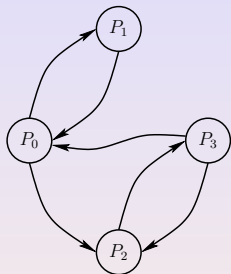
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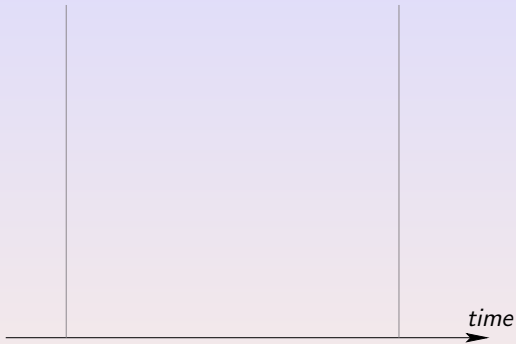
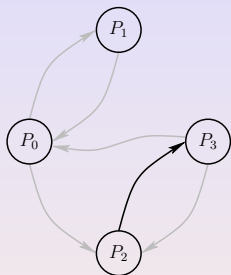
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- ▶ General case: affine model (includes latencies)
- ▶ Common variant: sending and receiving processors busy during whole transfer

Models

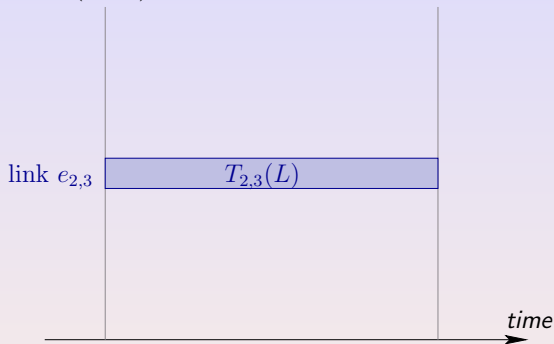
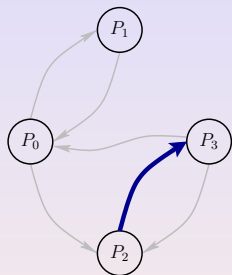
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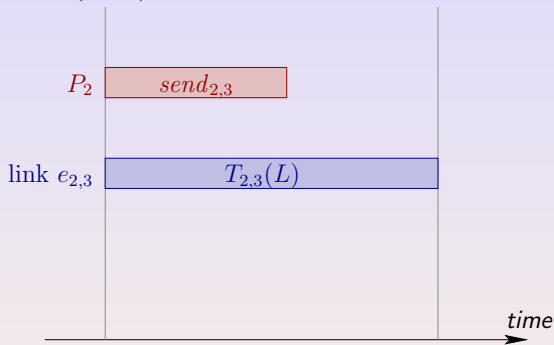
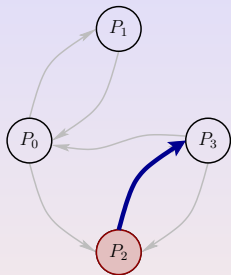
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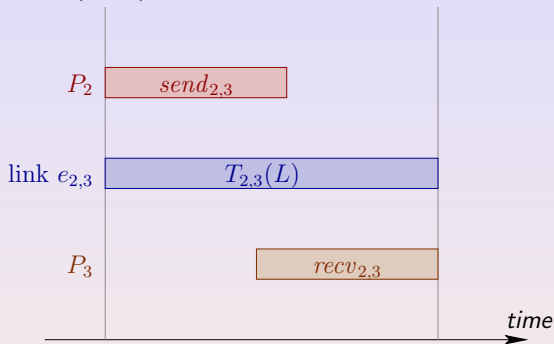
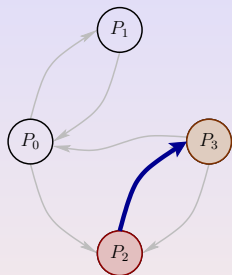
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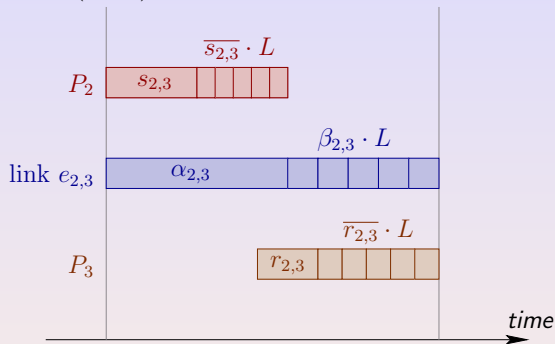
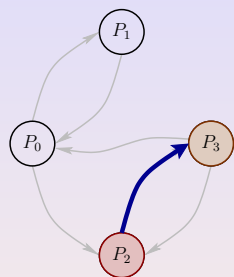
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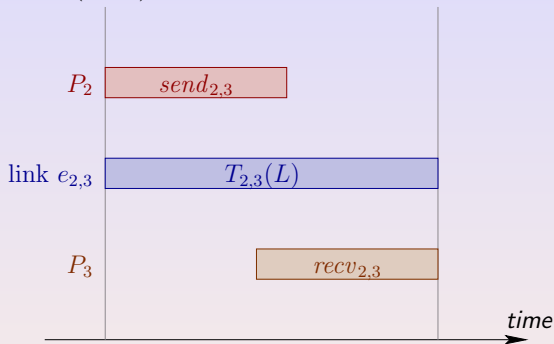
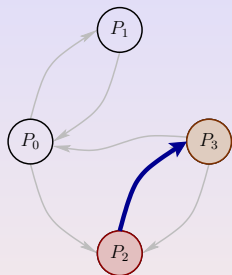
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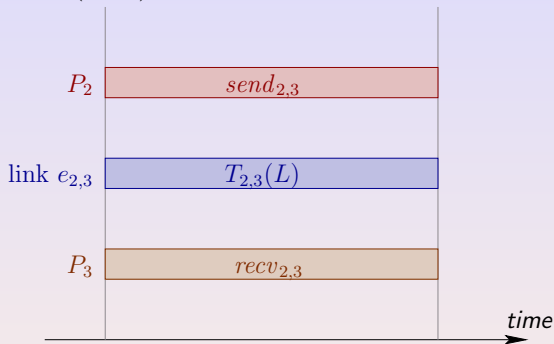
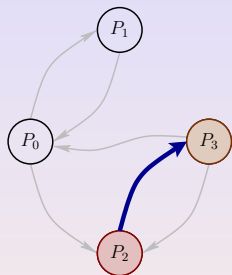
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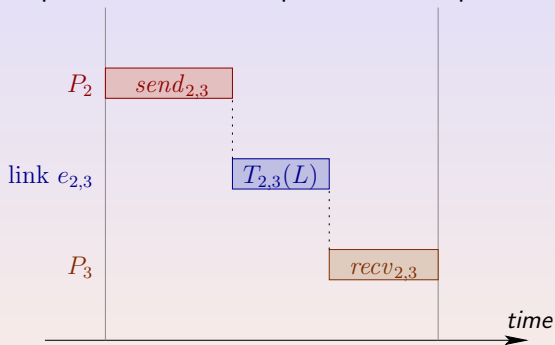
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Multi-port

- ▶ Banikazemi et al.
no overlap between link and processor occupation:

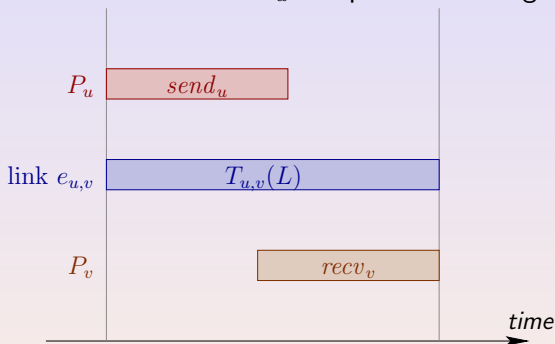


⇒ methodology to instantiate parameters

Multi-port

► Bar-Noy et al.

occupation time of sender P_u independent of target P_v

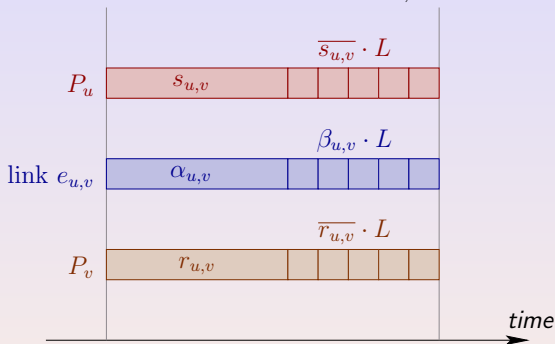


not *fully* multi-port model, but allows for starting a new transfer from P_u without waiting for previous one to finish

One-port

► Bhat et al.

same parameters for sender P_u , link $e_{u,v}$ and receiver P_v



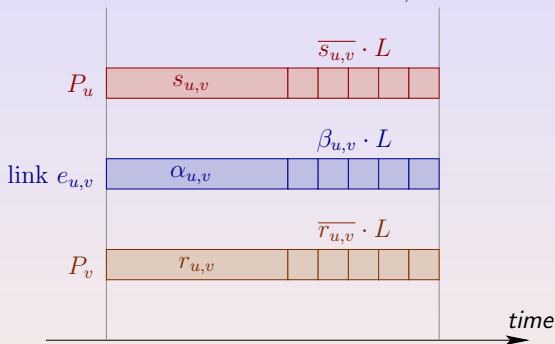
Two flavors:

- bidirectional: simultaneous send *and* receive transfers allowed
- unidirectional: only one send or receive transfer at a given time-step

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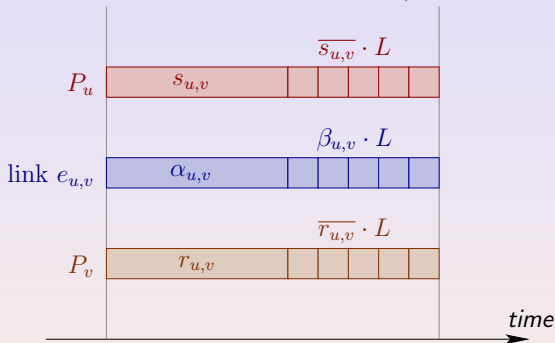
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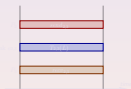
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Framework

- ▶ Platform graph $\mathcal{P} = (V, E)$
- ▶ Source processor P_{source}
- ▶ Goal: broadcast a series of messages to all other nodes
- ▶ Transfers of successive messages are pipelined
- ▶ Send messages along a spanning tree
- ▶ Find a spanning tree with good throughput (neglect initialization and clean-up phases)

- ▶ Bidirectional one-port model:



- ▶ Multi-port model:



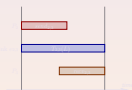
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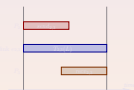
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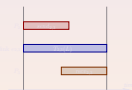
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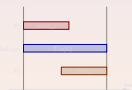
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- ▶ Duration of a transfer = f (link $e_{u,v}$)

$$send_{u,v}(L) = recv_{u,v}(L) = T_{u,v}(L) = T_{u,v}.$$

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Simple Platform Pruning

- ▶ Idea: delete edges of maximum weight, until we have a tree
- ▶ Algorithm:

SIMPLE-PLATFORM-PRUNING(\mathcal{P}, P_{source})

$TreeEdges \leftarrow$ all edges of E

while $|TreeEdges| > n - 1$ **do**

$L \leftarrow$ edges of $TreeEdges$ sorted by non-increasing weight $T_{u,v}$

for each edge $e \in L$ **do**

if the graph $(V, TreeEdges \setminus \{e\})$ is still connected **then**

$TreeEdges \leftarrow TreeEdges \setminus \{e\}$

return $(V, TreeEdges)$

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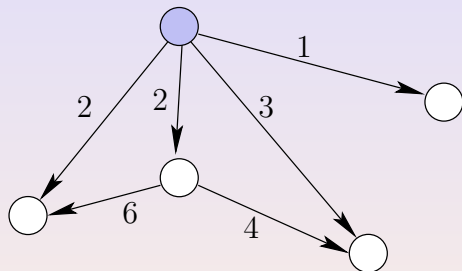
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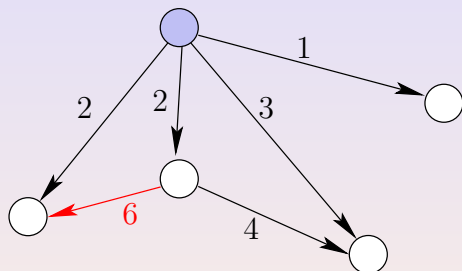
- ▶ Example of simple pruning:



Topology, costs of edges $T_{u,v}$

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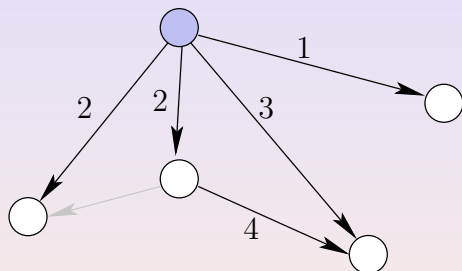
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Choosing and pruning edge of maximum weight

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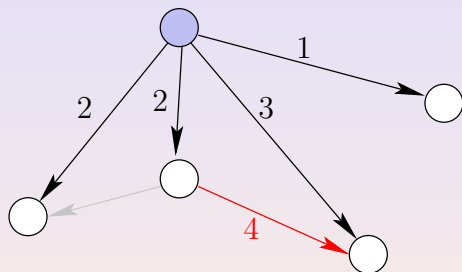
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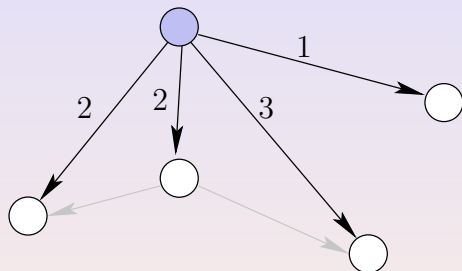
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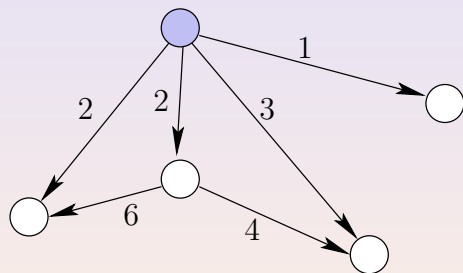
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Achievable throughput: $1/8$

Refined Platform Pruning

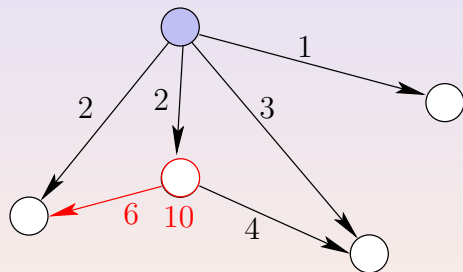
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 - ▶ at each step, compute the out-degree of each node
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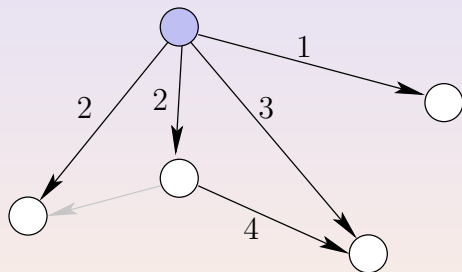
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Choosing maximum out-degree node, then maximum edge

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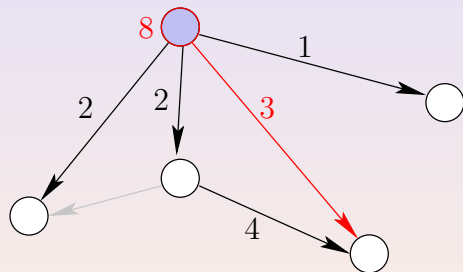
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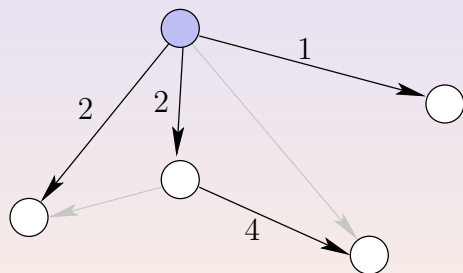
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Achievable throughput: $1/5$

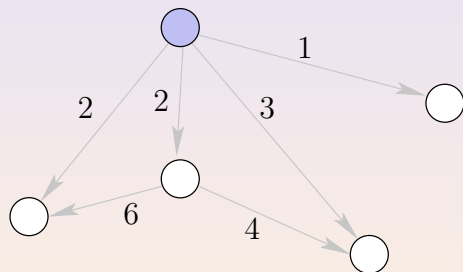
Refined Platform Pruning

REFINED-PLATFORM-PRUNING(\mathcal{P}, P_{source})

- 1: $TreeEdges \leftarrow$ all edges of E
- 2: **for** each $u \in V$ **do**
- 3: $OutDegree(u) \leftarrow \sum_{v, (u,v) \in E} T_{u,v}$
- 4: **while** $|TreeEdges| > n - 1$ **do**
- 5: $SortedNodes \leftarrow$ nodes sorted by non-increasing value of $OutDegree(u)$
- 6: **for** $u \in SortedNodes$ **do**
- 7: $L \leftarrow$ edges sorted by decreasing weight $T_{u,v}$
- 8: **for** each edge $e = (u, v) \in L$ **do**
- 9: **if** the graph $(V, TreeEdges \setminus \{e\})$ is still connected **then**
- 10: $TreeEdges \leftarrow TreeEdges \setminus \{e\}$
- 11: $OutDegree(u) \leftarrow OutDegree(u) - T_{u,v}$
- 12: **goto** 4
- 13: **return** $(V, TreeEdges)$

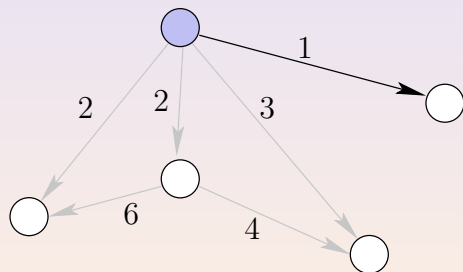
Growing a Minimum Weighted Out-Degree Tree

- ▶ Idea: grow a tree as in Prim's algorithm
- ▶ At each step, choose an edge optimizing metric
- ▶ Our metric:
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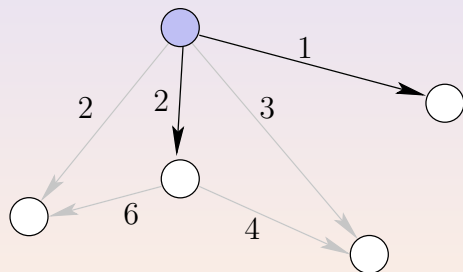
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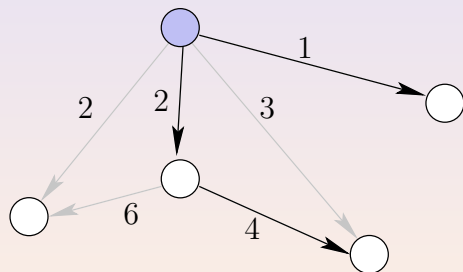
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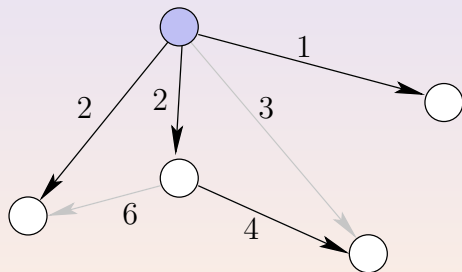
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Achievable throughput: $1/5$

Growing a Minimum Weighted Out-Degree Tree

```
GROWING-MINIMUM-WEIGHTED-OUT-DEGREE-TREE( $\mathcal{P}, P_{source}$ )
   $TreeEdges \leftarrow \emptyset$ 
   $TreeVertices \leftarrow \{P_{source}\}$ 
  for each edge  $e = (u, v)$  do
     $cost(u, v) \leftarrow T_{u,v}$ 
  while  $TreeVertices \neq V$  do
    choose the link  $(u, v)$  such that  $u \in TreeVertices$ ,
     $v \notin TreeVertices$  and  $(u, v)$  has minimum value  $cost(u, v)$ 
     $TreeVertices \leftarrow TreeVertices \cup \{v\}$ 
     $TreeEdges \leftarrow TreeEdges \cup \{(u, v)\}$ 
    for each edge  $(u, w) \notin TreeEdges$  do
       $cost(u, w) \leftarrow cost(u, w) + cost(u, v)$ 
  return ( $TreeVertices, TreeEdges$ )
```


Binomial tree heuristic

- ▶ For sake of comparison
- ▶ Close to MPI_Bcast
- ▶ Construct a binomial tree (without topological information)

Binomial tree heuristic

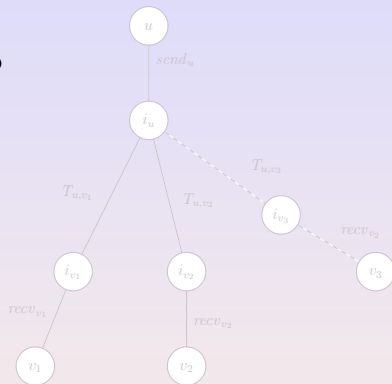
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Multi-port

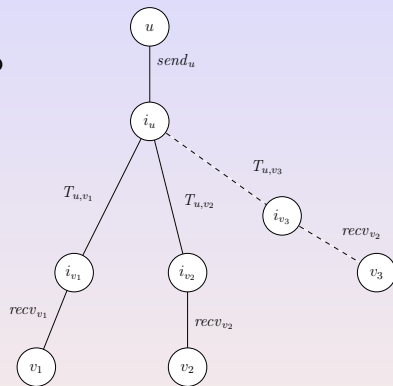
- ▶ Adapt the growing-tree heuristic to multi-port model
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 - ▶ the number of send operations from P_u ,
 - ▶ the length of a transfer $P_u \rightarrow P_v$
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$$T_{period} = \max \left(\delta_{out}(P_u) \times send_u, \max_i (T_{u,v_i}) \right)$$

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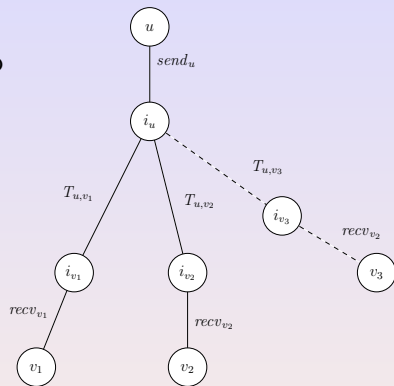
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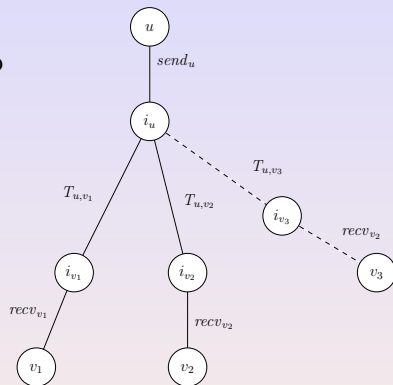
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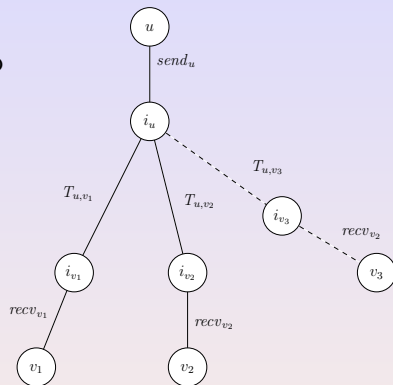
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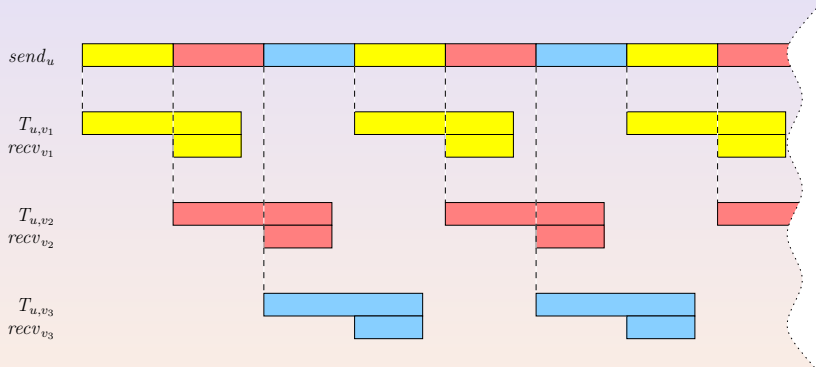
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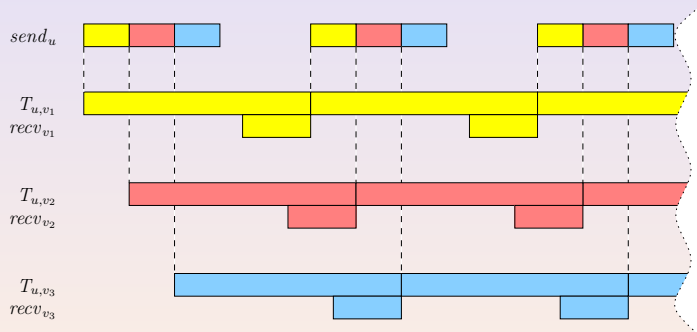
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- ▶ Case where throughput is bounded by the serialized $send_u$:
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- ▶ Solving the MTP problem with LP formulation:
 - ▶ variables: average number of messages going through each link
 - ▶ constraints: one-port model constraints, link occupation
- ▶ Solution of LP \Rightarrow network utilization to reach best throughput
- ▶ Complicated algorithm to reconstruct optimal set of trees for MTP, not needed here
- ▶ Use results output by LP, optimal solution \mathcal{S}_{opt} :
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Simulations using both the one-port and multi-port models

1. random generation of platforms, with parameters:

number of nodes : 10, 20, ..., 50

density : 0.04, 0.08, ..., 0.20

$T_{u,v}$: Gaussian distribution

: (mean=100MB/s, deviation=20MB/s)

$send_{u,v}$: $0.80 \cdot \min_{w, (u,w) \in E} \{T_{u,w}\}$

(for each set of parameters, 10 different configurations generated)

2. realistic platforms generated by Tiers:

- ▶ 100 platforms with 30 nodes
- ▶ 100 platforms with 65 nodes
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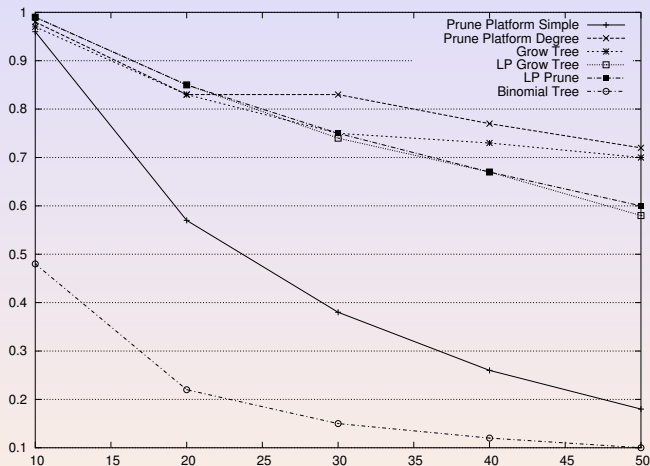
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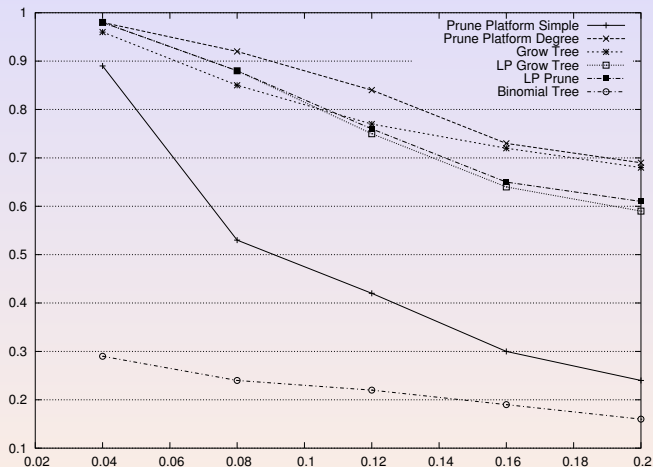
► Performance versus number of nodes



Y axis: relative average performance compared to the optimal solution for MTP

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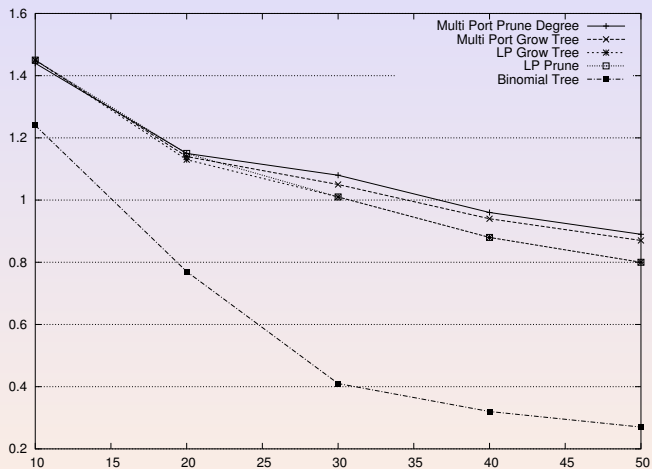
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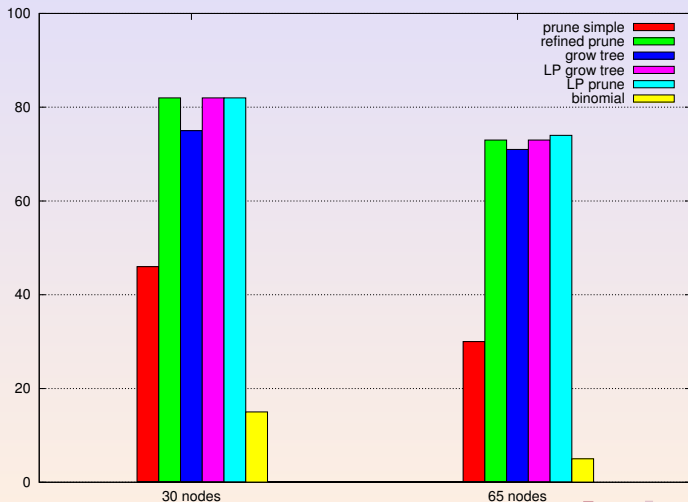
Results, multi-port, random platforms

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Results, one-port, realistic platforms

- ▶ Performance of the one-port heuristics on two types of platforms generated by TIERS



Analysis

- ▶ For the one-port model:
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 - ▶ large platforms: “advanced” heuristics within 60% of the optimal
 - ▶ simple pruning heuristic: not scalable
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- ▶ Designing efficient algorithms to broadcast data
- ▶ Use pipelining techniques, focus on steady-state
- ▶ Using multiple trees (MTP): polynomial algorithm, but difficult to enforce in practice
- ▶ Using a single tree (STP): NP-Complete
- ▶ Design heuristics for STP, possibly using MTP linear program
- ▶ Avoid binomial approach (MPI)